

Fisher distributionfisvec( $\theta, \phi, \text{isced}, \text{kappa}$ )

$$\frac{1}{2\pi} \frac{k}{2\sinh k} \exp(k \cos \theta) \sin \theta \, d\theta \, d\phi$$

$$\int_0^\pi \exp(k \cos \theta) \sin \theta \, d\theta = \int_{-1}^1 \exp(ky) \, dy = \frac{1}{k} \exp(ky) \Big|_{-1}^1 = \frac{2}{k} \sinh k$$

$$P(x) dx = P(x(\theta)) \frac{dy}{d\theta} d\theta = P(\theta) d\theta$$

P(x) white on [0, 1]

$$x = \int_0^\pi \frac{k}{2\sinh k} \exp(k \cos \theta) \sin \theta \, d\theta$$

$$y = \cos \theta \quad dy = -\sin \theta \, d\theta$$

$$\theta = 0 \quad y = 1$$

$$\theta = \pi \quad y = -1$$

$$\theta \quad y = \cos \theta$$

$$x = \frac{k}{2\sinh k} \int_{\cos \theta}^1 \exp(ky) \, dy = \frac{k}{2\sinh k} \left[ \frac{1}{k} \exp(ky) \right]_{\cos \theta}^1$$

$$= \frac{1}{2\sinh k} \left[ \exp(k) - \exp(k \cos \theta) \right]$$

$$\theta = \arccos \left[ \frac{1}{k} \log \left[ \exp(k) - 2x \sinh k \right] \right]$$

$$x = 0 \quad \theta = \arccos(k^{-1} k) = \arccos(1) = 0$$

$$\begin{aligned} x = 1 \quad \theta &= \arccos(k^{-1} \log(\exp(k) - \exp(k) + \exp(-k))) \\ &= \arccos(k^{-1} \log(\exp(-k))) = \arccos((k^{-1})(-k)) \\ &= \arccos(-1) = \pi \end{aligned}$$

kappa.5.0.01

small kappa

$$\theta = \arccos \left[ \frac{1}{\kappa} \log \left\{ \exp(\kappa) - x [\exp(\kappa) - \exp(-\kappa)] \right\} \right]$$

$$\approx \arccos \left[ \frac{1}{\kappa} \log \left\{ 1 + (\kappa - 2x\kappa) \right\} \right]$$

$$\approx \arccos \left[ \frac{1}{\kappa} (\kappa - 2x\kappa) \right]$$

$$\approx \arccos (1 - 2x)$$

which is the correct transformation for a white distribution

large kappa

$$\theta = \arccos \left[ \frac{1}{\kappa} \log \left\{ \exp(\kappa) - x [\exp(\kappa) - \exp(-\kappa)] \right\} \right]$$

$$\approx \arccos \left[ \frac{1}{\kappa} \log \left\{ \exp(\kappa) (1-x) \right\} \right]$$

$$\log ab = \log a + \log b$$

$$\approx \arccos \left[ \frac{1}{\kappa} (\kappa + \log(1-x)) \right]$$

$$\approx \arccos \left( 1 + \frac{1}{\kappa} \log(1-x) \right)$$

not uniformly asymptotic. screws up if  $x \neq 0$ . must check:

$$-1 \neq 1 + \frac{\log(1-x)}{\kappa}$$

*[Handwritten signature]*

κ/N	5	10	20	30	50	100
0	-503,+503 -487,+487	-443,+443 -357,+357	-428,+428 -252,+252	-424,+424 -205,+205	-423,+423 -160,+160	-423,+423 -114,+114
1	-423,+549 -417,+571	-238,+485 -253,+442	-056,+447 -156,+344	+051,+431 -113,+301	+141,+418 -065,+251	+217,+405 -020,+207
5	-499,+629 +307,+868	-292,+474 +426,+811	-164,+349 +487,+765	-116,+304 +516,+745	-066,+257 +543,+720	-018,+214 +571,+697
10	-575,+641 +619,+931	-362,+471 +686,+902	-236,+330 +721,+877	-184,+278 +836,+866	-130,+224 +753,+852	-080,+177 +769,+839
20	-623,+651 +798,+963	-400,+471 +834,+948	-274,+319 +852,+936	-218,+266 +861,+929	-165,+207 +870,+924	-111,+159 +877,+918
50	-649,+656 +913,+984	-432,+468 +929,+978	-292,+313 +935,+972	-240,+259 +940,+970	-185,+199 +941,+969	-130,+147 +943,+968
100	-651,+660 +957,+993	-436,+471 +957,+990	-296,+312 +935,+987	-241,+257 +913,+986	-183,+198 +938,+985	-125,+140 +957,+983