Let $p(x,t)$ be The probability The particle is at position x at time t.
Let $\Psi(x,t)$ be The corresponding wavefunction
Only The choice $p = \Psi^* \Psi$ preserves The usual addition rule for probability, when applied to a modal representation.
Modal representation (ie sum of states):

$$\Psi(x,t) = \sum_n a_n \psi_n(x) e^{i\omega_n t}$$

note orthonomality of modes (= states): $\int \psi_n \psi_m dx = \delta_{nm}$
note role for conjugation $(x \cdot y)^* = x^* y^*$.

Then The probability That a particle is somewhere is:

$$\int p(x,t)\, dx = 1 = \int \Psi \Psi^* dt = \sum_n \sum_m a_n a_m^* \int \psi_n \psi_m^* dx\, e^{i(\omega_n - \omega_m)t}$$

$$= \sum_n \sum_m a_m^* a_n \delta_{nm} e^{i(\omega_n - \omega_m)t} = \sum_n a_n^* a_n$$

so $1 = \sum_n a_n^* a_n$, meaning The probability That The particle is somewhere is The probability That it is in one of its modes (= states). This is The usual rule for addition of probabilities. Note That it would not be true if $p = |\Psi^* \Psi|^{1/2}$, because one could then not commute The integral and The sum in The above representation.

Hence in my way of looking at Things, the choice $p = \Psi^* \Psi$ is required for p to behave like a probability.

Bill Menke
Feb 20, 06