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MRN081

Isotropic tensors

invariant under rotation R_{ij}

Then invariant under infinitesimal rotation $R_{ij} = \delta_{ij} + \epsilon C_{ij}$

note C_{ij} must be antisymmetric to preserve length: $V'_i V'_i = V_i V_i$

$V'_i V'_i =$

$(\delta_{ij} + \epsilon C_{ij}) V_j \ (\delta_{ik} + \epsilon C_{ik} V_k) V_k =$

$(\delta_{jk} + \epsilon C_{jk} + \epsilon C_{kj} + O(\epsilon^2)) V_j V_k = V_j V_k$

so since $V_j V_k$ symmetric C_{jk} must be antisym.

Then for vector u_i

$$(\delta_{ij} + \epsilon C_{ij}) u_j = u_i + O(\epsilon^2)$$

implies

$$C_{ij} u_j = 0 \longrightarrow \left|\begin{array}{l} 0 + C_{12} u_2 + C_{13} u_3 = 0 \\ -C_{12} u_1 + 0 + C_{23} u_3 = 0 \\ C_{13} u_1 + C_{23} u_2 + 0 = 0 \end{array}\right.$$

$C_{ij} = \epsilon_{ijk} a_k$

$C_{ij} u_j = \epsilon_{ijk} a_k u_j = \underline{a} \times \underline{u} = 0$ for all $\underline{a}$.

$u = 0$

so no isotropic vectors.

Then for tensor u_{ij}

$$u'_{ij} = (\delta_{ip} + \epsilon C_{ip})(\delta_{jq} + \epsilon C_{jq}) u_{pq}$$

$$= (\delta_{ip}\delta_{jq} + \epsilon \delta_{ip} C_{jq} + \epsilon \delta_{jq} C_{ip}) u_{pq}$$

$$= u_{ij} + \epsilon (C_{jq} u_{iq} + C_{ip} u_{pj})$$

implies $C_{jq} u_{iq} + C_{ip} u_{pj} = 0$

case $i \neq j$ $i=1, j=2$ $C_{2q} u_{1q} + C_{1p} u_{p2} = 0$

$0 = C_{21} u_{11} + \cancel{C_{22} u_{12}}^{0} + C_{23} u_{13} + \cancel{C_{11} u_{12}} + C_{12} u_{22} + C_{13} u_{32}$

$C_{21}(u_{11} - u_{22}) + C_{23} u_{13} + {}^{\circ} C_{12} u_{32}$

for any values of C_{ij}

so $u_{11} = u_{22}$ $u_{13} = u_{32} = 0$ & etc