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variance goes down as one adds data

$\underline{\underline{G}}\,\underline{m} = \underline{d} \qquad \text{cov}\, d = C = \begin{bmatrix} \sigma_1^2 & & \\ & \sigma_2^2 & \\ & & \ddots \end{bmatrix}$

min $E = e^T C^{-1} e = (d - Gm)^T C^{-1} (d - Gm)$

leads to ls soln $m = (G^T C^{-1} G)^{-1} G^T C^{-1} d$

suppose $G = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ then $G^T C^{-1} G = \sum \sigma_i^{-2}$

and $G^T C^{-1} d = \dfrac{\sum \sigma_i^{-2} d_i}{\sum \sigma_i^{-2}} = \dfrac{[1\ 1\ 1\ 1 \cdots]\, C^{-1}}{\sum \sigma_i^{-2}}$

now if $m = G^{-g} d$ then $\text{cov}\, m = G^{-g}\, \text{cov}\, d\, G^{-gT}$

so $\text{cov}\, m = \dfrac{[1\ 1\ 1\ 1\ 1]\, \cancel{C^{-1}}}{\sum \sigma_i^{-2}} \cancel{C}\, C^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \dfrac{1}{\sum \sigma_i^{-2}}$

$= \dfrac{1}{\sum \sigma_i^{-2}} \sum \sigma_i^{-2} \dfrac{1}{\sum \sigma_i^{-2}} = \dfrac{1}{\sum \sigma_i^{-2}}$

$\text{cov}\, m = \dfrac{1}{\sum \sigma_i^{-2}} = \dfrac{\sigma_1^2}{1 + \sum_{i=2} \left(\dfrac{\sigma_1^2}{\sigma_i^2} \right)}$

but since $\sigma > 0$ adding data only drives down cov m