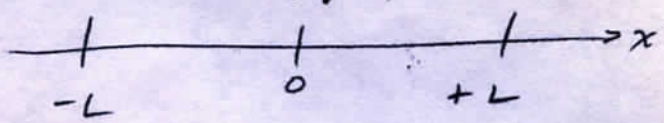
Pressure in slab driven by sinusoidal variation of face - Menke Sept 08

MRN083

$$k \frac{d^2p}{dx^2} = \frac{dp}{dt}$$



BC $p(x=\pm L) = P_0 \cos(\omega t)$

note solution symmetric about origin, so involves only cosines.

let $p(x,t) = \underset{\pm}{p}(x)\, e^{\pm i\omega t}$

$$k \frac{d^2p}{dx^2} e^{i\omega t} = i\omega\, p(x)\, e^{\pm i\omega t}$$

$$\frac{d^2p}{dx^2} = \frac{\pm i\omega}{k} p(x) = c^2 p(x)$$

$$p(x) = A e^{cx} \qquad c = \sqrt{\frac{\omega}{k}}\sqrt{\pm i} = \pm\sqrt{\frac{2\omega}{k}}(1 \pm i) = \pm a(1 \pm i)$$

$$\left.\begin{matrix} P_+(x) e^{i\omega t} \\ + \\ P_-(x) e^{-i\omega t} \end{matrix}\right\} = \left(A_{++} e^{+a(1+i)x} + A_{-+} e^{-a(1+i)x}\right) e^{i\omega t}$$
$$+ \left(A_{+-} e^{+a(1-i)x} + A_{--} e^{-a(1-i)x}\right) e^{-i\omega t}$$

BC symmetric about zero. only cosine allowed $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ so $A_{++} = A_{+-}$ $A_{-+} = A_{--}$

BC $p(x=\pm L) = P_0 \cos(\omega t)$

BC: symmetric about zero at all times

implies eqn invariant on $x \to -x$

so $A_{++} = A_{-+}$ and $A_{+-} = A_{--}$

$$P(x,t) = A_1 \left(e^{+a(1+i)x} + e^{-a(1+i)x} \right) e^{i\omega t} + A_2 \left[e^{+a(1-i)x} + e^{-a(1-i)x} \right) e^{-i\omega t}$$

BC $P(x=L) = P_0 \cos(\omega t)$ use $P_0 \cos(\omega t) = \frac{P_0}{2}(e^{i\omega t} + e^{-i\omega t})$

so $A_1 = \frac{P_0}{2} /$ (exps mult A_1)

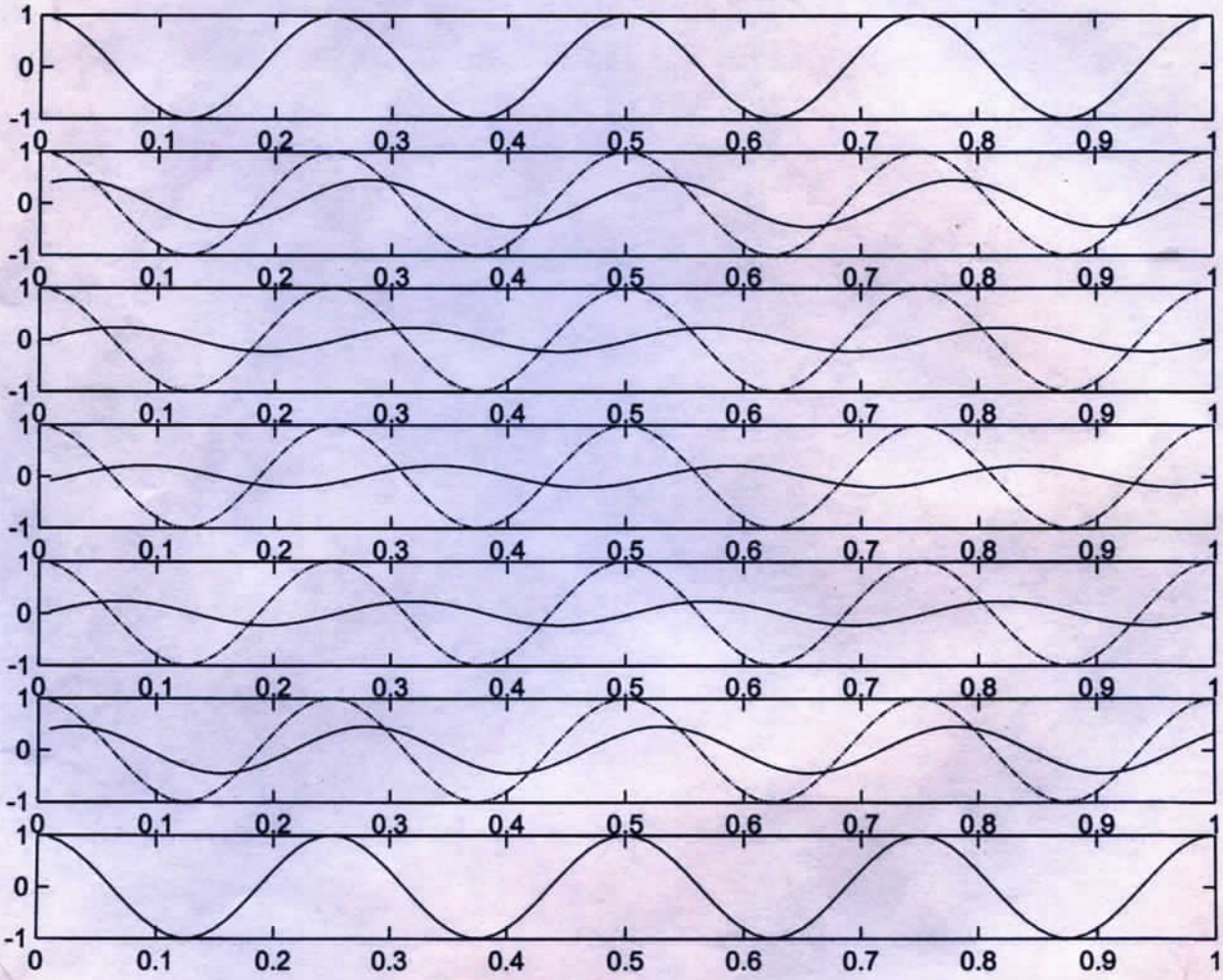
$A_2 = \frac{P_0}{2} /$ (exp mult A_2)

no doubt These expressions could be reduced to simpler ones containing sin, cos, sinh and cosh. But I plan just to use MATLAB to do complex arithmetic

```
p0 = 1.0;
w = 4*2*pi;
k = 1000.0;
a = sqrt(2*w/k);
L = 10;
dt = 0.01;
t = dt*[1:100]';

f1 = exp( (1+i)*a*L ) + exp( -(1+i)*a*L );
f2 = exp( (1-i)*a*L ) + exp( -(1-i)*a*L );
A1 = (p0/2) / f1;
A2 = (p0/2) / f2;


figure(1);
clf;
for j = [1:7]
    subplot(7,1,j);
    x = -L + (j-1)*(L/3);
    p = A1*(exp((1+i)*a*x)+exp(-(1+i)*a*x))*exp(i*w*t)+A2*(exp((1-i)*a*x)+exp(-(1-i)*a*x) ↙
)*exp(-i*w*t);
    plot(t,p);
    hold on
    plot(t,p0*cos(w*t),'r-.');
    hold on
end
```



dotted: cosine for reference.