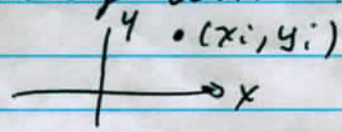
For Tolstoy, Sept 14, 2005. Using tripartite array to descriminate 2 incomming azimuths, when medium velocity is known.

1. $\underline{p}_j$, $j=1,2$ horizontal slowness of wave function of know velocity, v_j and unknown azimuth, θ_j

$$\underline{p}_j = \frac{1}{v}\begin{pmatrix} \sin\theta_j \\ \cos\theta_j \end{pmatrix}$$

2. 3-element array with elements at (x_i, y_i)
$i=1,3$



3. Signal at origin is $a_1(t) + a_2(t) = s(t)$
where each a has it's own slowness

4. Signal at (x_i, y_i)

$$s_i(t) = \sum_{j=1}^{2} a_j(t + \tau_{ij}) \qquad \tau_{ij} = \underline{p}_j \cdot \underline{x}_i$$

5. Fourier Transform $s_i(\omega) = \sum_{j=1}^{2} E_{ij}\, a_j(\omega)$

with $E_{ij} = e^{i\omega\tau_{ij}} = \exp(i\omega\, \underline{x}_i \cdot \underline{p}_j)$

6. so $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ etc. for $\begin{pmatrix} s_1 \\ s_3 \end{pmatrix}$

and $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}^{-1} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$

7. Note for a single pair of stations, a solution exists for any choice of (θ_1, θ_2). However since we have 3 stations, the pairs $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$ and $\begin{pmatrix} s_2 \\ s_3 \end{pmatrix}$ and $\begin{pmatrix} s_1 \\ s_3 \end{pmatrix}$ should yield the same solutions. Thus one finds (θ_1, θ_2) by minimizing the difference between the 3 estimates of $\underline{a}$.

sta locs
$$\begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cc} x & y \end{array}$$
$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$T_{ij} = X_i P_j$

$T_{12} =$

$$p = \begin{bmatrix} p_x(\theta_1) & p_y(\theta_1) \\ p_x(\theta_2) & p_y(\theta_2) \end{bmatrix}$$

$$\begin{bmatrix} P_x(\theta_1) & P_y(\theta_1) \\ P_x(\theta_2) & P_y(\theta_2) \end{bmatrix} \begin{bmatrix} x_A & x_B \\ y_A & y_B \end{bmatrix} =$$

$$\begin{bmatrix} [P_x\theta_1 x_A + P_y(\theta_1) y_A] & [P_x\theta_1 x_B + P_y\theta_1 y_B] \\ [P_x\theta_2 x_A + P_y\theta_2 y_A] & [P_x\theta_2 x_B + P_y\theta_2 y_B] \end{bmatrix}$$