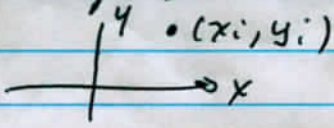


For Tolstoy, Sept 14, 2005. Using triplicate array to discriminate 2 incoming azimuths, when medium velocity is known.

1. \underline{p}_j , $j=1, 2$ horizontal slowness of wave function of known velocity, v_j and unknown azimuth, θ_j

$$\underline{p}_j = \frac{1}{v_j} \begin{pmatrix} \sin \theta_j \\ \cos \theta_j \end{pmatrix}$$

2. 3-element array with elements at (x_i, y_i)
 $i=1, 3$
- 

3. Signal at origin is $a_1(t) + a_2(t) = s(t)$
 where each a has its own slowness

4. Signal at (x_i, y_i)

$$s_i(t) = \sum_{j=1}^2 a_j(t + \tau_{ij}) \quad \tau_{ij} = \underline{p}_j \cdot \underline{x}_i$$

5. Fourier Transform $s_i(\omega) = \sum_{j=1}^2 E_{ij} a_j(\omega)$

with $E_{ij} = e^{i\omega \tau_{ij}} = \exp(i\omega \underline{x}_i \cdot \underline{p}_j)$

6. so $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ etc. for $\begin{pmatrix} s_1 \\ s_3 \end{pmatrix}$

and $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}^{-1} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$

7. Note for a single pair of stations, a solution exists for any choice of (θ_1, θ_2) . However since we have 3 stations, the pairs $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$ and $\begin{pmatrix} s_2 \\ s_3 \end{pmatrix}$ and $\begin{pmatrix} s_1 \\ s_3 \end{pmatrix}$ should yield the same solutions. Thus one finds (θ_1, θ_2) by minimizing the difference between the 3 estimates of \underline{a} .

Velocity is known. Use trigonometric array to determine measurement weights, when position coordinates are known.

$$T_{ii} = x_i P_i$$

$$T_{12} =$$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} P_x(\theta_1) & P_y(\theta_1) \\ P_x(\theta_2) & P_y(\theta_2) \end{pmatrix}$$

2-point at origin is $a(x) + b(y) = z(x)$ where each has its own standard deviation

$$\begin{bmatrix} P_x(\theta_1) & P_y(\theta_1) \\ P_x(\theta_2) & P_y(\theta_2) \end{bmatrix} \begin{bmatrix} x_A & y_A \\ x_B & y_B \end{bmatrix}$$

$$\begin{bmatrix} P_x \theta_1 x_A + P_y \theta_1 y_A \\ P_x \theta_2 x_A + P_y \theta_2 y_A \end{bmatrix} = \begin{bmatrix} P_x \theta_1 x_B + P_y \theta_1 y_B \\ P_x \theta_2 x_B + P_y \theta_2 y_B \end{bmatrix}$$

with $E_i = \exp(i \omega x_i - P_i)$

$$\begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}$$
 etc for $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$
 and $\begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}^{-1} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

7. Note for a single pair of stations a solution exists for any choice of (θ_1, θ_2) . However since we have 3 stations, the pairs $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ and $\begin{pmatrix} z_2 \\ z_3 \end{pmatrix}$ should yield the same solutions. Thus we find (θ_1, θ_2) by minimizing the difference between the 3 estimates of a .