MRN088

Q: Is the double-difference technique capable of determining the absolute locations of a pair of earthquakes, presuming that the velocity structure is known and that the differential traveltimes, ΔT are known everywhere on the earth's surface?

A: Yes.

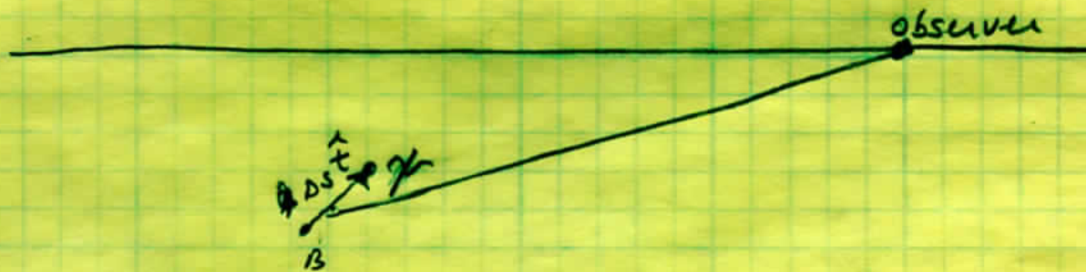
B. Menke

12-17-03

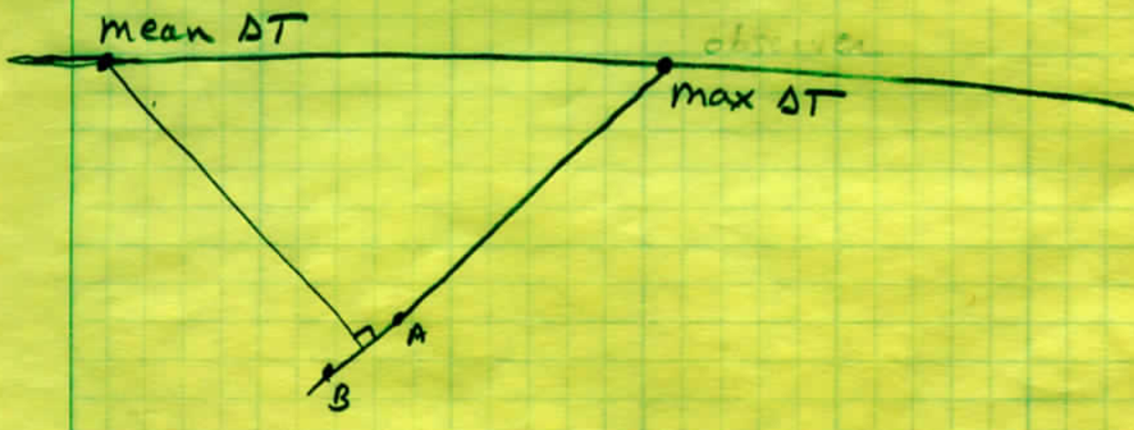
Double-Difference locations in a Homogenous Earth, of known velocity, v

Bill Menke, Dec. 16, 2003

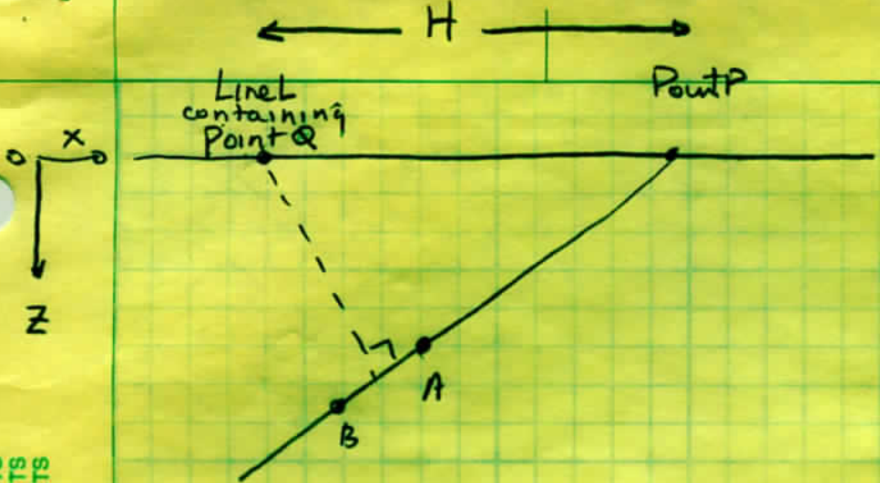
1. The traveltime difference between earthquakes A and B, separated by $\Delta s\,\hat{t}$ is $\Delta T = (t^B - t^A) + \frac{\Delta s}{v}\cos\psi$ where t^A, t^B are origin times.

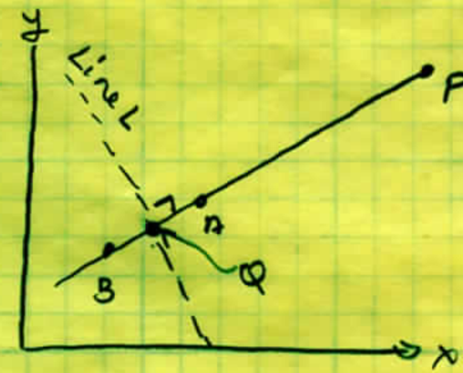


2. The maximum ΔT occurs when the line to the observer is parallel to $\hat{t}$. The mean $(t^B - t^A)$ when $\perp$ to $\hat{t}$



3. in 3D, The max ΔT is a line That intersects The earth's surface at a point. The mean ΔT is a plane That intersects The surface at a line.





4) Let's assume ΔT is known everywhere on the surface of the earth (including sides),

so $\Delta T_{max} = (t^B - t^A) + \frac{\Delta s}{V}$, $\Delta T_{min} = (t^B - t^A) - \frac{\Delta s}{V}$

and $\Delta T_{mean} = (t^B - t^A)$ can be measured.

note $\frac{\Delta s}{V} = (\Delta T_{max} - \Delta T_{min})/2$

Then

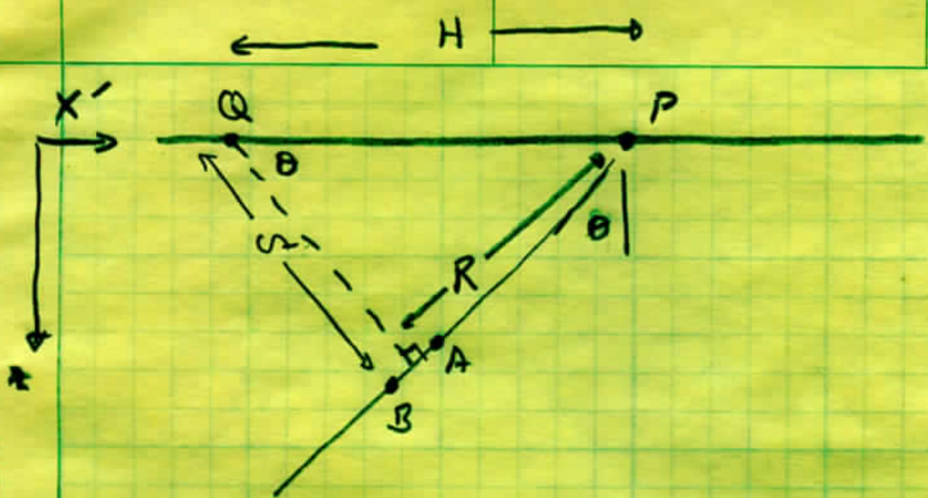
Point P: where $\Delta T(x, y, z=0)$ is max

Line L: where $\Delta T(x, y, z=0) = \frac{1}{2}(\Delta T_{max} + \Delta T_{min})$

Point Q: point on L closest to P.

note that the distance H from Q to P is known

5) Note that $\Delta T(x, y, z=0)$ has mirror symmetry about line $\overline{QP}$. So the problem is 2D in a coordinate system aligned so $x' \parallel$ to $\overline{QP}$.



6. while points Q and P are known, the orientation θ of line $\overline{BAP}$, and the length, R, are unknown (since the earthquakes have unknown location).

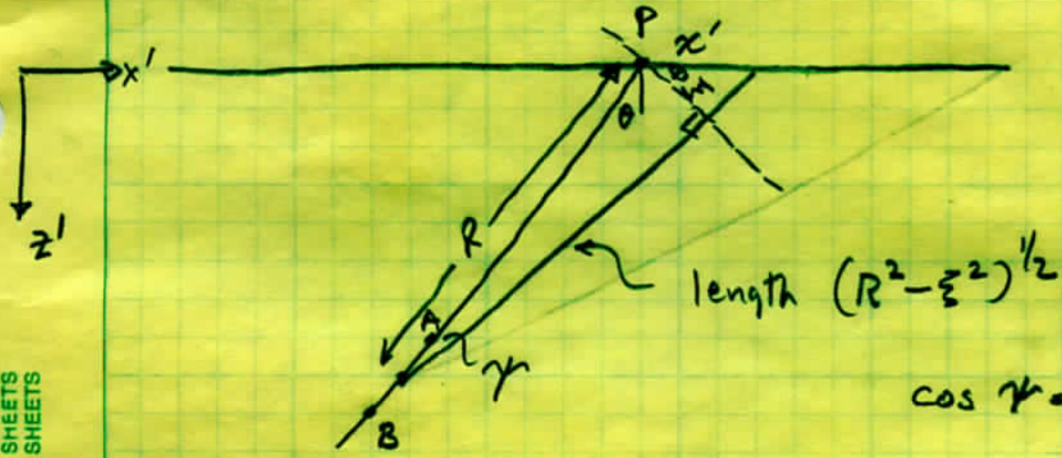
7. Since ΔQBP is right, R and S are functions of θ: $\sin\theta = R/H$ or $R = R(\theta) = H\sin\theta$

$\cos\theta = S/H$, $S = S(\theta) = H\cos\theta$. So there is really only one unknown, θ.

8. The second derivative $\left.\partial^2 \Delta T / \partial x'^2\right|_{x'=P}$ can be measured.

For fixed θ it will clearly decrease as R increases, That is, as the earthquakes are farther from P. It must therefore contain information about R.

9. The derivative in the ξ direction is:



$$\cos\psi = \frac{(R^2-\xi^2)^{1/2}}{R}$$

$$\Delta T = (t^B - t^A) + \frac{\Delta s}{V}\cos\psi$$

$$= (t^B - t^A) + \frac{\Delta s}{V}\frac{(R^2-\xi^2)^{1/2}}{R}$$

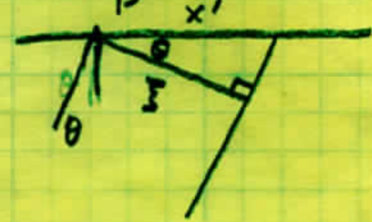
$$\frac{\partial \Delta T}{\partial \xi} = -\frac{\Delta s}{VR}\left(\tfrac{1}{2}\right)(-2\xi)(R^2-\xi^2)^{-1/2} = -\frac{\Delta s}{VR}\,\xi\,(R^2-\xi^2)^{-1/2}$$

$$\frac{\partial^2 \Delta T}{\partial \xi^2} = -\frac{\Delta s}{VR}\left\{(R^2-\xi^2)^{-1/2} + \left(-\tfrac{1}{2}\right)(-2\xi)(R^2-\xi^2)^{-3/2}\right\}$$

$$\left.\frac{\partial^2 \Delta T}{\partial \xi^2}\right|_{\substack{\xi=0\\ (\text{at } P)}} = -\frac{\Delta s}{VR^2}$$

10. The derivative in the x' direction is

$$\cos\theta = \frac{\xi}{x'}$$



$$\xi = \cos\theta \, x'$$

$$\frac{\partial \xi}{\partial x'} = \cos\theta$$

$$\frac{\partial}{\partial x'} = \frac{\partial \xi}{\partial x'} \frac{\partial}{\partial \xi} = \cos\theta \frac{\partial}{\partial \xi}$$

$$\frac{\partial^2}{\partial x'^2} = \cos^2\theta \frac{\partial^2}{\partial \xi^2}$$

so

$$\frac{\partial^2 \Delta T}{\partial x'^2} = \cos^2\theta \frac{\partial^2 \Delta T}{\partial \xi^2} = -\cos^2\theta \frac{\Delta s}{V R^2}$$

but $R = H \sin\theta$ (from 7) so

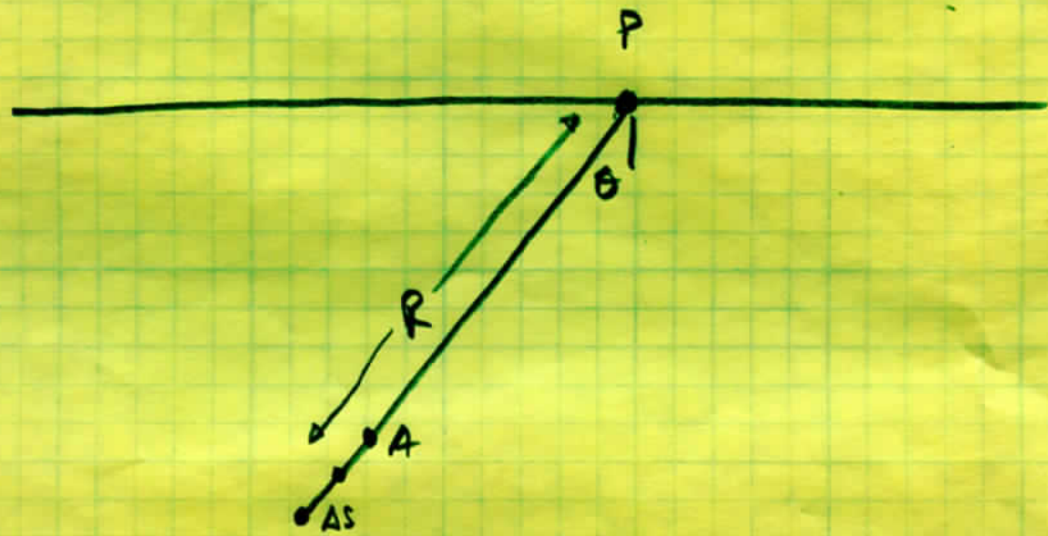
$$\frac{\partial^2 \Delta T}{\partial x'^2} = - \frac{\cos^2\theta \, \Delta s}{H^2 \sin^2\theta \, V}$$ or

$$\tan^2\theta = \left(\frac{\Delta s}{V}\right) \frac{1}{H^2} \Big/ \frac{\partial^2 \Delta T}{\partial x'^2}$$

note that r.h.s. contains only known quantities, so θ known

11. So measurements of ΔT on the surface of the earth suffice to estimate P, θ, R, ΔS.

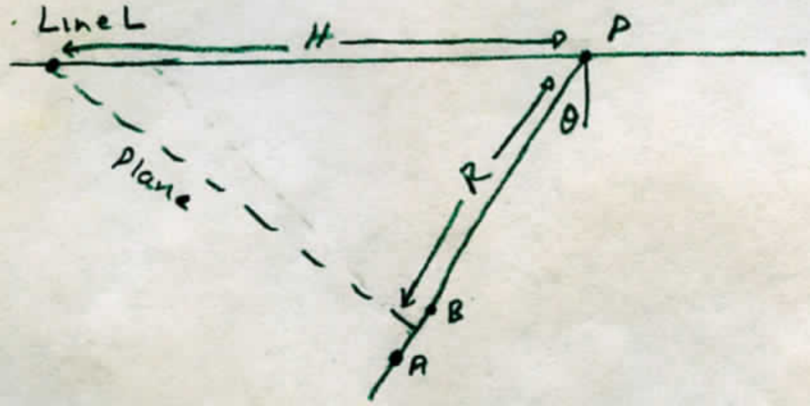
We can thus locate earthquakes A and B



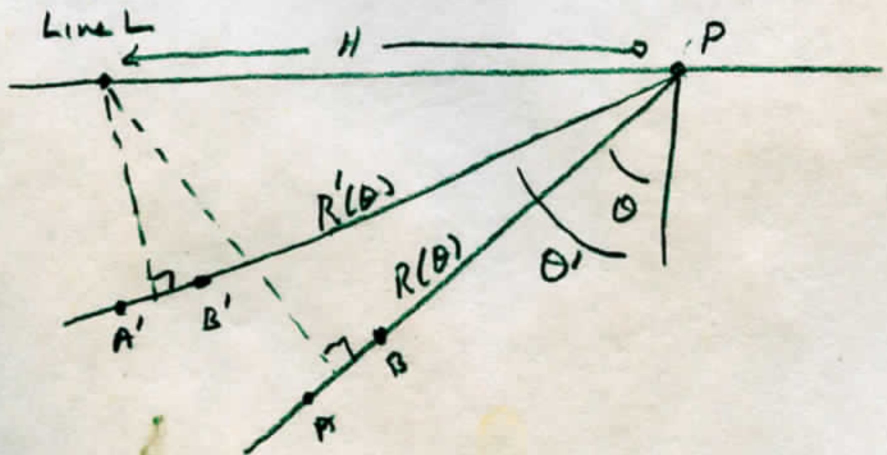
Double-Difference locations, Addendum, W Menke, 12-17-03

Generalizat of Methodology to a vertically stratified earth, $v(z)$

1. In the homogeneous earth, the directions of mean differential traveltime were a plane that intersected the earth's surface at Line, L.

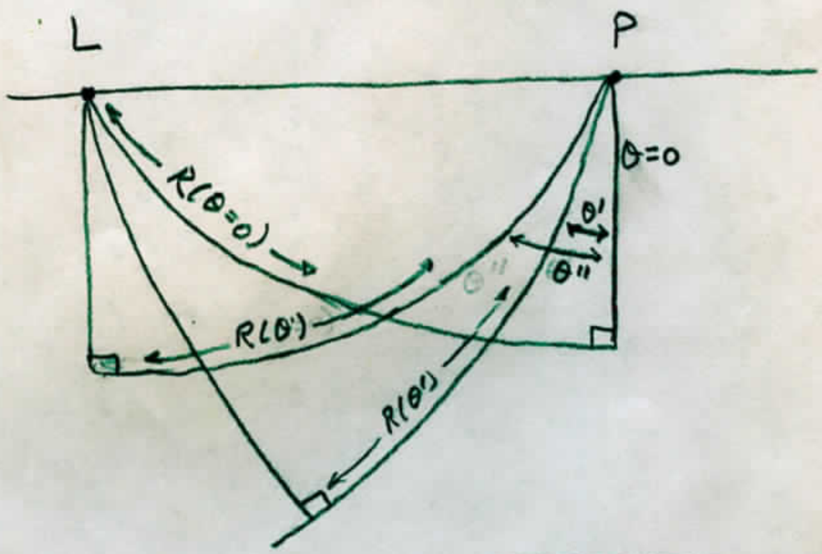


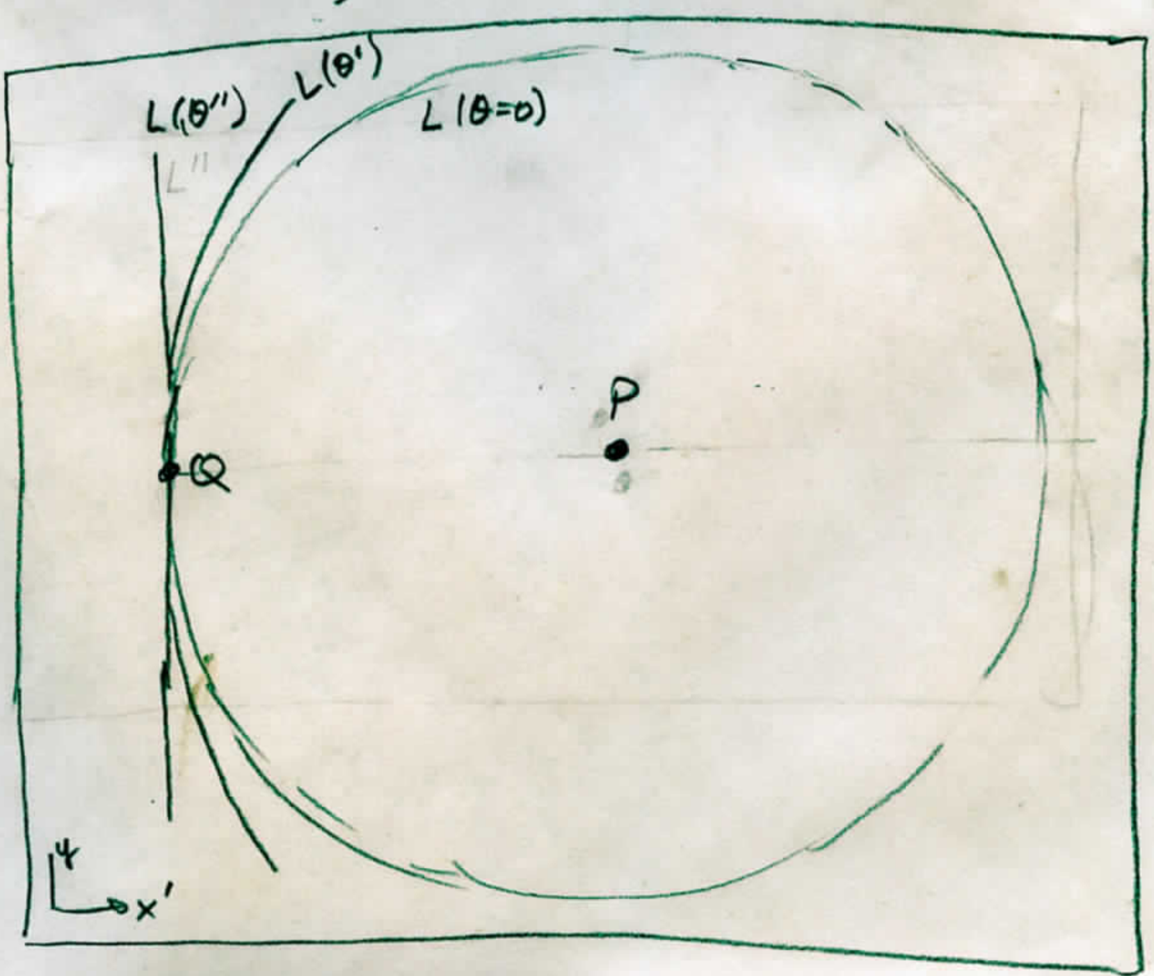
This allowed multiple choices of θ all to match a given position of L.



So the distance H did not determine the range, R.

2. When v varies with depth, z, the pattern of mean differential times is not a line, but rather a curve on The surface of The earth. The shape of This curve is different for different θ's. (Note The problem still has mirror symmetry about a line PQ).





So the shape of $L(\theta)$ is different for each θ. Thus a measurement of L determines θ, and hence $R(\theta)$. Unlike the homogenous case, calculation of $\frac{\partial^2 T}{\partial x'^2}$ is not needed to located the pair of earthquakes.

3. This result carrys over to the general $V(x, y, z)$ case, except that the differential traveltime is no longer symmetric about the line PQ. (Q now defined as the point of closest approach of L to P).