01-24-02

find radial direction (polarization direction) by min energy on on other comp.

$$r = cN + se \qquad r\cdot r = c^2N^2 + s^2e^2 + 2cs(n\cdot e)$$

$$t = sN - ce \qquad t\cdot t = s^2N^2 + c^2e^2 - 2cs(n\cdot e)$$

$$\frac{dt}{d\phi} = cN + se \qquad r^2 + t^2 = n^2 + e^2$$

min $\frac{1}{2} t\cdot t$ wrt ϕ

$$0 = \frac{\partial}{\partial t}\left(\tfrac{1}{2}t\cdot t\right) = \tfrac{1}{2}\cdot 2\cdot t\cdot \frac{\partial t}{\partial \phi} = t\cdot\frac{\partial t}{\partial\phi} = 0$$

$$0 = (sN - ce)\cdot(cN + se) = \sout{sc N^2 + s^2 (ne) - c^2(ne)}$$

$$= sc n^2 + s^2(n\cdot e) - c^2(n\cdot e) - sc e^2$$

$$= sc(n^2 - e^2) - (c^2 - s^2)(n\cdot e)$$

$$= \tfrac{1}{2} s2 (n^2 - e^2) - c2 (n\cdot e)$$

but $c^2 - s^2 = c2$

$sc = \frac{1}{2} s2$

(CRC p 230)

$$t(2\phi) = \frac{2(n\cdot e)}{(n^2 - e^2)}$$

$$2\phi = t^{-1}\left(\frac{2(n\cdot e)}{n^2 - e^2}\right)$$

$$\phi = \tfrac{1}{2} t^{-1}\left(\frac{2(n\cdot e)}{n^2 - e^2}\right)$$

but use $\frac{1}{2}$ atan2($2(n\cdot e)$, $n^2 - e^2$) in practice