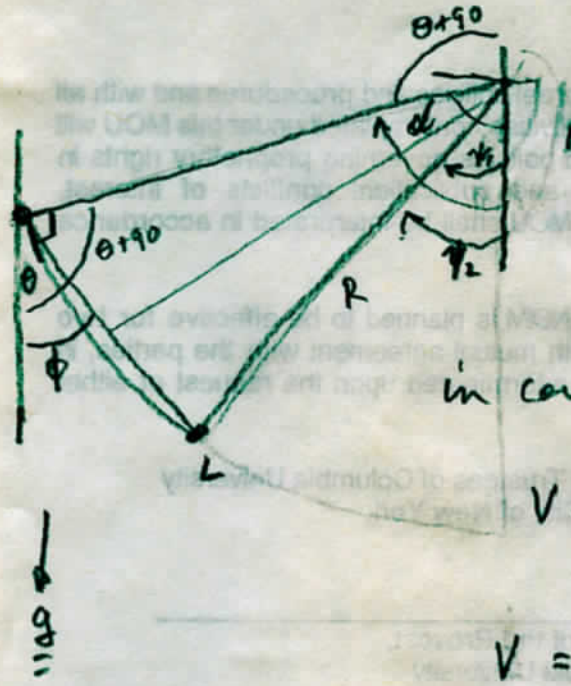
Traveltime along circular ray

MRN097

5/7/01



$180-\theta-90 = 90-\theta = \psi_2$

$90-\theta-2\alpha = \psi_1$

$\theta - \phi - \alpha$

in coord system centered on center of circle with $g_z \uparrow z$

$V = V_0 + g_z z$ but $V_0 = 0$ since ray vert for $\psi = 90$

$V = g_z R \cos\psi$

$$T = \int_{\psi_1}^{\psi_2} \frac{ds}{v} = \int_{\psi_1}^{\psi_2} \frac{g_z R\, d\psi}{g_z R \cos\psi} = \frac{1}{g_z}\int_{\psi_1}^{\psi_2} \frac{d\psi}{\cos\psi}$$

$$T = \frac{1}{g_z} \log \tan\left(\frac{\pi}{4} + \frac{\psi}{2}\right)\Big|_{\psi_1}^{\psi_2}$$

$\frac{\pi}{4} + \frac{\psi_1}{2} = \frac{\pi}{4} + \frac{\pi}{4} - \frac{\theta}{2} - \alpha$

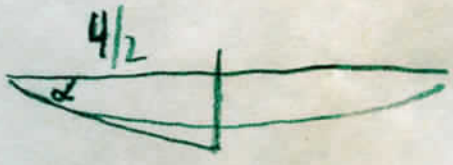
$= \frac{\pi}{2} - \frac{\theta}{2} - \alpha = \frac{\pi}{2} + \frac{\phi}{2} + \frac{\alpha}{2} - \alpha$

$= \frac{\pi}{2} + \frac{\phi}{2} - \frac{\alpha}{2}$

$\frac{\pi}{4} + \psi_2 = \frac{\pi}{4} + \frac{\pi}{4} - \frac{\theta}{2} =$

$= \frac{\pi}{2} - \frac{\theta}{2}$

$= \frac{\pi}{2} - \frac{\phi}{2} + \frac{\alpha}{2}$



$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan\left(\frac{\pi}{4} + \psi_2\right) = \frac{\tan\left(\frac{\pi}{2} - \frac{\phi}{2}\right) + \frac{\alpha}{2}}{1 - \tan\left(\frac{\pi}{2} - \frac{\phi}{2}\right)\frac{\alpha}{2}}$

$(B + \frac{\alpha}{2})(1 - \frac{\alpha}{2}B)^{-1}$

$= (B + \frac{\alpha}{2})(1 + \frac{\alpha}{2}B)$

$= B + \frac{\alpha}{2}(1+B)$