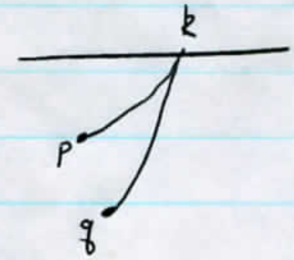
Differential Times



(1) Given events p, q and station k

τ_p = origin time

t_{pk} = arrival time

T_{pk} = travel time

Then

$$\tau_p + T_{pk} = t_{pk}$$

(2) Define differential time $\delta t_{pqk} = t_{pk} - t_{qk}$

note: $\delta t_{pqk} = -\delta t_{qpk}$

and
$$\begin{aligned} \delta t_{pqk} &= \delta t_{pik} - \delta t_{qik} \\ &= t_{pk} - t_{ik} - t_{qk} + t_{ik} \\ &= t_{pk} - t_{qk} \end{aligned}$$

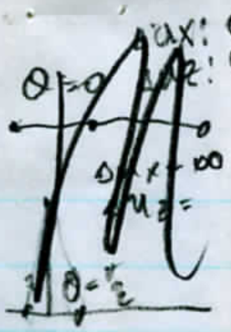
(3) given N_k observations, $t_{1k}, t_{2k} \dots t_{Nk}$

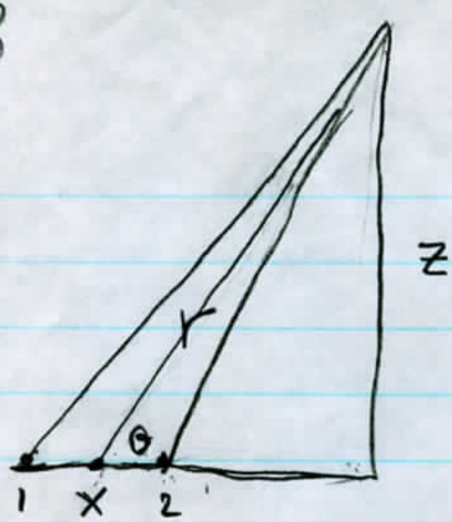
The linear combinations

$$\bar{t}_k = \frac{1}{N_k}\{t_{1k} + t_{2k} + \dots t_{N_k k}\}$$

$$\begin{aligned} &\delta t_{12k} \\ &\delta t_{23k} \\ &\delta t_{34k} \\ &\dots \end{aligned}$$

Are a complete set.





$$r^2 = x^2 + z^2$$

$$\frac{x}{r} = \cos\theta \qquad \frac{z}{r} = \sin\theta$$

$$\left[(x \pm h)^2 + z^2\right]^{-1/2} = \left[r^2 \pm 2xh\right]^{-\frac{1}{2}} = r^{-1}\left[1 \mp \frac{xh}{r^2}\right]$$

$$\frac{x+h}{\left((x+h)^2+z^2\right)^{\frac{1}{2}}} = \frac{1}{r}(x+h)\left(1-\frac{xh}{r^2}\right) = \frac{1}{r}\left(x - \frac{x^2}{r^2}h + h\right)$$

$$\frac{x-h}{\left((x-h)^2+z^2\right)^{1/2}} = \frac{1}{r}(x-h)\left(1+\frac{xh}{r^2}\right) = \frac{1}{r}\left(x + \frac{x^2h}{r^2} - h\right)$$

subtract $\frac{1}{r}\left(-2h\left(1-\frac{x^2}{r^2}\right)\right)$

$\left[-\frac{h}{r}\sin^2\theta\right]$ $\quad -\frac{2xh}{rV} \approx -2h\cos\theta/V$

$$\frac{z}{\left((x+h)^2+z^2\right)^{1/2}} = \frac{z}{r}\left(1-\frac{xh}{r^2}\right)$$

$$\frac{z}{\left((x-h)^2+z^2\right)^{1/2}} = \frac{z}{r}\left(1+\frac{xh}{r^2}\right)$$

subtract $\frac{z}{r}\left(\frac{2xh}{r^2}\right) \approx \frac{2h}{r}\sin\theta\cos\theta$

$$\Delta u_z \approx \frac{-2h}{r}\sin\theta/V$$

<u>proof</u> The matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & \cdots \\ 1 & -1 & & & & \\ & 1 & -1 & & & \\ & & 1 & -1 & & \\ & & & 1 & -1 & \\ & & & & & \cdots \end{bmatrix}$$

can be diagonalized as follows: First take linear combinations of rows to get a row [1 0 0 0 0]. This is achieved by summing the first row, (N-1) times the second row, (N-2) times the second row, ... 1 times the last row to get [N 0 0 0 0 ... 0]. Now divide by N to get [1 0 0 0 ... 0]. Now subtract second row from this to get [0 1 0 0 0 ... 0]. Now subtract third row from this to get [0 0 1 0 0 ... 0]. Apply recursively,

4) suppose we linearize traveltime equation about a trial location x_p^0:

$$\tau_p + \nabla_p T_{pk} \cdot \Delta \underline{x}_p = t_{pk} - T_{pk}^0$$

where ∇_p are differential w.r.t. x_p evaluated at x_p^0

$x_p = x_p^0 + \Delta x_p$

$T_{pk}^0 = T_{pk}(x_p^0)$

5. Suppose a ray leaves source p heading towards station k with slowness $\underline{S}_{pk}$. Then Geiger's method gives

$$\nabla_p T_{pk} = \underline{S}_{pk}$$

so

$$\tau_p + \underline{S}_{pk} \cdot \Delta \underline{x}_p = t_{pk} - T^0_{pk}$$

6. Now suppose we represent Δx_p as a centroid position $\underline{c}$ plus a deviation Δr_p:

$$\Delta x_p = \underline{c} + \Delta \underline{r}_p$$

with the constraint $\sum_p \Delta \underline{r}_p = 0$.

Note: if $\sum_p \Delta r_p = 0$ then $\underline{V} \cdot \sum_p \Delta r_p = \sum_p \underline{V} \cdot \Delta \underline{r}_p = 0$

7. No suppose we write the slowness S_{pk} as the slowness at the centroid S_{ck} plus a deviation ΔS_{pk}: $\quad S_{pk} = S_{ck} + \Delta S_{pk}$

8. The traveltime equation is

$$\tau_p + (\underline{S}_{ck} + \Delta \underline{S}_{pk}) \cdot (\underline{c} + \Delta \underline{r}_p) = t_{pk} - T^0_{pk}$$

$$\tau_p + \underline{S}_{ck} \cdot \underline{c} + \underline{S}_{ck} \cdot \Delta \underline{r}_p + \Delta \underline{S}_{pk} \cdot \underline{c} + \Delta \underline{S}_{pk} \cdot \Delta \underline{r}_p = t_{pk} - T^0_{pk}$$

$1 \longrightarrow 2 \longrightarrow m \quad 3$

$1 \quad d_1 = \frac{\sigma}{N} + \frac{(N-1)}{N}\delta_{12} + \frac{(N-2)}{N}\delta_{23} + \frac{(N-3)}{N}\delta_{34} + \dots$

$\downarrow n \quad 2 \quad d_2 = \frac{\sigma}{N} + \left\{\frac{(N-1)}{N} - 1\right\}\delta_{12} + \frac{N-2}{N}\delta_{23} + \frac{N-3}{N}\delta_{34} + \cdots$

$3 \quad d_3 = \frac{\sigma}{N} + \left\{\frac{N-1}{N} - 1\right\}\delta_{12} + \left\{\frac{N-2}{N} - 1\right\}\delta_{23} + \cdots$

$\frac{N}{N} - \frac{1}{N} - 1$

$\frac{\partial d_1}{\partial \sigma} = \frac{1}{N} \qquad \frac{\partial d_2}{\partial \sigma} = \frac{1}{N} \qquad$ etc

$\frac{\partial d_1}{\partial \delta_{12}} = \frac{N-1}{N} \qquad \frac{\partial d_2}{\partial \delta_{12}} = \left(\frac{N-1}{N} - 1\right)$

$$\frac{\partial d_n}{\partial \delta_{m,m+1}} = \cancel{\begin{cases} \frac{N-1}{N} & \text{if } m \geq n \\ -\frac{1}{N} & \text{otherwise} \end{cases}} = C_{nm}$$

$$\frac{\partial d_n}{\partial \sigma} = \frac{1}{N}$$

$$\frac{\partial T_p}{\partial \sigma} = \sum_q \underbrace{\frac{\partial T_p}{\partial x_q}}_{u_p \delta_{pq}} \underbrace{\frac{\partial x_q}{\partial \sigma}}_{\frac{1}{N}} = \frac{u_p}{N}$$

$$\frac{\partial T_p}{\partial \delta_{m,m+1}} = \sum_q \frac{\partial T_p}{\partial x_q} \frac{\partial x_q}{\partial \delta_{m,m+1}} = u_p C_{pm}$$

$\tau_p + T_p = t_p$ $\quad \rightarrow \cancel{T_p + \nabla T_p \cdot \Delta x_p} =$

$\tau_q + T_q = t_q$

$\sigma = x_p + x_q$

$\delta = x_p - x_q$

$x_p = \frac{\sigma}{2} + \frac{\delta}{2}$

$x_q = \frac{\sigma}{2} - \frac{\delta}{2}$

$x_q = x_p - \delta$

$$\frac{\partial T_p}{\partial \sigma} = \frac{\partial T_p}{\partial x_p}\frac{\partial x_p}{\partial \sigma} = \frac{u_p}{2}$$

$$\frac{\partial T_p}{\partial \delta} = \frac{\partial T_p}{\partial x_p}\frac{\partial x_p}{\partial \delta} = \frac{u_p}{2}$$

$$\frac{\partial T_q}{\partial \sigma} = \frac{\partial T_q}{\partial x_q}\frac{\partial x_q}{\partial \sigma} = \frac{u_q}{2}$$

$$\frac{\partial T_q}{\partial \delta} = \frac{\partial T_q}{\partial x_q}\frac{\partial x_q}{\partial \delta} = -\frac{u_q}{2}$$

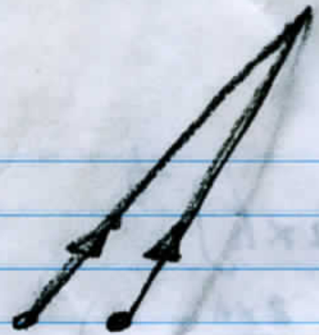
$$\tau_p + \frac{\partial T_p}{\partial \sigma}\Delta\sigma + \frac{\partial T_p}{\partial \delta}\Delta\delta = t_p - T_p^0$$

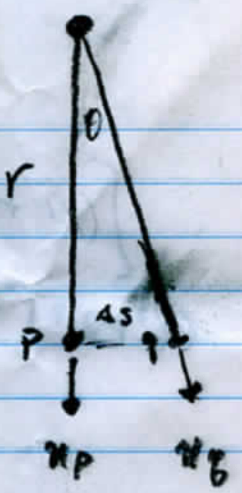
$$\tau_q + \frac{\partial T_q}{\partial \sigma}\Delta\sigma + \frac{\partial T_q}{\partial \delta}\Delta\delta = t_q - T_q^0$$

$(T_p - T_q)$

$$\delta\tau_{pq} + \frac{1}{2}[u_p - u_q]\cdot\Delta\sigma + \frac{1}{2}[u_p + u_q]\cdot\Delta\delta = \delta t_{pq}$$

$$\Sigma\tau = \frac{1}{2}[u_p + u_q]\cdot\Delta\sigma + \frac{1}{2}[u_p - u_q]\cdot\Delta\delta = \Sigma t - \Sigma T^0$$





Δu_x

$v u_p = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad v(u_p - u_s) = \begin{pmatrix} \sin\theta \\ 1-\cos\theta \end{pmatrix}$

$v u_s = \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$ $\quad = \begin{pmatrix} \Delta s / r \\ \frac{1}{2}\left(\frac{\Delta s}{r}\right)^2 \end{pmatrix}$

$v(u_p + u_s) = \begin{pmatrix} \sin\theta \\ 1+\cos\theta \end{pmatrix} = \begin{pmatrix} \Delta s / r \\ 2 + \frac{1}{2}\left(\frac{\Delta s}{r}\right)^2 \end{pmatrix}$

$\overrightarrow{v u_p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad v u_q = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \frac{1}{2}(u_p - v u_q) = 0$

$\frac{1}{2}(u_p + u_q) = 1/v$

$$\begin{pmatrix} 0 & \frac{1}{v} \end{pmatrix}$$

$$\left[(x^2+z^2)-2xh\right]^{-\frac{1}{2}}$$

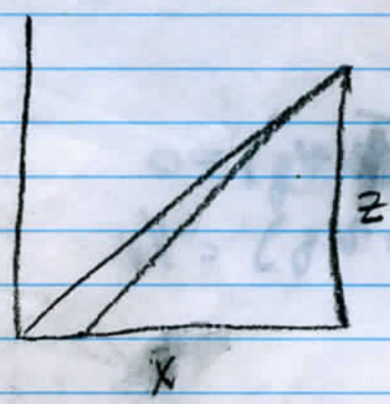
$$(x^2+z^2)^{-\frac{1}{2}}\left(1-\frac{2xh}{(x^2+z^2)}\right)^{-\frac{1}{2}}$$

$$(x^2+z^2)^{-\frac{1}{2}}\left(1+\frac{1}{2}\frac{2xh}{(x^2+z^2)}\right)$$

$$u_x = \pm\frac{1}{v}\sin\theta = \pm\frac{\Delta s}{2vr}$$

$$u_z = \frac{1}{v}\cos\theta = \frac{1}{v}\left(1-\frac{1}{2}\left(\frac{\Delta s}{2r}\right)^2\right)$$

$$(u_p - u_p) = \begin{pmatrix} -\Delta s/vr \\ 0 \end{pmatrix}$$

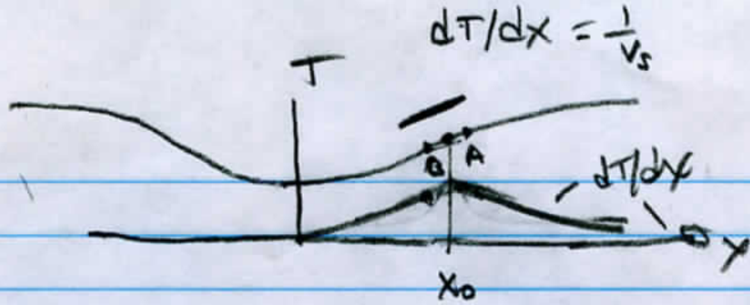


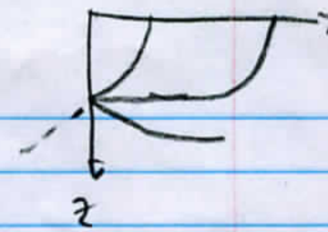
$$\begin{pmatrix} x\,(x^2+z^2)^{-1/2} \\ z\,(x^2+z^2)^{-1/2} \end{pmatrix}$$

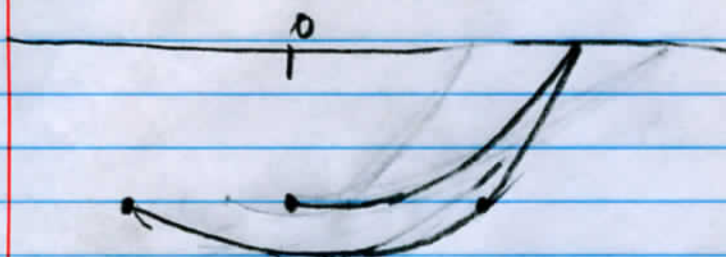
$$\approx x+\frac{x^2h}{r^2}-h$$

$$\begin{pmatrix} (x-h)\left((x-h)^2+z^2\right)^{-1/2} \\ z\left((x-h)^2+z^2\right)^{-1/2} \end{pmatrix}\hat{n}\begin{pmatrix} (x-h)\left(1+\frac{xh}{r^2}\right) \\ z\left(1+\frac{xh}{r^2}\right) \end{pmatrix}-\begin{pmatrix} x \\ z \end{pmatrix}$$

$$= \begin{pmatrix} h\left(\frac{x^2}{r^2}-1\right) \\ xzh/r^2 \end{pmatrix}$$







$$\delta T = T(x_0 + \Delta x) - T(x_0 - \Delta x)$$

$$T(x_0) + \frac{\partial T}{\partial x}\Big|_{x_0} \Delta x + \frac{1}{2}\frac{\partial^2 T}{\partial x^2}\Big|_{x_0} \Delta x^2 + \frac{1}{6}\frac{\partial^3 T}{\partial x^3}\Big|_{x_0} \Delta x^3 + \cdots$$

$$- T(x_0) + \frac{\partial T}{\partial x} \Delta x - \frac{1}{2}\frac{\partial^2 T}{\partial x^2}\Big|_{x_0} \Delta x^2 - \cdots$$

$$= (2\Delta x)\frac{\partial T}{\partial x}\Big|_{x_0} + \frac{1}{6}(2\Delta x)^3 \frac{\partial^2 T}{\partial x^3}\Big|_{x_0} \cdots$$

max δT when $2\Delta x \frac{\partial^2 T}{\partial x^2}\Big|_{x_0} + \frac{1}{6}(2\Delta x)^3 \frac{\partial^4 T}{\partial x^3}\Big|_{x_0} \cdots = 0$

or

$\approx$

$\frac{\partial T}{\partial x}\Big|_{x_0}$ is max

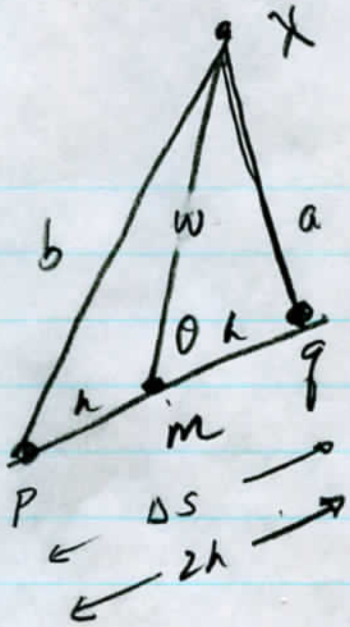
very high order

$$\frac{\partial d_n}{\partial \delta \dot{x}_{m,m+1}} = \frac{(N-m)}{N} \quad \text{if} \quad m \geq$$

$$= 1 - \frac{m}{N}$$

$$= 1 - \frac{m}{N} - 1$$

$$= -\frac{m}{N} \quad \text{if} \quad m < n$$



$$a^2 = w^2 + h^2 - 2wh\cos\theta$$

$$b^2 = w^2 + h^2 - 2wh\cos(\pi - \theta)$$
$$= w^2 + h^2 + 2wh\cos\theta$$

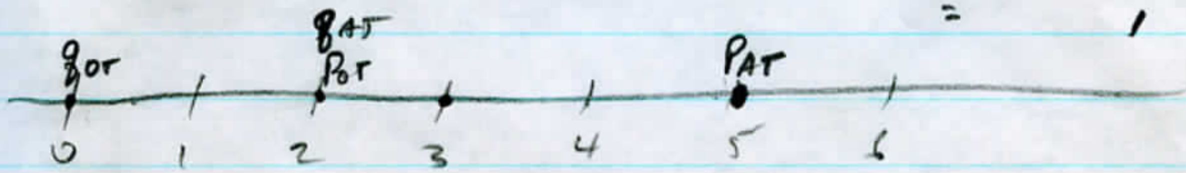
$$b^2 - a^2 = 4wh\cos\theta$$

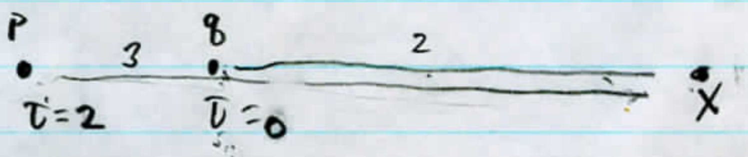
$$(b-a)(b+a) \approx (b-a)2w$$

$$(b-a)(2w) = 4wh\cos\theta$$
$$(b-a) = 2h\cos\theta = \Delta s\cos\theta$$

$$\cos(\pi - \theta) = \cos(h - \theta) = -\cos(\theta)$$

$$p_{AT} - q_{AT} = 3$$
$$= \quad 1$$





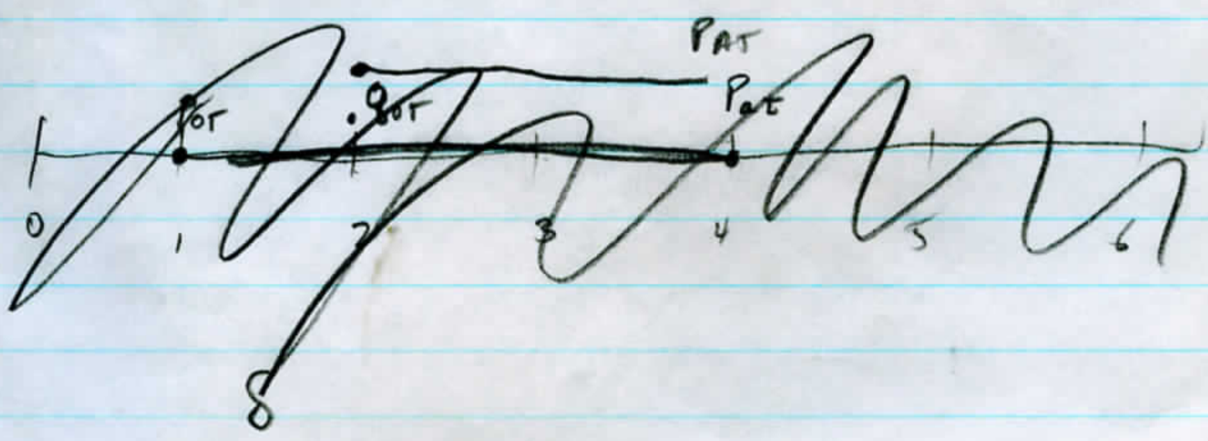
$$t_p = \tau_p + T_p$$
$$t_q = \tau_q + T_q$$

$$\delta t_{pq} = (\tau_p - \tau_q) + (T_p - T_q)$$

$$\begin{matrix} 2-0 & & 3-2 \\ 2 & & 1 \end{matrix}$$

$$3$$

$$\delta t_{pq} = t_p - t_q$$



x_0, z_0

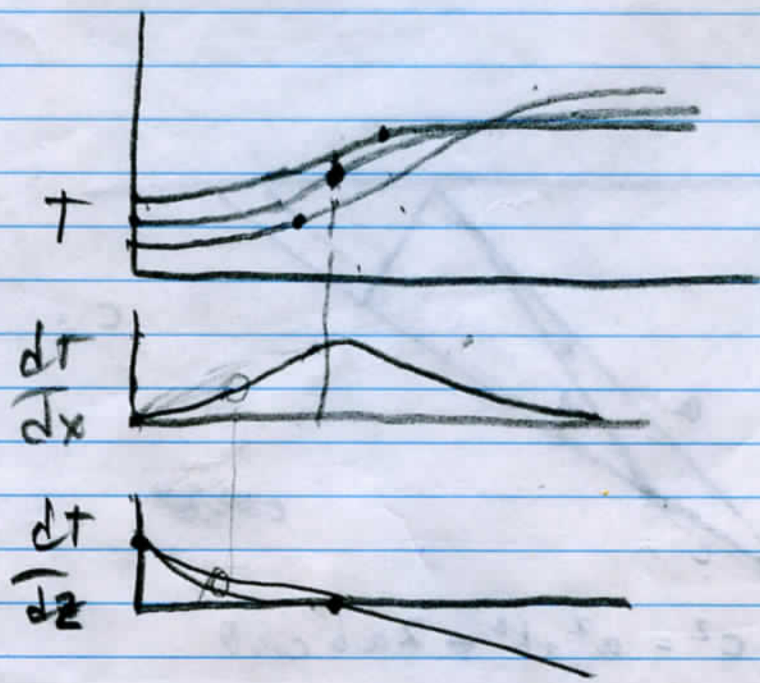
$$\delta T = T(x_0+\Delta x, z+\Delta z) - T(x_0-\Delta x, z_0-\Delta z)$$

$$T(x_0,z_0) + \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial z}\Delta z + \frac{1}{2}\frac{\partial^2 T}{\partial x^2}\Delta x^2 + \frac{1}{2}\frac{\partial^2 T}{\partial z^2}\Delta z^2 + \frac{\partial^2 T}{\partial x^2}\Delta x\Delta z$$

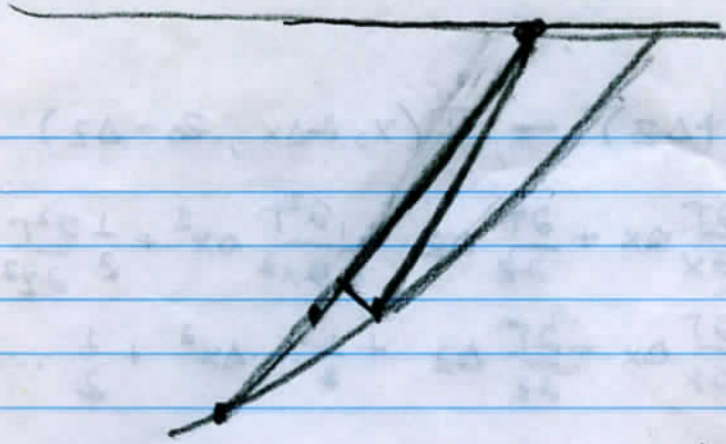
$$-\left[T(x_0,z_0) - \frac{\partial T}{\partial x}\Delta x - \frac{\partial T}{\partial z}\Delta z + \frac{1}{2}\cdots\Delta x^2 + \frac{1}{2}\cdots\Delta z^2 + \cdots\Delta x\Delta z\right.$$

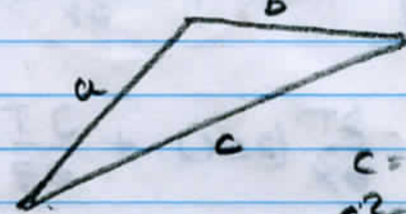
$$= \frac{\partial T}{\partial x}(2\Delta x) + \frac{\partial T}{\partial z}2\Delta z + O(\Delta x)^3$$

δT max when $\frac{\partial T}{\partial x} + \frac{\partial T}{\partial z}$ is max



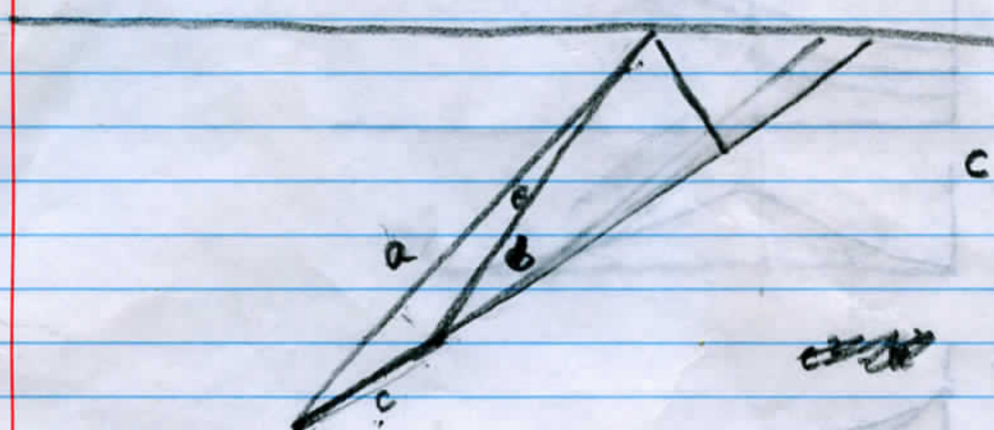
$$\left.\frac{dT}{dx}\right| = \frac{\sin\theta(x)}{v} + \frac{1-\cos^2\theta(x)}{v}$$





$c = a + b$

$c^2 = a^2 + b^2 + 2a \cdot b$



$c =$

$c^2 = a^2 + b^2 - 2ab\cos\theta$

$(b-a)^2 = b^2 + a^2 - 2ab$

$= c^2 + 2ab\cos\theta - 2ab$

$= c^2 - \underbrace{2ab(1-\cos\theta)}_{+}$

$(b-a)^2 < c^2$