MRN102

MENKE 20 APR 2005

Investigations of the equation

EQN 1 $$t_{ij} = t^0_i + \sum_k d_{ijk} s_k$$

note: I introduce the symbol $w_i = 1$ for all i in order to "match indices" in the equation

so

EQN 2 $$t_{ij} = t^0_i w_j + \sum_k d_{ijk} s_k$$

now eliminate t^i_0

$\sum_j$: $$\sum_j t_{ij} = N t^0_i + \sum_k \sum_j d_{ijk} s_k \quad \text{with } N = \sum_j w_j$$

solve: $$t^i_0 = \frac{1}{N} \sum_j t_{ij} - \frac{1}{N} \sum_k \sum_j d_{ijk} S_k \quad \text{eqn 3}$$

sub: $$t_{ij} - \frac{w_j}{N} \sum_k t_{ik} = \sum_k \left(d_{ijk} - \frac{w_j}{N} \sum_p d_{ipk} \right) S_k$$

$$= \sum_k \left(\sum_p \delta_{jp} d_{ipk} - \frac{w_j}{N} \sum_p d_{ipk} \right) S_k$$

$$= \sum_p \left(\delta_{jp} - \frac{w_j w_p}{N} \right) \left(\sum_k d_{ipk} S_k \right)$$

note addition of $w_p = 1$ is just to match indices.
and δ_{jp} = Kronecker delta

$$= \frac{1}{N} \sum_p \left(N \delta_{jp} - w_j w_p \right) \left(\sum_k d_{ipk} S_k \right) \quad \text{eqn 4}$$

note matrixes ~~[illegible]~~ in the form

$$A_{ij} = \frac{1}{N} \sum_p B_{ip} C_{pj}$$

$$B_{ip} = \sum_k d_{ipk} S_k \quad \text{and} \quad C_{pj} = C_{jp} = N\delta_{jp} - w_j w_p$$

Note That C is the symmetric matrix

$$C = \begin{pmatrix} (N-1) & -1 & -1 & -1 & \cdots \\ -1 & (N-1) & -1 & -1 & \cdots \\ -1 & -1 & (n-1) & -1 & \cdots \\ \cdots \end{pmatrix}$$

Now lets consider when $BC=0$. Clearly if B has constant rows then This is true: $B = b_i w_p$
where b is an arbitrary vector.

$$0 \overset{?}{=} \sum_p B_{ip} C_{pj} = \sum_p b_i w_p (N \delta_{jp} - w_j w_p)$$

$$= \sum_p b_i w_p N \delta_{j,p} - \sum_p b_i w_j w_p^2 \qquad \text{note } \sum_p w_p^2 = N$$

$$= N b_i w_j - N b_i w_j \overset{\checkmark}{=} 0$$

Now lets construct The S_k that solves

$$B_{ip} = b_i w_p = \sum_k d_{ipk} S_k$$

Let's define d^{-1} to be the inverse of d, in The sense That $\sum_i \sum_p d^{-1}_{qip} d_{ipk} = \delta_{qk}$. Then

$$\sum_i \sum_p d^{-1}_{qip} b_i w_p = \sum_k \sum_i \sum_p d^{-1}_{qip} d_{ipk} S_k$$

$$= \sum_k \delta_{qk} S_k = S_q \qquad \text{Eqn 5}$$

So any solution of The form $S_q = \sum_i \sum_p d^{-1}_{qip} b_i w_p$ where b_i is an arbitrary vector is a <u>null solution</u> of The equation.

if a solution S^0_k is perturbed by adding a null solution

$$S_k = S^0_k + S^{NULL}_k = S^{new}$$

and if the corresponding t^i_0 is recalculated using equation 3, $t_0^{i\,NEW} = f(S^{new})$

Then these new solutions also satisfy equation 1.

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Note that the existence of d^{-1} is implied by the equation $t_{ij} = \sum_k d_{ijk} S_k$ (ie: Eqn 1 without the t^i_0 term) being solvable. In the tomography case, that's the tomography w/o the source statics case. That seems like a fairly "weak" = reasonable condition