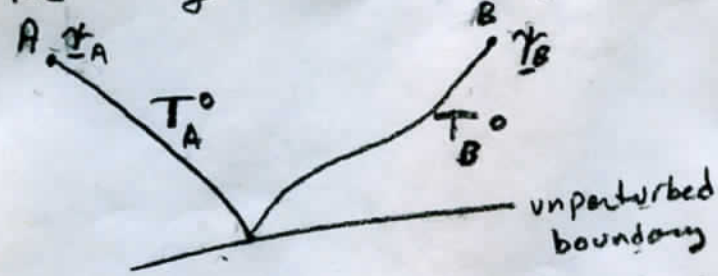
Calculating The Kernel for reflections

Bill Menke 9-28-00

$\frac{\partial T}{\partial m}$ — controls elevation of moho.

(1) Find the ray in the unperturbed case



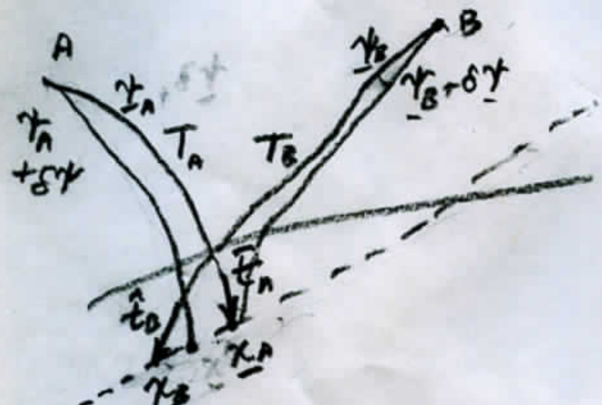
$\underline{\gamma} = [\theta, \phi]^T$ takeoff angles

T = traveltime

(2) Perturb the medium by δm so the boundary changes.
Trace 3 rays from A to boundary $\underline{\gamma}_A$, $\underline{\gamma}_A + [\delta\theta, 0]^T$, $\underline{\gamma}_A + [0, \delta\phi]^T$, and 3 similar ones from B to boundary.

(3) work in a coordinate system $[x, y, z]^T = (\underline{x}^H, z)$ in the plane of the perturbed boundary:

(6 half shoots)



Calculate the point of intersection of the rays with the interface, $\underline{x}^A$, $\underline{x}^B$.

Calculate the ray tangent $[t_x, t_y, t_z]^T = (\underline{t}^H, t_z)$ in this coordinate system, and their partial derivatives $\frac{\partial \underline{t}_A^H}{\partial \underline{\gamma}}$, $\frac{\partial \underline{t}_B^H}{\partial \underline{\gamma}}$. Calculate the partial derivative $\frac{\partial \underline{x}_A^H}{\partial \underline{\gamma}}$, $\frac{\partial \underline{x}_B^H}{\partial \underline{\gamma}}$. (I.E. use finite-difference derivatives).

④ Calculate $\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H}$ and $\frac{\partial \underline{t}_B^H}{\partial \underline{x}_B^H}$ by the chain

rule $\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H} = \frac{\partial \underline{t}_A^H}{\partial \underline{\gamma}} \left[\frac{\partial \underline{x}_A^H}{\partial \underline{\gamma}}\right]^{-1}$ and similarly for B.

Note this requires a 2×2 matrix inversion.

⑤ Expand $\underline{t}_A^H(\underline{x}^H)$ in a taylor series about $\underline{X}_A^H$;

$$t_A^H(\underline{x}^H) = t_A^H(X_A^H) + \left[\frac{\partial \underline{t}_A^H}{\partial X_A^H}\right] \cdot (\underline{x}^H - \underline{x}_A^H)$$

similarly

$$t_B^H(\underline{x}^H) = t_B^H(X_B^H) + \left[\frac{\partial \underline{t}_B^H}{\partial X_B^H}\right] \cdot (x^H - x_B^H)$$

note. in a typical implimentation $t_a = -t_b$ for snell's law so this eqn is different by sign

⑥ find approximately the $\underline{X}_P^H$ that satisfies snell's law $\left(\underline{t}_A^H(\underline{x}_P^H) = \underline{t}_B^H(\underline{x}_P^H) \right)$ by solving the 2×2 linear system :

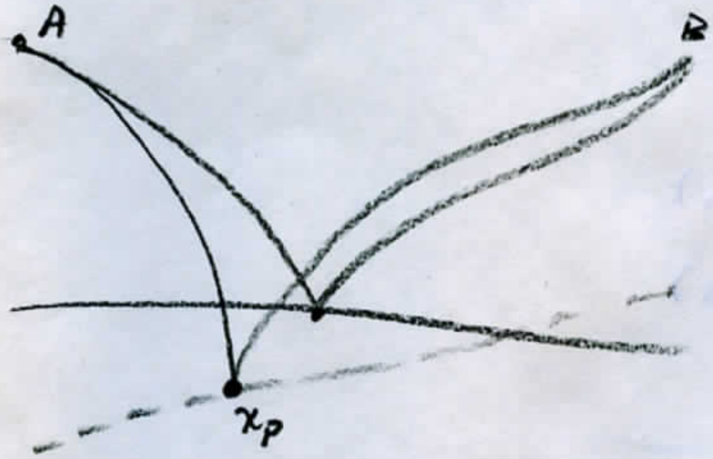
$$\left[\underline{t}_A^H(x_A^H) - \underline{t}_B^H(x_B^H)\right] - \left[\frac{\partial \underline{t}_A^H}{\partial x_A^H} \cdot \underline{x}_A^H - \frac{\partial \underline{t}_B}{\partial \underline{x}_B} \cdot x_B^H\right] =$$

$$- \left[\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H} - \frac{\partial \underline{t}_B^H}{\partial x_B^H}\right] \underline{x}_P^H$$

solve for approx bounce point

⑦ Compute traveltime $T_A(X_H)$, $T_B(X_H)$ and its derivatives $\frac{\partial T_A}{\partial X_A}$ and $\frac{\partial T_B}{\partial X_B}$. Plug into taylor series to estimate $T_A(X_P^H)$ and $T_B(X_P^H)$.

8.



Kernel $\frac{\partial T}{\partial m}$ is approximately

$$\frac{\partial T}{\partial m} \approx \frac{T_A(\underline{x}_p) + T_B(\underline{x}_p) - (T_A^0 + T_B^0)}{\delta m}$$

MRN 112

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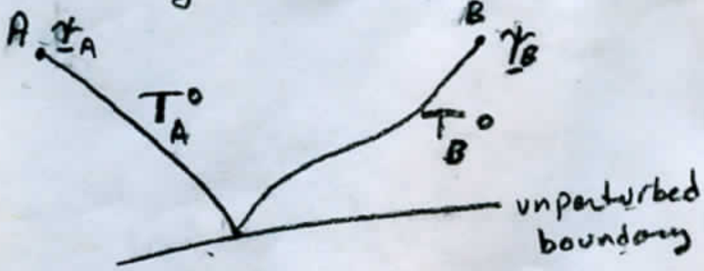
(1)

$\frac{\partial T}{\partial m}$ ✓ controls elevation of moho.

(1) Find the ray in the unperturbed case



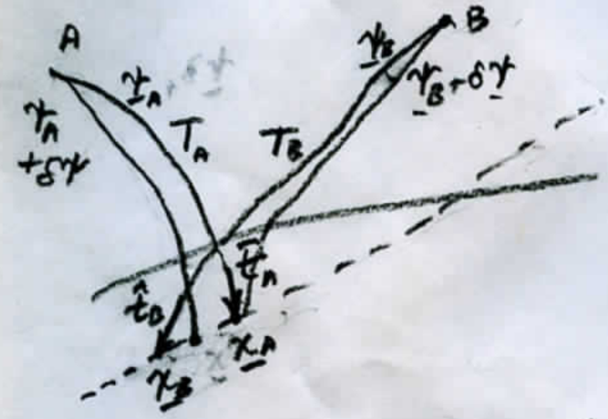
$\underline{\gamma} = [\theta, \phi]^T$ takeoff angles

T = traveltime

(2) Perturb the medium by δm so the boundary changes. Trace 3 rays from A to boundary $\underline{\gamma}_A$, $\underline{\gamma}_A + [\delta\theta, 0]^T$, $\underline{\gamma}_A + [0, \delta\phi]^T$, and 3 similar ones from B to boundary.

(3) work in a coordinate system $[x, y, z]^T = (\underline{x}^H, z)$ in the plane of the perturbed boundary:

(6 half sheets)



Calculate the point of intersection of the rays with the interface, $\underline{x}^A$, $\underline{x}^B$.

Calculate the ray tangent $[t_x, t_y, t_z]^T = (\underline{t}^H, t_z)$ in this coordinate system, and their partial derivatives $\frac{\partial \underline{t}_A^H}{\partial \underline{\gamma}}$, $\frac{\partial \underline{t}_B^H}{\partial \underline{\gamma}}$. Calculate the partial derivative $\frac{\partial \underline{x}_A^H}{\partial \underline{\gamma}}$, $\frac{\partial \underline{x}_B^H}{\partial \underline{\gamma}}$. (I.E. use finite-difference derivatives).

(4) Calculate $\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H}$ and $\frac{\partial \underline{t}_B^H}{\partial \underline{x}_B^H}$ by the chain rule

$$\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H} = \frac{\partial \underline{t}_A^H}{\partial \underline{\gamma}} \left[\frac{\partial \underline{x}_A^H}{\partial \underline{\gamma}}\right]^{-1}$$ and similarly for B.

Note this requires a 2×2 matrix inversion.

(5) Expand $\underline{t}_A(\underline{x}^H)$ in a taylor series about $\underline{x}_A^H$;

$$t_A^H(\underline{x}^H) = t_A^H(X_A^H) + \left[\frac{\partial \underline{t}_A^H}{\partial X_A^H}\right] \cdot (\underline{x}^H - \underline{x}_A^H)$$

similarly

$$t_B^H(\underline{x}^H) = t_B^H(X_B^H) + \left[\frac{\partial \underline{t}_B^H}{\partial X_B^H}\right] \cdot (x^H - x_B^H)$$

note: in a typical implementation $t_a = -t_b$ for snell's law so this eqn is different by sign

(6) find approximately the $\underline{x}_P^H$ that satisfies snell's law ($\underline{t}_A^H(\underline{x}_P^H) = \underline{t}_B^H(\underline{x}_P^H)$) by solving the 2×2 linear system:

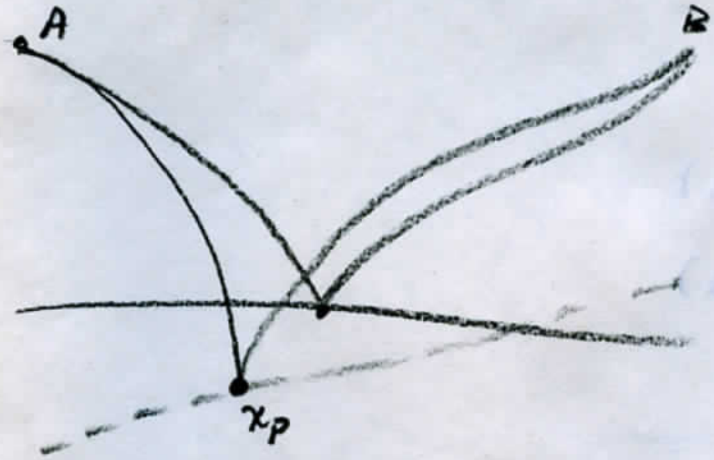
$$\left[\underline{t}_A^H(X_A^H) - \underline{t}_B^H(X_B^H)\right] - \left[\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H} \cdot \underline{x}_A^H - \frac{\partial \underline{t}_B}{\partial \underline{x}_B} \cdot \underline{x}_B^H\right] =$$

$$- \left[\frac{\partial \underline{t}_A^H}{\partial \underline{x}_A^H} - \frac{\partial \underline{t}_B^H}{\partial \underline{x}_B^H}\right] \underline{x}_P^H$$

solve for approx bounce point

(7) Compute traveltime $T_A(X_H)$, $T_B(X_H)$ and its derivatives $\frac{\partial T_A}{\partial X_A}$ and $\frac{\partial T_B}{\partial X_B}$. Plug into taylor series to estimate $T_A(X_P^H)$ and $T_B(X_P^H)$.

8.



Kernel $\frac{\partial T}{\partial m}$ is approximately

$$\frac{\partial T}{\partial m} \simeq \frac{T_A(\underline{x}_p) + T_B(\underline{x}_p) - (T_A^0 + T_B^0)}{\delta m}$$

$$t_A = t_A^0 + \nabla_A (x - x_A)$$

$$t_B = t_B^0 + \nabla_B (x - x_B)$$

$$t_A = -t_B$$

$$t_A^0 + \nabla_a (x - x_a) = -t_B^0 - \nabla_B (x - x_b)$$

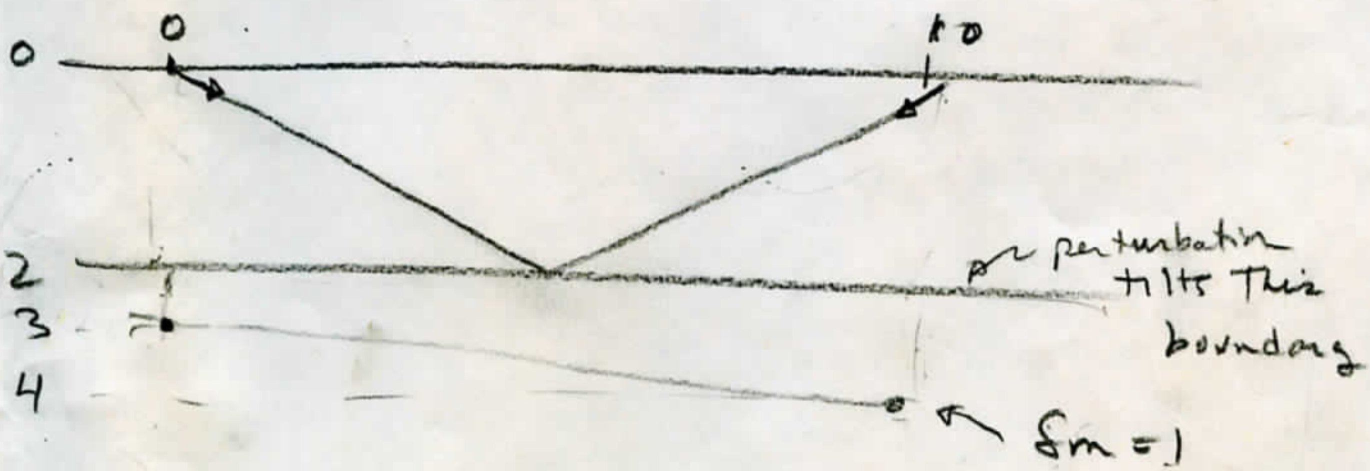
$$(t_A^0 + t_B^0) - \nabla_a x_a - \nabla_B x_B = -\nabla_B x - \nabla_A x$$

Small Perturbation 5%

$$T_r = \frac{\partial T}{\partial m}\,\delta m + T_0$$

```
refltest dm 0.100000
source at: 0.000000 0.000000 0.000000
receiver at: 10.000000 0.000000 0.000000
plane at: (0.000000 0.000000 2.000000)  (10.000000 0.000000 2.000000)  (0.000000 10.000000
 2.000000)
reflected ray info:
    takeoff angles (68.198591 -0.000000)
    traveltime 10.770330
    bounce point (5.000000 -0.000000 2.000000)
    tan 1 (0.928477 0.000000 0.371391)
    tan 2 (0.928477 -0.000000 -0.371391)
perturbation 0.010000
perturbed plane at: (0.000000 0.000000 2.010000)  (10.000000 0.000000 2.020000)  (1.000000
 10.000000 2.030000)
A derivatives:
    angles: 68.198591 -0.000000
    tangent 1: 0.928477 0.000000 0.371391
    tangent a: 0.928477 -0.000000 0.371391
    dtha_d_psi: 0.002984 -0.005736 0.014340 0.007547
    dxha_d_psi: 0.119052 -0.226199 0.077999 0.040579
    dtha_d_xha: 0.149717 0.065606 0.065606 0.059888
    dTa_d_psi: 0.237428 0.000451
    dTa_d_xha: 0.433232 -0.821626
B derivatives:
    angles: 68.198591 180.000000
    tangent 2: 0.928477 -0.000000 -0.371391
    tangent b: 0.928477 -0.000000 -0.371391
    dthb_d_psi: 0.003054 -0.005736 0.014340 0.007547
    dxhb_d_psi: -0.118454 0.225062 -0.077608 -0.041317
    dthb_d_xhb: -0.149976 -0.065367 -0.065367 -0.059890
    dTb_d_psi: 0.236046 -0.000449
    dTb_d_xhb: -0.431637 0.821626
linear system:
    -0.29969  -0.13097        -0.11809
    -0.13097  -0.11978         0.22440
approximate xh of bounce point (rotated) 2.322796 -4.413375
approximate x of bounce point (unrotated) 4.985297 -0.003853 2.014978
dTdm 1.112533
perturbation 0.100000
perturbed plane at: (0.000000 0.000000 2.100000)  (10.000000 0.000000 2.200000)  (1.000000
 10.000000 2.300000)
approximate new traveltime 10.881583
reflected ray info for perturbed plane:
   takeoff angles (66.167780 -0.480876)
   traveltime 10.884461
   bounce point (4.862243 -0.040809 2.147847)
traveltime error 0.002878
```

Predicted (10.881583) — Actual (10.884461)



$\frac{\partial T}{\partial m}$ calculated for $\delta m = 0.01$

medium perturbation 15%

```
refltest dm 0.300000
source at: 0.000000 0.000000 0.000000
receiver at: 10.000000 0.000000 0.000000
plane at: (0.000000 0.000000 2.000000)  (10.000000 0.000000 2.000000)  (0.000000 10.000000
 2.000000)
reflected ray info:
    takeoff angles (68.198591 -0.000000)
    traveltime 10.770330
    bounce point (5.000000 -0.000000 2.000000)
    tan 1 (0.928477 0.000000 0.371391)
    tan 2 (0.928477 -0.000000 -0.371391)
perturbation 0.010000
perturbed plane at: (0.000000 0.000000 2.010000)  (10.000000 0.000000 2.020000)  (1.000000
 10.000000 2.030000)
A derivatives:
    angles: 68.198591 -0.000000
    tangent 1: 0.928477 0.000000 0.371391
    tangent a: 0.928477 -0.000000 0.371391
    dtha_d_psi: 0.002984 -0.005736 0.014340 0.007547
    dxha_d_psi: 0.119052 -0.226199 0.077999 0.040579
    dtha_d_xha: 0.149717 0.065606 0.065606 0.059888
    dTa_d_psi: 0.237428 0.000451
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B derivatives:
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    tangent 2: 0.928477 -0.000000 -0.371391
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     -0.13097  -0.11978        0.22440
approximate xh of bounce point (rotated) 2.322796 -4.413375
approximate x of bounce point (unrotated) 4.985297 -0.003853 2.014978
dTdm 1.112533
perturbation 0.300000
perturbed plane at: (0.000000 0.000000 2.300000)  (10.000000 0.000000 2.600000)  (1.000000
 10.000000 2.900000)
approximate new traveltime 11.104090
reflected ray info for perturbed plane:
    takeoff angles (62.265181 -1.717404)
    traveltime 11.127496
    bounce point (4.620956 -0.138552 2.430731)
traveltime error 0.023406
```

predicted

actual

Big perturbation 50%

```
refltest dm 1.000000
source at: 0.000000 0.000000 0.000000
receiver at: 10.000000 0.000000 0.000000
plane at: (0.000000 0.000000 2.000000)  (10.000000 0.000000 2.000000)  (0.000000 10.000000
 2.000000)
reflected ray info:
    takeoff angles (68.198591 -0.000000)
    traveltime 10.770330
    bounce point (5.000000 -0.000000 2.000000)
    tan 1 (0.928477 0.000000 0.371391)
    tan 2 (0.928477 -0.000000 -0.371391)
perturbation 0.010000
perturbed plane at: (0.000000 0.000000 2.010000)  (10.000000 0.000000 2.020000)  (1.000000
 10.000000 2.030000)
A derivatives:
    angles: 68.198591 -0.000000
    tangent 1: 0.928477 0.000000 0.371391
    tangent a: 0.928477 -0.000000 0.371391
    dtha_d_psi: 0.002984 -0.005736 0.014340 0.007547
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linear system:
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approximate x of bounce point (unrotated) 4.985297 -0.003853 2.014978
dTdm 1.112533
perturbation 1.000000
perturbed plane at: (0.000000 0.000000 3.000000)  (10.000000 0.000000 4.000000)  (1.000000
 10.000000 5.000000)
approximate new traveltime 11.882863
reflected ray info for perturbed plane:
    takeoff angles (50.716522 -8.941256)
    traveltime 12.078274
    bounce point (3.957966 -0.622721 3.277480)
traveltime error 0.195412
```