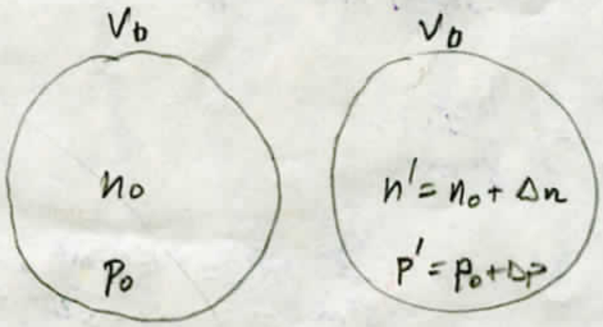
ISOTHERMAL IDEAL GAS

$pV = nRT$

A.



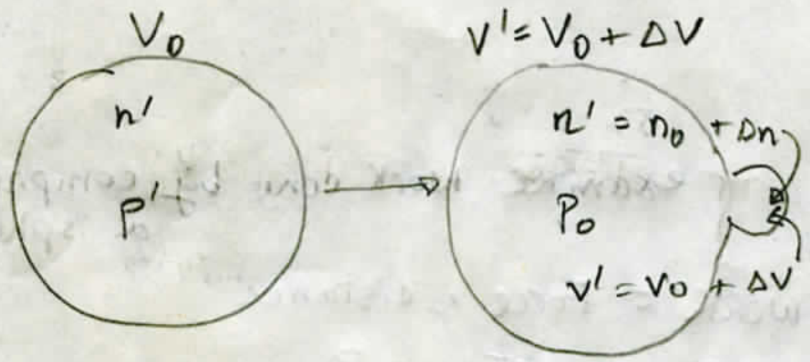
How much does pressure change if number of moles is increased (at constant volume)

$p_0 V_0 = n_0 RT$
$p' V_0 = n' RT$ so $\boxed{\frac{p'}{p_0} = \frac{n'}{n_0}}$

$\Delta n = n' - n_0 = \left(\frac{p'}{p_0} - 1\right) n_0$

now, since we know $p_0 = 100$ MPa and $\Delta p = 5$ MPa, we can infer Δn

B.



How much does volume change if volume is expanded enough to reduce pressure by Δp? (at const number of moles)

$p' V_0 = n' RT$ $\quad V' = \frac{n' RT}{p_0} = n_0 \frac{p'}{p_0} \frac{RT}{p_0} = \boxed{\frac{n_0 RT p'}{p_0^2} = V'}$

C. How much energy is released when n' moles of gas expanded from V_0 to V'?

$$E = \int_{V_0}^{V'} p(V)\, dV \overset{\text{sub. in ideal gas law}}{=} n'RT \int_{V_0}^{V'} \frac{dV}{V} = n'RT(\ln V' - \ln V_0) = n'RT \ln \frac{V'}{V_0} =$$

$$\frac{V'}{V_0} = \frac{n_0 RT p'}{p_0^2} \frac{p_0}{n_0 RT} = \frac{p'}{p_0}$$

$$\boxed{E = n'RT \ln \frac{p'}{p_0}}$$

D. How many moles, n', of gas are in a volume V_0 at pressure p'?

$p' V_0 = n'RT$ $\quad n' = \frac{p' V_0}{RT}$ $\quad$ so $\boxed{E = p' V_0 \ln \frac{p'}{p_0}}$

E. Energy Calculation   2.5% porosity

$V_o = \frac{4}{3}\pi (1500)^3 \cdot 0.025\ m^3 = 3.53\times10^8\ m^3$

$p' = 134\times10^6\ Pa$ $\frac{kg\,m}{s^2 m^2} = \frac{kg}{s^2 m}$

$p_0 = 130\times10^6\ Pa$

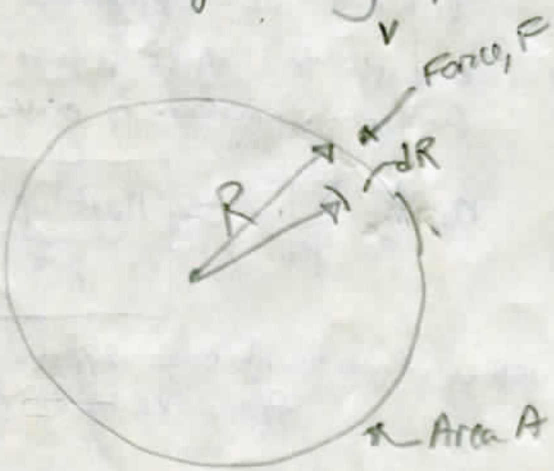
$\frac{p'}{p_0} = \frac{134}{130}$

$pV \to \frac{kg}{s^2 m} m^3 = kg\frac{m^2}{s^2} = J$

$\boxed{\text{note } J = \frac{kg\,m^2}{s^2}}$

$E = p' V_0 \ln\left(\frac{p'}{p_0}\right) = (134\times10^6)(3.53\times10^8)(3.03\times10^{-2}) = 1.43\times10^{15}\ J$

F. Proof of $E = \int_V p\,dV$ · examine work done by compressing a sphere.



WORK = Force · distance

$dE = F \cdot dR$ since $F = pA$

$= pA \cdot dR$

$= p\,AdR = p\,dV$

since $dV = A\,dR$

G. Note $\ln(1+\varepsilon) \approx \varepsilon$ so $\ln\left(\frac{p_1}{p_0}\right) = \ln\left(\frac{p_0+\Delta p}{p_0}\right) = \ln\left(1+\frac{\Delta p}{p_0}\right) \approx \frac{\Delta p}{p_0}$

so $E = p' V_0 \ln\left(\frac{p'}{p_0}\right) \approx \frac{p'}{p_0}\Delta p V_0 \approx \Delta p V_0$

$(4\times10^6)(3.5\times10^8) = 1.4\times10^{15}\ J$

G. Energy Delivered by Seismic WAVE

plane SH wave $\underline{u} = A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i\omega(\frac{x}{\beta} - t)}$

of strain $= \varepsilon = 10^{-5}$

velocity $\dot{u}_y = -i\omega A e^{\cdots}$

strain $\varepsilon_{21} = \frac{1}{2} u_{y,x} = \frac{i\omega}{2\beta} A e^{\cdots} = \varepsilon e^{\cdots}$

$\varepsilon = \frac{i\omega A}{2\beta}$ so $i\omega A = 2\beta\varepsilon$

(note not dependent on prestress)

$\dot{u}_y = -2\beta\varepsilon e^{\cdots}$

mean energy flux $= \rho\beta\langle \dot{u}_y^2 \rangle = \frac{1}{2}\rho\beta \dot{u}_{y\,max}^2 = \frac{1}{2}\rho\beta 4\beta^2\varepsilon^2 = 2\rho\beta^3\varepsilon^2$

energy thru disk of Radius, R in time T

$$E = 2\rho\beta^3\varepsilon^2 \pi R^2 T < 2\pi\rho\beta^3\varepsilon^2 R^2 T$$

$$= 2\pi(2500)(3000)^3(10^{-5})^2(1500)^2\, 30 = 2.86 \times 10^{12}\ j$$

$$\frac{kg}{m^3} \quad \frac{m^3}{s^3} \quad m^2 \quad s = \frac{kg\,m^2}{s^2} = j$$

H. Results

$E_{\text{seismic wave}} \ll E_{\text{compressed gas}} \not\approx E_{\text{compressed magma}}$

$\varepsilon = 10^{-5}$	$2.9 \times 10^{12}\ j$	$1.43 \times 10^{15}\ j$	9.7×10^{14}
m=8 eq at 150 Km	$1.5 \times 10^{14}\ j$		

H. Energy in The magma fluid

$$\text{Energy density} = \frac{1}{2} T_{ij} \varepsilon_{ij} = \frac{3}{2} c p = \frac{3}{2} \frac{p^2}{\lambda} = \frac{3}{2} \frac{p^2}{\rho \alpha^2}$$

$$\varepsilon_{ij} = c\,\delta_{ij}$$

$$T_{ij} = p\,\delta_{ij}$$

$$p = \lambda c \qquad c = p/\lambda$$

$$\rho \alpha^2 = \lambda$$

$$\Delta \mathcal{E} = \frac{3}{2} \frac{\left[(134 \times 10^6)^2 - (130 \times 10^6)^2\right]}{(2500)(3000)^2}$$

$$= \frac{3}{2} \cdot \frac{1056 \times 10^{12}}{2500 \cdot (3000)^2}$$

$$= 7.04 \times 10^4 \; j/m^3$$

E = Volume · Energy density

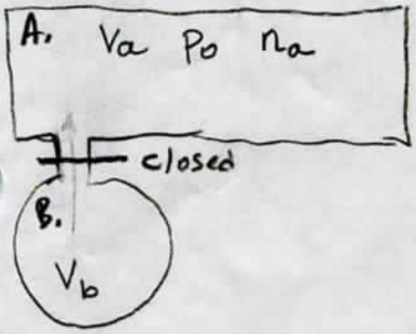
$$= \frac{4}{3} \pi (1500)^3 \underbrace{(0.975)}_{1-\text{porosity}} 7.04 \times 10^4 = 9.70 \times 10^{14} j$$

I. Radiated energy according to Richter

$\log E = 5.8 + 2.4m$ in ergs

gives 10^{18} j for m=8 say Eq is 150 km from volcano

$$E_{Mc} = \frac{10^{18} \cdot \pi (1500)^2}{\frac{2}{3} \pi (150000)^2} = \frac{3}{2} \left(\frac{1.5}{150}\right)^2 \cdot 10^{18} = \frac{3}{2} \times 10^{14} = 1.5 \times 10^{14} j$$

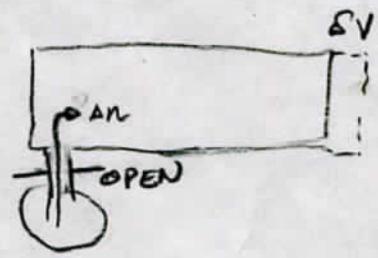


① How many moles, Δn, must one move from B. to A. to increase pressure in A. from P_o to $P_o + \Delta P$?

$$P_o V_a = n_a RT$$
$$(P_o + \Delta P) V_a = (n_a + \Delta n) RT$$

$$\rightarrow \Delta P\, V_a = \Delta n\, RT$$
$$\Delta P \frac{V_a}{n_a RT} = \frac{\Delta n}{n_a}$$
$$\frac{\Delta P}{P_o} = \frac{\Delta n}{n_a}$$

② How much one must expand a to move Δn moles from b to a? Say this causes a pressure drop, δp and volume increase δV



Before
$$P_o V_a = n_a RT$$
$$P_o V_b = n_b RT$$
$$P_o V_T = n_T RT \;;\quad V_T = V_a + V_b,\quad n_T = n_a + n_b$$

after
$$(P_o - \delta p)(V_T + \delta v) = n_T RT$$
$$P_o - \delta p = \frac{n_T RT}{V_T}\left(1 + \frac{\delta V}{V_T}\right)^{-1} \approx P_o\left(1 - \frac{\delta V}{V_T}\right)$$
$$\frac{\delta p}{P_o} = \frac{\delta V}{V_T}$$

the change in moles in b is $\frac{\delta p}{P_o} = \frac{\Delta n}{n_b}$ (from #1)

$$\frac{\delta p}{P_o} = \frac{\Delta n}{n_b} = \frac{n_a}{n_b}\frac{\Delta P}{P_o} = \frac{\delta V}{V_T} \qquad \delta v = \frac{n_a}{n_b}\frac{\Delta P}{P_o} V_T$$

$$V_T = V_a + V_b$$
$$\frac{n_a}{n_b} = \frac{V_a}{V_b}$$

note $\frac{n_a}{n_b} V_T = \frac{V_a}{V_b}(V_a + V_b)$
$$= \frac{V_a^2}{V_b} + V_a = V_a\left(1 + \frac{V_a}{V_b}\right)$$

$$\frac{\delta V}{V_a} = \frac{\Delta P}{P_o}\left(1 + \frac{V_a}{V_b}\right)$$

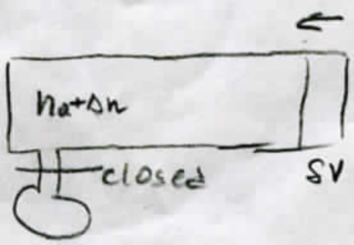
③ How much work is done by this expansion?

$$E_1 = \int_{V_T}^{V_T+\delta V} p\, dV = n_T RT \int_{V_T}^{V_T+\delta V} \frac{dV}{V} = n_T RT \ln\left(\frac{V_T+\delta V}{V_T}\right) = n_T RT \ln\left(1+\frac{\delta V}{V_T}\right)$$

$$\approx n_T RT \frac{\delta V}{V_T} = p_0 \delta V = p_0 V_T \frac{n_a}{n_b}\frac{\Delta P}{p_0} = \frac{n_a}{n_b} V_T \Delta P$$

$$= V_a \Delta P \left(1+\frac{V_a}{V_b}\right)$$

④ Valve closed and A compressed back to volume V_a. This brings the pressure to $P_0+\Delta P$ in A. How much work?



in A

$$p = \frac{(n_a+\Delta n)RT}{V}$$

$$E_2 = -\int_{V_a}^{V_a+\delta V} p\, dV = -(n_a+\Delta n)RT\int_{V_a}^{V_a+\delta V}\frac{dV}{V} = -(n_a+\Delta n)RT \ln\left(\frac{V_a+\delta V}{V_a}\right)$$

$$\approx -(n_a+\Delta n)RT\frac{\delta V}{V_a}$$

but $\dfrac{(n_a+\Delta n)RT}{V_a} = P_0+\Delta P$ so

$$E_2 = -(P_0+\Delta P)\,\delta V = -(P_0+\Delta P)V_T\frac{n_a}{n_b}\frac{\Delta P}{P_0}$$

⑤ TOTAL E

$$E = E_1 + E_2 = P_0 V_T \frac{n_a}{n_b}\frac{\Delta P}{P_0} - (P_0+\Delta P)V_T\frac{n_a}{n_b}\frac{\Delta P}{P_0} =$$

$$-\Delta P\, V_T \frac{n_a}{n_b}\frac{\Delta P}{P_0} = -\Delta P\, V_a\left(1+\frac{V_a}{V_b}\right)\frac{\Delta P}{P_0}$$

Case of infinite V_b

1. moles needed to increase pressure at constant volume

$$(P_0+\Delta P)V_a = (n_a+\Delta n)RT \quad \text{(after)}$$
$$P_0 \quad V_a = n_a \quad RT \quad \text{(before)}$$

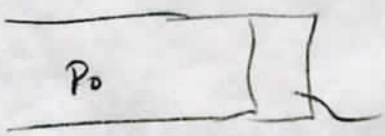
subtract $\quad \Delta P\, V_a = \Delta n\, RT$

$$\boxed{\Delta P = \frac{\Delta n\, RT}{V_a}} = \frac{\Delta n}{n_a}\frac{RT n_a}{V_a} = \frac{\Delta n}{n_a}P_0$$

$$\frac{\Delta P}{P_0} = \frac{\Delta n}{n_a}$$

note $\Delta n\, RT = V_a \Delta P$

2. isobaric expansion by δV to bring in Δn moles



$$P_0\,\delta V = \Delta n\, RT = \Delta P\, V_a$$
$$\frac{\Delta P}{P_0} = \frac{\delta V}{V_a} \qquad \boxed{\delta V = \frac{\Delta P}{P_0}V_a}$$

3. work done on expansion

$$E_1 = \int_{V_a}^{V_a+\delta V} P\, dV = P_0\int_{V_a}^{V_a+\delta V} dV = P_0\,\delta V$$

4. work done in compression, reservoir disconnected

$$E_2 = -\int_{V_a}^{V_a+\delta V} P(V)\,dV = -(n_a+\Delta n)RT\ln\left(1+\frac{\delta V}{V_a}\right)$$

but $(P_0+\Delta P)V_a = (n_a+\Delta n)RT$

$$\approx -(P_a+\Delta P)V_a\frac{\delta V}{V_a}$$
$$= -P_0\,\delta V - \Delta P\,\delta V$$

5. $(E_1+E_2) = -\Delta P\,\delta V = -\Delta P\dfrac{\Delta P}{P_0}V_a$

This checks with complete calc.

$$\lim_{V_b\to\infty} -\Delta P\frac{\Delta P}{P_0}V_a\left(1+\frac{V_a}{V_b}\right) = -\Delta P\, V_a\left(\frac{\Delta P}{P_0}\right)$$

$\ddot{u}_i = -\frac{1}{\rho} p_{,i} + \frac{1}{\rho} f_i$

$-\ddot{u}_{i,i} = +\frac{1}{\rho} p_{,ii} + \frac{1}{\rho} f_{i,i}$ (ρ const)

$u_{i,i} = -\frac{p}{\lambda}$

$\ddot{\left(\frac{p}{\lambda}\right)} \equiv \ddot{q} = -\ddot{u}_{i,i} = \frac{1}{\rho} p_{,ii} + \frac{1}{\rho} f_{i,i}$

$\ddot{q} = \frac{1}{\rho} \nabla^2 p + \frac{1}{\rho} \nabla \cdot f \qquad q = \frac{p}{\lambda}$

$pV = nRT$

inject gas

before

$d\underline{f} = \delta p\, \hat{n}\, dS$

$\delta p = \delta n \frac{RT}{V_b}$

suppose $\nabla \cdot \underline{f} = S\, \delta(\underline{x})$

then $\int_V \nabla \cdot f\, dV = S$

but $\int_V \nabla \cdot f\, dV = \int_A f \cdot n\, dS = \delta p\, A_b$

so $S \approx \delta p\, A$

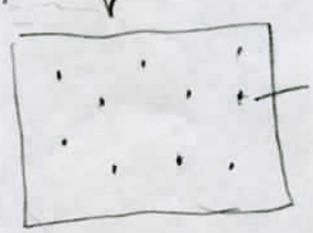
$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

$\frac{V}{A} = \frac{1}{3} r$

$\frac{A}{V^2} = \frac{4\pi r^2}{\frac{16}{9}\pi^2 r^6}$

V



given ϕ, V_b

$V_g = \phi V$

$N = \frac{\phi V}{V_b}$

$\overline{\nabla \cdot f} = \frac{1}{V} \int \sum_i^N \nabla \cdot f_i\, dV = \frac{NS}{V}$

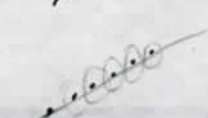
$= \frac{\phi V}{V_b} \frac{S}{V} = \frac{\phi S}{V_b}$

$= \phi \frac{A_b}{V_b} \delta p$

$= \phi \frac{A_b}{V_b^2} \delta n RT$

$=$

$\frac{x}{\sigma_x}$



$$\ddot{u}_i = -\frac{1}{\rho} p_{,i}$$

$$-\ddot{u}_{i,i} = +\frac{1}{\rho} p_{,ii} \qquad (\rho \text{ constant})$$

$$u_{k,k} = -\frac{p}{\lambda} = -cp$$

$$\frac{d}{dt} = \frac{dp}{dt}\frac{\partial}{\partial p}$$

$$\frac{d^2}{dt^2} = \frac{dp}{dt}\frac{\partial}{\partial p}\frac{dp}{\partial t}\frac{\partial}{\partial p}$$

$$= \left(\frac{\partial p}{\partial t}\right)^2 \frac{\partial^2}{\partial p^2} \ ?$$

$$-\dot{u}_{k,k} = \dot{c}p + \dot{p}c$$

$$-\ddot{u}_{k,k} = c\ddot{p} + 2\dot{c}\dot{p} + \ddot{c}p$$

$$\dot{c} = \frac{\partial c}{\partial p}\dot{p}$$

$$\ddot{c} = \frac{\partial^2 p}{\partial c^2}$$

$$c\ddot{p} + 2\dot{c}\dot{p} + \ddot{c}p = \frac{1}{\rho}\nabla^2 p$$

$$=$$

$$-\ddot{u}_{k,k} = \ddot{\left(\frac{p}{\lambda}\right)} = \frac{1}{\rho}\nabla^2 p$$

$$\nabla^2(cp) = ((cp)_{,i})_{,i} = (cp_{,i} + c_{,i}p)_{,i}$$

$$= cp_{,ii} + 2c_{,i}p_{,i} + c_{,ii}p$$

$$\nabla^2(cp) = c\nabla^2 p + 2\nabla c\cdot\nabla p + p\nabla^2 c$$

$$\nabla^2 p = \frac{1}{c}\nabla^2(cp) - \frac{2}{c}\nabla c\cdot\nabla p - \frac{p}{c}\nabla^2 c$$

$$\nabla(cp) = c\nabla p + p\nabla c \quad \text{so} \quad \nabla p = \frac{1}{c}\nabla(cp) - \frac{p}{c}\nabla c$$

$$\nabla^2 p = \frac{1}{c}\nabla^2(cp) - \frac{2}{c^2}\nabla c\cdot\nabla(cp) + \frac{2(cp)}{c^3}\nabla c\cdot\nabla c - \frac{(cp)}{c^2}\nabla^2 c$$

$$p/\lambda = Q \qquad p = \lambda Q$$

$$Q_3 - 2Q_2 + Q_1 = h^2 S$$

$$Q_3 = h^2 S + 2Q_2 - Q_1$$

$$P_3 = \lambda_3 Q_3$$

$$= (\lambda_2 + \dot{\lambda}\,dt)\,Q_3$$

in gas

$$\tau_{ij} = -p\delta_{ij}$$

$$c_{ijpq} = \lambda \delta_{ij} \delta_{pq} \qquad \text{fluid}$$

$$\tau_{ij} = c_{ijpq}\,\varepsilon_{pq} = \lambda \delta_{ij}\varepsilon_{pp} = -p\delta_{ij}$$

$$-p = \lambda \varepsilon_{pp} \qquad \varepsilon_{pp} = \text{volumetric strain} = \frac{\delta V}{V}$$

$$-\Delta p = \lambda \frac{\Delta V}{V}$$

$$-\frac{1}{V}\frac{\Delta V}{\Delta p} = \frac{1}{\lambda}$$

$$V = nRT p^{-1}$$

$$-\frac{1}{V}\frac{dV}{dp} = \frac{p}{nRT}\, nRT p^{-2} - \frac{p}{nRT}\frac{RT}{p}\frac{dn}{dp}$$

$$\frac{1}{\lambda_b} = p^{-1} - \frac{1}{n}\frac{dn}{dp} = \frac{1}{p}\left(1 - \frac{p}{n}\frac{dn}{dp}\right)$$

$$\lambda_b \approx p\left(1 + \frac{p}{n}\frac{dn}{dp}\right) = p\left(1 + \frac{1}{n}\frac{dn}{dt} \Big/ \frac{1}{p}\frac{dp}{dt}\right)$$

$\lambda_T = \left(\frac{(1-\phi)}{\lambda_m} + \frac{\phi}{\lambda_s}\right)^{-1}$ say for mixture, so

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$$\rho \frac{\partial^2 p}{\partial t^2} = \lambda_T(n,p)\frac{\partial^2 p}{\partial x^2}$$

$$\lambda_T = \left[\frac{(1-\phi)}{\lambda_m} + \frac{\phi}{\lambda_b}\right]^{-1} \text{ and } \frac{1}{\lambda_b} = p\left(1 + \frac{\frac{1}{n}\frac{dn}{dt}}{\frac{1}{p}\frac{dp}{dt}}\right)$$

$$\frac{dn}{dt} = \begin{cases} a\, n_s(t)\,[p_s(t) - p(t)] & p < p_s \\ -b\, n(t)\,[p(t) - p_s(t)] & p \geq p_s \end{cases}$$

$$n_s(t) = n_t - n(t)$$

$$p_s(t) = p^* n_s(t) / n_s(t=0)$$

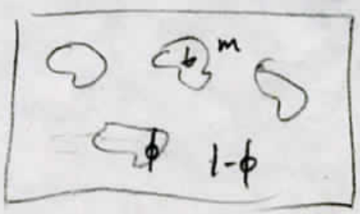
May 5, 2000

I have reviewed the proposal ``Spontaneous Dynamic Rupture Sequences on Non-Planar Fault: Examining Segmentation Hypotheses of Hazard Estimates" by Bruce Shaw and find it worthy of submission.

Christopher Scholz
Professor

$$-\frac{1}{V}\frac{dV}{dp} = \frac{1}{\lambda}$$

→ dp same everywhere



quasi-static ϕ = volume fraction of bubbles

$$dV_b = -\frac{V_b}{\lambda_b}\,dp = -\frac{\phi V_T\,dp}{\lambda_b}$$

$$dV_s = -\frac{V_m}{\lambda_m}\,dp = -\frac{(1-d)V_T\,dp}{\lambda_m}$$

$$dV_T = (dV_b + dV_s) = -\left(\frac{\phi}{\lambda_b} + \frac{(1-\phi)}{\lambda_m}\right)V_T\,dp$$

so $$\lambda_T^{-1} = \left(\frac{\phi}{\lambda_b} + \frac{(1-\phi)}{\lambda_m}\right)$$

$$\lambda_T = \left(\frac{(1-\phi)}{\lambda_m} + \frac{\phi}{\lambda_b}\right)^{-1} = \frac{\lambda_m}{(1-\phi)}\left(1 + \frac{\lambda_m}{\lambda_b}\frac{\phi}{(1-\phi)}\right)^{-1}$$

$$= \frac{\lambda_m}{1-\phi}\left(1 - \frac{\phi}{(1-\phi)}\lambda_m\frac{1}{p}\right.$$

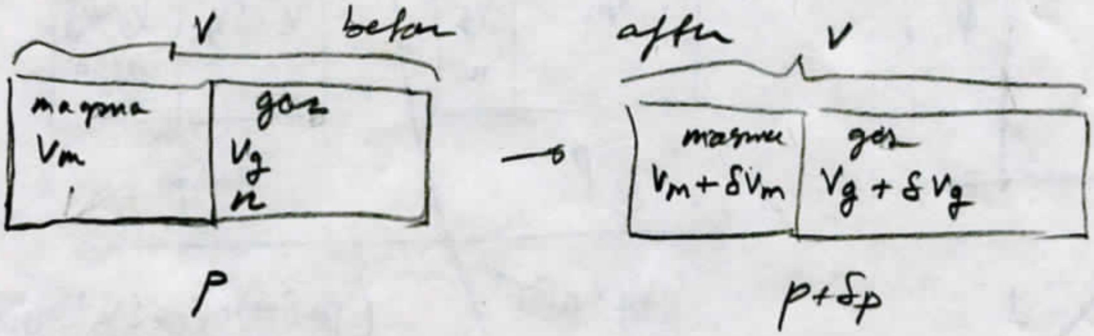
$$\frac{1}{V}\frac{\delta V}{\delta p} = -\frac{1}{\lambda}$$

$$pV = nRT$$

$$(p+\delta p)(V+\delta V) = (n+\delta n)RT$$

$$V\delta p + p\delta V = \delta n RT$$

$$\delta V = -\frac{V}{\lambda}\delta p$$



<u>magma</u>

$$\delta V_m = -\frac{V_m}{\lambda}\delta p$$

<u>gas</u>

$$p\delta V_g + V_g\delta p = \delta n RT$$

but $\delta V_m = -\delta V_g$ and $V_m = (1-\phi)V$ and $V_g = \phi V$

$$\delta V_g = -\delta V_m = \frac{V_m}{\lambda_m}\delta p$$

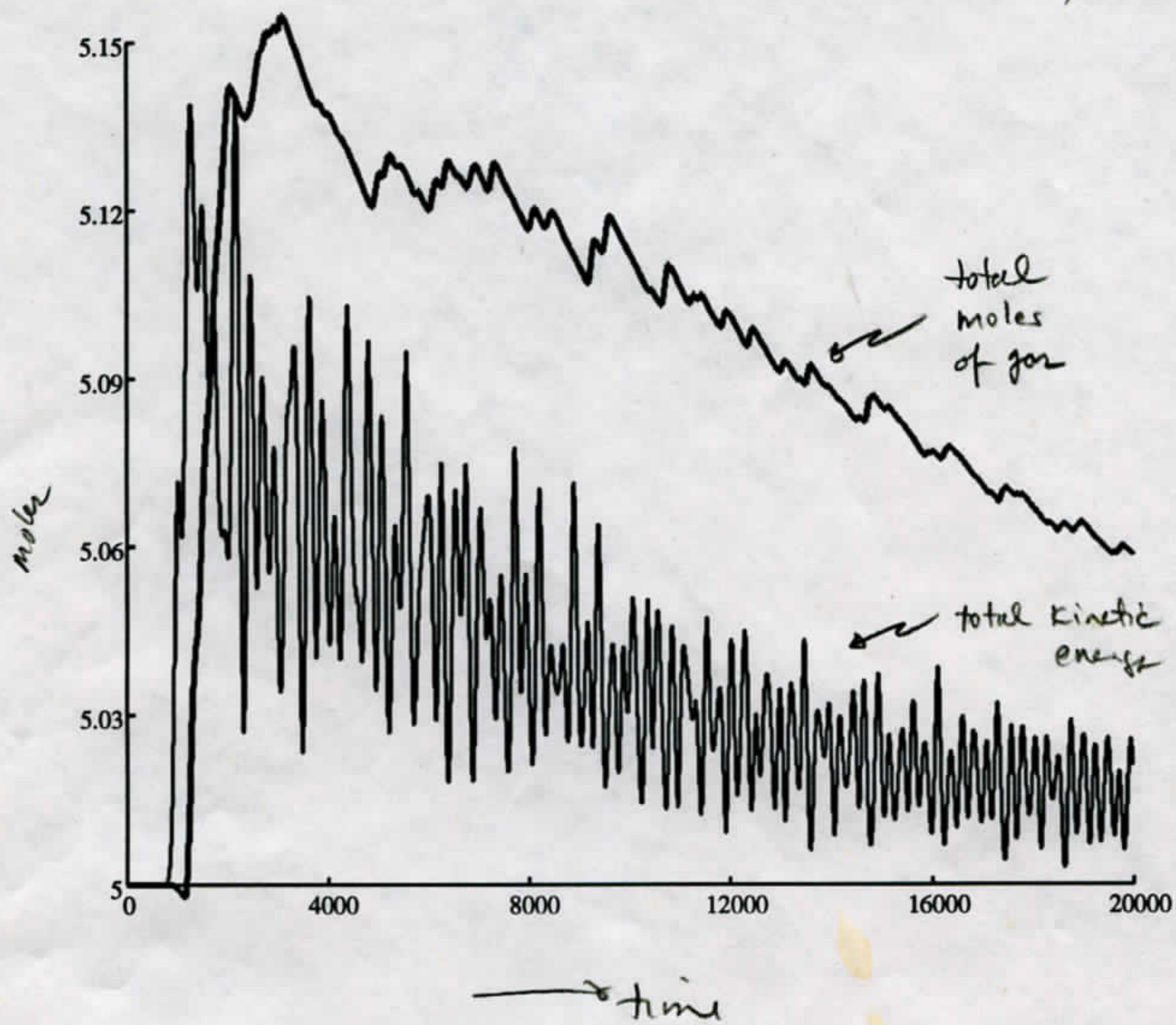
$$p\frac{V_m}{\lambda_m}\delta p + p\frac{V_g}{p}\delta p = \delta n RT$$

$$p\left(\frac{V_m}{\lambda_m} + \frac{V_g}{p}\right)\delta p = \delta n RT$$

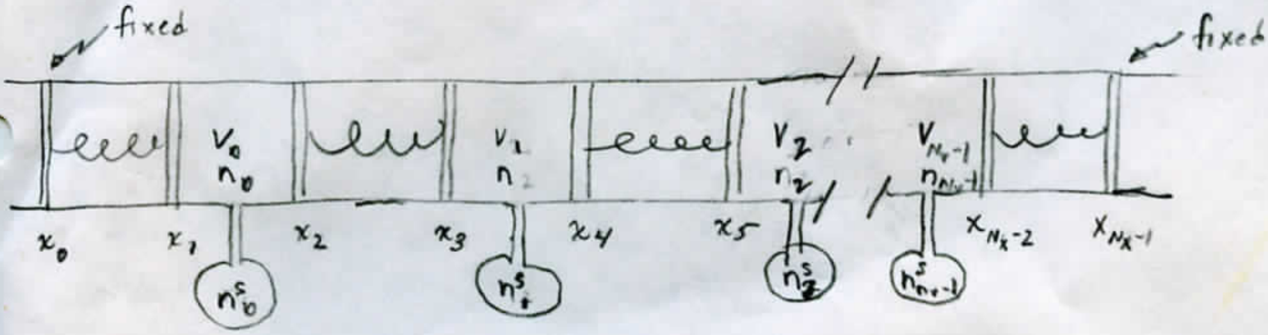
$$\boxed{p\delta p = \frac{\delta n RT}{\frac{V_m}{\lambda} + \frac{V_g}{p}}} = \frac{\delta n RT}{V\left(\frac{1-\phi}{\lambda} + \frac{\phi}{p}\right)} = \frac{\frac{\delta n}{n} nRT}{\left(\frac{1-\phi}{\lambda} + \frac{\phi}{p}\right)V} = p\frac{\frac{\delta n}{n}}{\left(\frac{1-\phi}{\lambda} + \frac{\phi}{p}\right)}$$

$$\boxed{\delta p = \frac{\frac{\delta n}{n}}{\frac{1-\phi}{\lambda_m} + \frac{\phi}{p}}} \qquad \lim_{\phi \to 1} \quad \frac{\delta p}{p} = \frac{\delta n}{n}$$

5e-3 1e-4



77 ek
79
mt
1



$$\frac{dn}{dt} = \begin{cases} a \cdot n_s \cdot (P_s - P) & P_s \geq P \\ -b \cdot n \cdot (P - P_s) & P_s < P \end{cases} \qquad \text{where} \quad P_s = p^* \cdot n_s / n_s(0)$$

P_s = vapor pressure of reservoir

Henry's law: vapor pressure proportional to mole fraction

Henry's law

$$p = K x \qquad x = \text{mole fraction} = \frac{n}{n_{ALL}}$$

$$p = \frac{nK}{n_{ALL}}$$

compare ideal gas

$$p = n\frac{RT}{V} \qquad \text{so} \qquad \frac{K}{n_{ALL}} = \frac{RT}{V}$$

so a box of n_{ALL} moles of Henry's Law solute is "equivalent" to a volume $V = \dfrac{n_{ALL} RT}{K}$ of ideal gas.