Error in P wave angle of incidence due to the free surface

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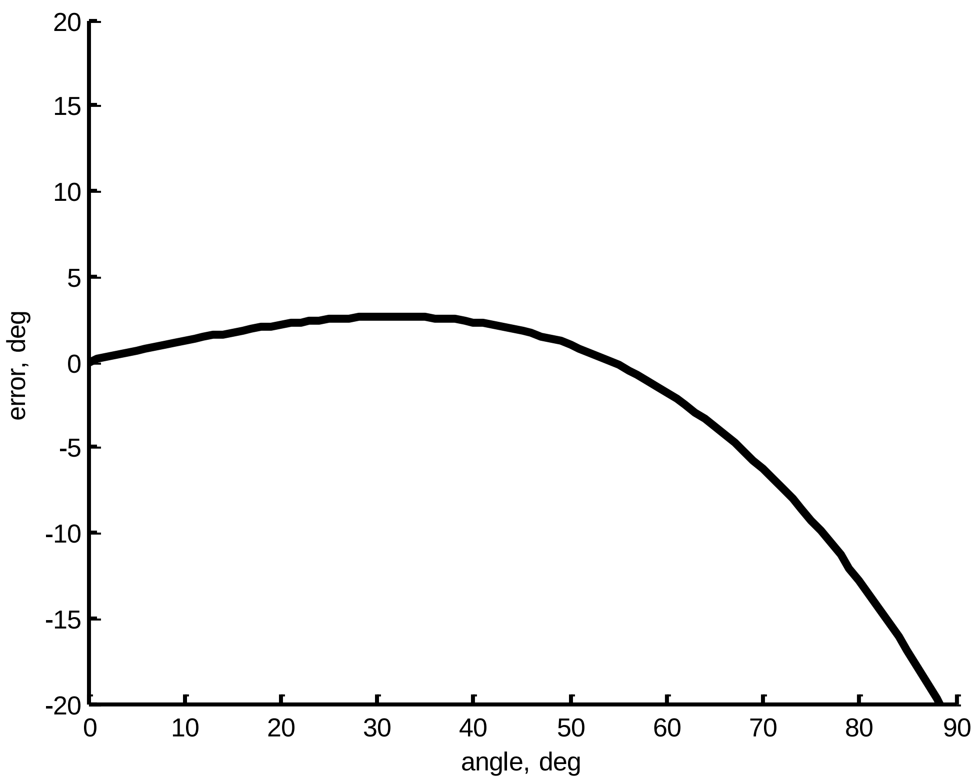
As is pointed out in Aki and Richards, Quantitative Seismology, Vol. 1, 1980, Page 190, Problem 5.6, the displacement **U** of the free surface is not the same as the polarization **P** of an incident P wave because of the interfering effect of the reflected P wave and converted S wave.

P and **U** have the same azimuth, since all the phases are polarized in the plane containing the source and receiver. Furthermore, **U** is always linear, since the S wave must be steeper than the P wave and this can never be evanescent.

However, they have different angles of incidence. Here is the error (deviation between the two angles) for rock with Vp=6.5 km/s and Vp/Vs=1.78: The error is defined as positive when the angle of incidence of the displacement U is numerically larger (= shallower) than the angle of incidence of the P wave polarization.



Note that at small angles of incidence, the converted S wave has a sub-horizontal (=shallow) angle of incidence, and thus the error is positive, but a large angles of incidence the converted S wave has a sub-vertical (=steep) angle of incidence, so the error is negative.

My Matlab script is attached.

```
clear all;

a = 6.5;
b = a/1.78;

A = zeros(90,1);
qU = zeros(90,1);
qP = zeros(90,1);
E = zeros(90,1);

for i=[1:90]

ti = i-1 % angle of incidence of incident P wave
A(i) = ti;

sti = sin( (pi/180)*ti ); % sin of ti
cti = cos( (pi/180)*ti ); % cos of ti
tj = (180/pi) * asin( b * sti / a );
stj = sin( (pi/180)*tj ); % sin of tj
ctj = cos( (pi/180)*tj ); % cos of tj

% polarization vector of incident S wave
Px = sti;
Py = 0;
Pz = cti;
LP = sqrt( Px*Px + Py*Py + Pz*Pz );
qP(i) = (180/pi) * atan2( Px, Pz );


% free surface displacement
% see Aki and Richards, Quantitative Seismology, Vol 1, Page 190
% Problem 5.6.
p = sti/a;  % horizontal slowness
ctioa = cti/a; ctjob = ctj/b; % vertical slownesses
x = ( (1/(b*b))-(2*p*p) );
D = (x*x) + (4*p*p*ctioa*ctjob);
Ux = ( (4*a*p/(b*b)) * ctioa * ctjob ) / D;
Uy = 0;
Uz = ((-2*a/(b*b))*ctioa*x) / D;
LU = sqrt( Ux*Ux + Uy*Uy + Uz*Uz );

qU(i) = (180/pi) * atan2( -Ux, -Uz );
E(i) = qU(i)-qP(i);

end

figure(1);
clf;
set( gca, 'LineWidth', 2 );
hold on;
axis( [0, 90, -20, 20] );
plot( A, E, 'k-', 'LineWidth', 3 );
```

```
xlabel('angle, deg');
ylabel('error, deg');
```