Roger,

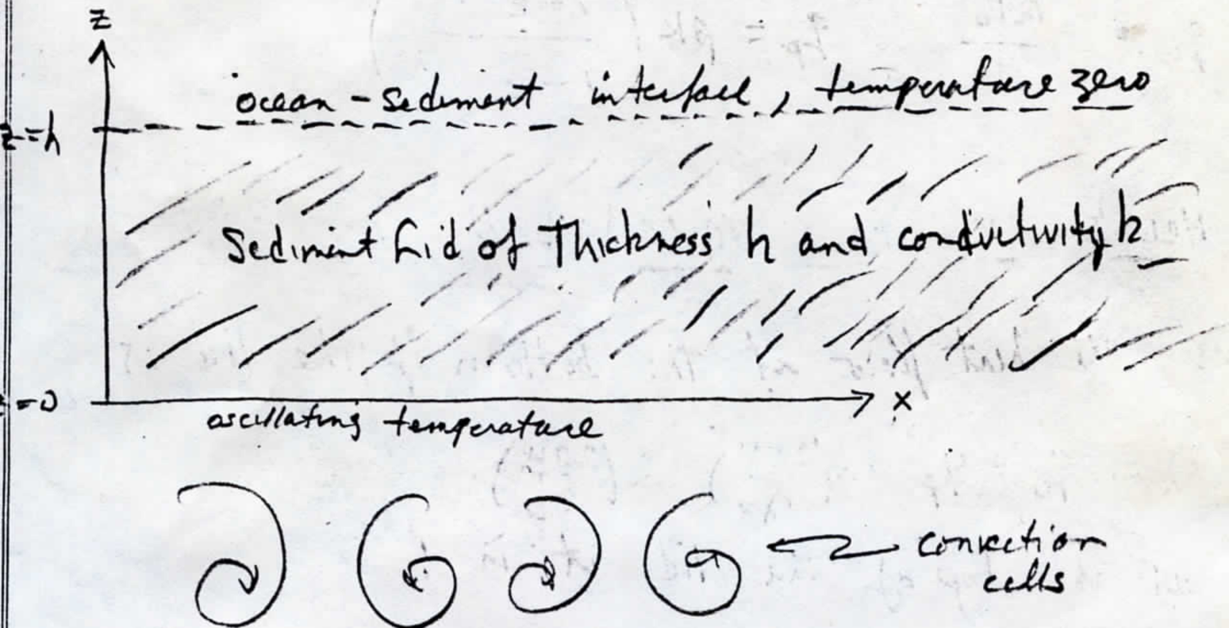
Here's the answer. Don't lose it this time!

cordially,
Bill Menke

P.S.

Damping Of a Horizontally Oscillating Heat Flow by a Sediment Lid:

I set the problem up like this,



Let the Temperature be zero on the ocean-sediment interface

Let the horizontally oscillating temperature at the bottom of the Lid have oscillations with wavelength λ_x in the x direction and λ_y in the y direction. if T_0 is an average temperature and T_p is the amplitude of the oscillation above and below average,

$$T(z=0) = T_0 + T_p \sin\left(\frac{2\pi x}{\lambda_x}\right) \sin\left(\frac{2\pi y}{\lambda_y}\right)$$

The heat flow equation is (time independent)

$$\rho c \frac{\partial T}{\partial t} = 0 = k \nabla^2 T$$

(over $\Longrightarrow$)

The solution is

$$T(x,y,z) = T_0\left(1-\frac{z}{h}\right) + T_p \sin\left(\frac{2\pi x}{\lambda_x}\right)\sin\left(\frac{2\pi y}{\lambda_y}\right)\left(\frac{e^{-\beta z} - e^{-\beta(2h-z)}}{1-e^{-2\beta h}}\right)$$

where $\beta^2 = \frac{4\pi^2}{\lambda_x^2} + \frac{4\pi^2}{\lambda_y^2}$

and the vertical heat flow q_z is

$$q_z(x,y,z) = \frac{kT_0}{h} + T_p\beta k \sin\left(\frac{2\pi x}{\lambda_x}\right)\sin\left(\frac{2\pi y}{\lambda_y}\right)\left(\frac{e^{-\beta z} + e^{-\beta(2h-z)}}{1-e^{-2\beta h}}\right)$$

we can define:

$$q_0 = \frac{kT_0}{h} \qquad q_p = T_p\beta k\left(\frac{1+e^{-2\beta h}}{1-e^{-2\beta h}}\right).$$

Now HERE'S THE PART YOUR really interested in:

Suppose the heat flow at the bottom of the lid is

$$q_z(z=0) = q_0 + q_p \sin\left(\frac{2\pi x}{\lambda_x}\right)\sin\left(\frac{2\pi y}{\lambda_y}\right)$$

Then at the top of the lid it is:

$$q_z(z=h) = q_0 + q_p \sin\left(\frac{2\pi x}{\lambda_x}\right)\sin\left(\frac{2\pi y}{\lambda_y}\right)\left[\frac{2e^{-\beta h}}{1+e^{-2\beta h}}\right]$$

THIS means the heat flow variation is damped by a factor of $\left[\dfrac{2e^{-\beta h}}{1+e^{-2\beta h}} \right]$ where $\beta = \sqrt{\dfrac{4\pi^2}{\lambda_x^2} + \dfrac{4\pi^2}{\lambda_y^2}}$

suppose that the Lid thickness is equal to the wavelength in both directions, ie. $h = \lambda_x = \lambda_y$ then $\beta = \dfrac{2\sqrt{2}\,\pi}{h} \approx \dfrac{8.89}{h}$

Then the damping factor is:

$$\text{DAMPING} = \frac{2 \cdot 0.00014}{1} = 0.00028$$

As the wavelength increases the damping approaches 1. How fast? see graph

$T_0 = \dfrac{h}{k}\, q_0'(\text{surface})$ $\quad T_p = \dfrac{1}{\beta k}\left(\dfrac{1-e^{-2\beta h}}{2e^{-\beta h}}\right) q_p'(\text{surface})$

DAMPING COEFFICIENT AS A Function of Wavelength