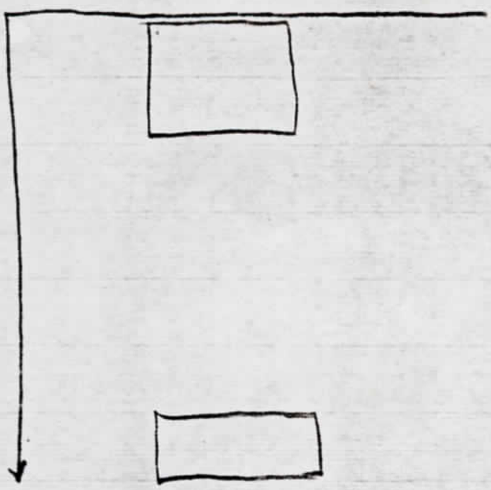


box moves down into sediment, its carbonate dissolving as it gets deeper.

<u>definitions</u>

$n(x)$ = porosity at depth x

$v(x)$ = volume of box at depth x

$w(x)$ = volume of water in box at depth x = constant

$s(x)$ = volume of non-carbonate in box at depth x = const

$c(x)$ = volume of carbonate in box at depth x = const

$V(x)$ = downward velocity of box at depth x

$f(x)$ = fraction of solid volume that is carbonate as a function of depth

ρ_c = density of carbonate

$d(x)$ = dissolution rate in $\frac{gm}{cm^3 s}$

between depths x and $x+dx$ box loses a volume of carbonate $l(x)$ is lost from box

$l(x)$ = (dissolution rate in grams per sec per volume) (volume of box) (time spent between x and dx) / (carbonate density)

$$= \frac{d(x)\, v(x)\, dx}{\rho_c\, V(x)} = \frac{\cancel{gm}}{\cancel{cm^3}\,\cancel{s}}\ \cancel{cm^3}\ \cancel{cm}\ \frac{cm^3}{\cancel{gm}}\ \frac{\cancel{s}}{\cancel{cm}} = cm^3$$

$$= \frac{d(x)}{\rho_c}\ \frac{v(x)}{V(x)}\ dx$$

now as volume of box shrinks it slows down:

$$V(x) = V_0\ \frac{v(x)}{v_0} \quad \text{so}$$

$$\frac{v_0}{V_0} = \frac{v(x)}{V(x)}$$

and $\displaystyle l(x) = \frac{d(x)}{\rho_c}\ \frac{v_0}{V_0}\ dx$ so total loss down to depth x is

$$L(x) = \int_0^x l(x) = \int_0^x \frac{d(x)\, v_0\, dx}{\rho_c\, V_0} = \frac{D(x)\, v_0}{\rho_c\, V_0}$$

where $D(x)$ = cumulative dissolution at depth x

now remaining volume of carbonate $c(x) = c_0 - L(x)$

total volume is
$$v(x) = w_0 + s_0 + c(x)$$
$$= w_0 + s_0 + c_0 - L(x)$$

porosity is
$$\frac{\text{volume of water}}{\text{total volume}} = n(x) = \frac{w_0}{w_0 + s_0 + c_0 - L(x)}$$

$$w_0 = v_0 n_0 \qquad s_0 = v_0 (1-n_0)(1-f_0) \qquad c_0 = v_0 (1-n_0) f_0$$

$$n(x) = \frac{v_0 n_0}{v_0 n_0 + v_0 (1-n_0)(1-f_0) + v_0 (1-n_0) f_0 - \dfrac{D(x)\, v_0}{\rho_c V_0}}$$

$$n(x) = \frac{n_0}{\cancel{n_0} + \underbrace{(1-n_0)(1-f_0)}_{(1-\cancel{n_0}) - \cancel{(1-n_0) f_0}} + (1-\cancel{n_0}) f_0 - \dfrac{D(x)\, \cancel{v_0}}{\rho_c V_0}}$$

$$n(x) = \frac{n_0}{1 - \dfrac{D(x)}{\rho_c V_0}}$$

increases with depth

$f(x)$ = volume fraction of solid material that is carbonate

$$f(x) = \frac{\text{volume of carbonate}}{\text{volume of siliceous} + \text{volume of carbonate}}$$

$$= \frac{C_0 - L(x)}{S_0 + C_0 - L(x)}$$

$$= \frac{\cancel{v_0}(1-n_0) f_0 - \frac{D(x)\cancel{v_0}}{V_0 \rho_c}}{\cancel{v_0}(1-n_0)(1-f_0) + \cancel{v_0}(1-n_0) f_0 - \frac{D(x)}{V_0 \rho_c}\cancel{v_0}}$$

$$= \frac{(1-n_0) f_0 - \frac{D(x)}{V_0 \rho_c}}{(1-n_0) - \frac{D(x)}{V_0 \rho_c}}$$

$$= \frac{f_0 - \frac{D(x)}{V_0 \rho_c (1-n_0)}}{1 - \frac{D(x)}{V_0 \rho_c (1-n_0)}}$$

...sume:

$$\frac{\text{loss of carbonate volume}}{\text{original volume of box}} = \frac{D(x)}{\rho_c V_0}$$

$$\text{porosity} = n(x) = \frac{n_0}{1 - \frac{D(x)}{\rho_c V_0}}$$ increases with depth

$$\text{volume fraction of solids that is carbonate } f(x) = \left[\frac{f_0 - \frac{D(x)}{\rho_c V_0 (1-n_0)}}{1 - \frac{D(x)}{\rho_c V_0 (1-n_0)}}\right]$$ (decreases with depth

$$\text{velocity at depth } x = V(x) = \frac{V_0\, v(x)}{v_0} = V_0 - \frac{D(x)}{\rho_c}$$

$$\text{age at depth } x = A(x) = \int_0^x \frac{dx}{V(x)} = \frac{1}{V_0}\int_0^x \frac{dx}{\left(1 - \frac{D(x)}{\rho_c V_0}\right)}$$

$$\text{critical } f_0 \text{ at which all carbonate is dissolved} = f_0^c = \frac{D_\infty}{\rho_c V_0 (1-n_0)}$$

$$\text{porosity at infinity for } f_\infty^c = \frac{n_0}{1 - f_0^c (1-n_0)}$$

summary $\qquad$ source $= a\,\delta^{p}$

Diffusion only

K = diffusion coefficient

$\delta(x)$ = Dissolution concentration at a Given Depth

$$\delta = \begin{cases} \delta_0 \exp\left\{-\left(\frac{a}{k}\right)^{1/2} x\right\} & p=1 \\ \left\{\left[\frac{p-1}{2}\right]\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}\right\}^{-\left[\frac{p-1}{2}\right]^{-1}} & p>1 \end{cases}$$

Total Dissolution

$$D(x) = \begin{cases} (ak)^{1/2}\,\delta_0 \left\{1 - \exp\left[-\left(\frac{a}{k}\right)^{1/2} x\right]\right\} & p=1 \\ \left(\frac{2ak}{p+1}\right)^{1/2} \left\{\delta_0^{\frac{p+1}{2}} - \left[\left(\frac{p-1}{2}\right)\left(\frac{2a}{k(p+1)}\right)^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}\right]^{-\frac{p+1}{p-1}}\right\} & p>1 \end{cases}$$

Total Dissolution

$$D(x=\infty) = \begin{cases} \delta_0\,(ak)^{1/2} & p=1 \\ \delta_0^{\left[\frac{p+1}{2}\right]} \left(\frac{2ak}{p+1}\right)^{1/2} & p>1 \end{cases}$$

downward velocity only

$$\delta = \begin{cases} \delta_0 \exp\left\{-\frac{a}{v} x\right\} & p=1 \\ \left\{\delta_0^{1-p} + (p-1)\frac{a}{v} x\right\}^{\frac{1}{1-p}} & p>1 \end{cases}$$

$$D(x) = \begin{cases} v\delta_0 \left\{ 1 - \exp\left[-\frac{a}{v}x\right]\right\} & p=1 \\ v\left\{ \delta_0 - \left[\delta_0^{1-p} + \frac{a(p-1)}{v}x\right]^{\frac{1}{1-p}}\right\} & p>1 \end{cases}$$

$$D(x=\infty) = v\delta_0 \qquad p \geq 1$$

1. EQN $\quad -v \dfrac{d\delta}{dx} = -k \dfrac{d^2\delta}{dx^2} + a\,\delta^p \qquad p > 0$

2. re-arrange $\quad \dfrac{d^2\delta}{dx^2} = \dfrac{v}{k}\dfrac{d\delta}{dx} + \dfrac{a}{k}\delta^p$

3. change variables $\quad y = \dfrac{d\delta}{dx}$ note $\dfrac{d^2\delta}{dx^2} = \dfrac{dy}{dx} = \dfrac{dy}{d\delta}\dfrac{d\delta}{dx} = y\dfrac{dy}{d\delta}$

$$y\frac{dy}{d\delta} = \frac{v}{k}y + \frac{a}{k}\delta^p$$

$$\frac{dy}{d\delta} = \frac{v}{k} + \frac{a}{k}\delta^p y^{-1}$$

4. case of $v=0$ $\quad \dfrac{dy}{d\delta} = \dfrac{a}{k}\delta^p y^{-1}$

$$y\,dy = \frac{a}{k}\delta^p\,d\delta$$

$$\frac{1}{2}y^2 = \frac{a}{k}(p+1)^{-1}\delta^{p+1} + \text{const}$$

4. evaluating constant $\quad$ as $x \to \infty$ both $y = \dfrac{d\delta}{dx}$ and $\delta \to 0$

so const $= 0$

5. solve for y

note minus branch of $\sqrt{\ }$

$$y = -\left[\frac{2a}{k(p+1)}\right]^{\frac{1}{2}} \delta^{\left(\frac{p+1}{2}\right)} = \frac{d\delta}{dx}$$

6. second integral $\quad \displaystyle\int \delta^{-\left(\frac{p+1}{2}\right)}\,d\delta = -\left[\frac{2a}{k(p+1)}\right]^{1/2}\int dx$

case $p > 1$

$$\left[1 - \frac{p+1}{2}\right]^{-1} \delta^{\left[1-\frac{p+1}{2}\right]} = -\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \text{const}$$

At $x=0$ $\delta = \delta_0$ so const $= \left[1-\frac{p+1}{2}\right]^{-1} \delta_0^{\left[1-\frac{p+1}{2}\right]}$

$$\delta = \left\{-\left[1-\frac{p+1}{2}\right]\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \delta_0^{\left[1-\frac{p+1}{2}\right]}\right\}^{\left[1-\frac{p+1}{2}\right]^{-1}}$$

at $x=0$ $\delta = \delta_0$ as $x \to \infty$ $x \to 0$

8\. case $p = 1$

$$\int \delta^{-1}\, d\delta = -\left[\frac{2a}{k(p+1)}\right]^{1/2} \int dx$$

$$\ln \delta = -\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \text{const}$$

At $x=0$ $\delta = \delta_0$ so const $= \ln \delta_0$

$$\delta = \delta_0 \exp\left\{-\left[\frac{2a}{k(p+1)}\right]^{1/2} x\right\}$$

$$\delta = \delta_0 \exp\left\{-\left[\frac{a}{k}\right]^{1/2} x\right\}$$

case of velocity dominating $\quad -v\frac{d\delta}{dx} = a\delta^p \quad p>0$

$v>0$

$v<0$ doesn't work since then diffusion never small

$$\frac{d\delta}{dx} = -\frac{a}{v}\delta^p$$

10. integrate $\quad \int \delta^{-p}\,d\delta = -\frac{a}{v}\int dx$

11 case $p>1$ $\quad (1-p)^{-1}\delta^{(1-p)} = -\frac{a}{v}x + \text{const}$

$x=0 \quad \delta=\delta_0$ so $\quad \text{const} = (1-p)^{-1}\delta_0^{(1-p)}$

$$\delta = \left\{\delta_0^{(1-p)} - (1-p)\frac{a}{v}x\right\}^{\frac{1}{(1-p)}}$$

12 case $p=1$ $\quad \int \delta^{-1}\,\delta\delta = -\frac{a}{v}\int dx$

$$\delta = \delta_0 \exp\left(-\frac{a}{v}x\right)$$

13. total dissolution $= \int_0^x a\,\delta^p\,dp$

$p=1 \quad = \int_0^x a\,\delta_0 \exp\left(-\frac{a}{v}\right)dx = \left[-v\delta_0 e^{-\frac{a}{v}x}\right]_0^x = \boxed{v\delta_0\left(1-e^{-\frac{a}{v}x}\right)}$

$p>1 \quad = a\int_0^\infty \left\{\delta_0^{(1-p)} - (1-p)\frac{ax}{v}\right\}^{\frac{p}{1-p}} dx$

let $y = \delta_0^{(1-p)} - \frac{a(1-p)}{v}x \qquad \frac{dy}{dx} = -\frac{a(1-p)}{v}$

$x=0 \;\; y=\delta_0^{(1-p)} \qquad x=\infty \;\; y=-\infty \qquad x=x \;\; y=\delta_0^{(1-p)} - \frac{a(1-p)}{v}x$

$$\frac{v}{a(1-p)} \; a \int_{\delta_0^{(1-p)}}^{\delta_0^{(1-p)} - \frac{a(1-p)}{v}x} y^{\frac{p}{1-p}} \, dy =$$

$$\frac{v}{(1-p)} \int_{\delta_0^{(1-p)} - \frac{a(1-p)}{v}x}^{\delta_0^{(1-p)}} y^{\frac{p}{1-p}} \, dy =$$

$$\left(\frac{v}{p-1}\right)\left(\frac{p}{1-p}+1\right)^{-1} \left[y^{\frac{p}{1-p}+1} \right]_{\delta_0^{(1-p)} - \frac{a(1-p)}{v}x}^{\delta_0^{(1-p)}}$$

$$\frac{p}{1-p} + \frac{1-p}{1-p} = \frac{1}{1-p}$$

$$\cancel{\left(\frac{v}{p-1}\right)\left(\frac{1}{1-p}\right)^{-1}} \left[y^{\frac{1}{1-p}} \right]_{\delta_0^{(1-p)} - \frac{a(1-p)}{v}x}^{\delta_0(1-p)}$$

$$= v \left[\delta_0 - \left\{ \delta_0^{(1-p)} + \frac{a(p-1)}{v}x \right\}^{\frac{1}{1-p}} \right]$$

14. limits $x \to \infty$

$p = 1$ Total dissolution $= v\delta_0$

$p > 1$ Total dissolution $= v\delta_0$

$$\frac{p+1}{2} - \frac{2}{2} = \frac{p-1}{2}$$

$$\frac{p}{\frac{p-1}{2}} = \frac{2p}{p-1}$$

Total dissolution
diffusion only

$$D = \int_0^x a\delta^p\,dx$$

$p=1$ $\quad \delta = \delta_0 \exp\left\{-\left(\frac{a}{k}\right)^{1/2} x\right\}$

$$D = \int_0^x a\delta_0 \exp\left\{-\left(\frac{a}{k}\right)^{1/2} x\right\} dx$$

$$= -a\delta_0\left(\frac{k}{a}\right)^{1/2}\left[\exp\left\{-\left(\frac{a}{k}\right)^{1/2} x\right\}\right]_0^x$$

$$= \delta_0 (ak)^{1/2}\left[1 - \exp\left\{-\left(\frac{a}{k}\right)^{1/2} x\right\}\right]$$

$p>1$ $\quad \delta = \left\{\left[\frac{p-1}{2}\right]\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}\right\}^{-\left[\frac{p-1}{2}\right]^{-1}}$

$$D = a\int_0^x \left\{\left[\frac{p-1}{2}\right]\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}\right\}^{-\frac{2p}{p-1}} dx$$

$$y = \left[\frac{p-1}{2}\right]\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}$$

$$= a\int_{\delta_0^{-\left[\frac{p-1}{2}\right]}}^{\left[\frac{p-1}{2}\right]\left[\frac{2a}{k(p+1)}\right]^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}} y^{-\frac{2p}{p-1}} \left[\frac{2}{p-1}\right]\left[\frac{k(p+1)}{2a}\right]^{1/2} dy$$

$$\frac{\;}{p-1} + 1 = \frac{2p}{p-1} + \frac{1-1}{p-1} = -\left(\frac{p+1}{p-1}\right)$$

$$D = -a \left[\frac{2}{p-1}\right]\left[\frac{k(p+1)}{2a}\right]^{1/2}\left[\frac{p-1}{p+1}\right]\left[y^{-\frac{p+1}{p-1}}\right]_{\text{limits}}$$

$$= \left(\frac{2ka}{p+1}\right)^{1/2} \cdot$$

$$\left\{ -\left[\left(\frac{p-1}{2}\right)\left(\frac{2a}{k(p+1)}\right)^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]} \right]^{-\frac{p+1}{p-1}} + \delta_0^{+\left[\frac{p+1}{2}\right]} \right\}$$

TOTAL DISSOLUTION
limit $x \to \infty$
diffusion only

$p = 1 \quad d_0 (ak)^{1/2}$

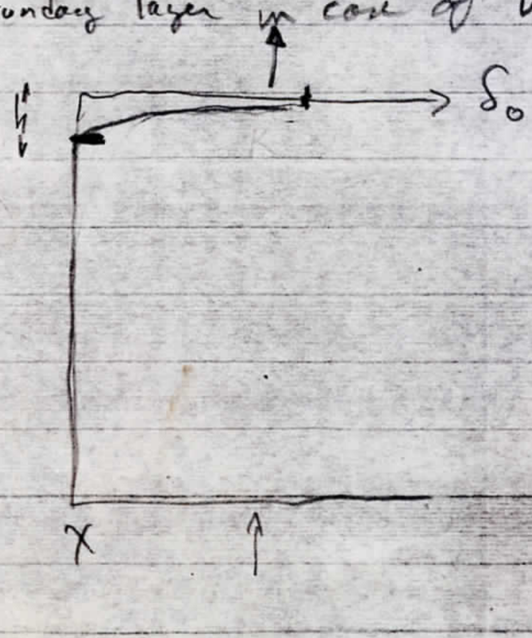
$p < 1 \quad \left(\frac{2ka}{p+1}\right)^{1/2} \delta_0^{\left[\frac{p+1}{2}\right]}$

source $= h_0 \left(\frac{\delta}{\delta_0}\right)^p$ - so $a = h_0 \delta_0^{-p}$

$$D(x) = \begin{cases} \left(\frac{h_0 k}{\delta_0^p}\right)^{1/2} d_0 \left\{1 - \exp\left[-\left(\frac{h_0}{\delta_0^p k}\right)^{1/2} x\right]\right\} & p=1 \\ \left(\frac{2 h_0 k}{(p+1)\delta_0^p}\right)^{1/2} \left\{ d_0^{\frac{p+1}{2}} - \left[\left(\frac{p-1}{2}\right)\left(\frac{2 h_0}{(p+1) k \delta_0^p}\right)^{1/2} x + \delta_0^{-\left[\frac{p-1}{2}\right]}\right]^{-\frac{p+1}{p-1}} \right\} & p> \end{cases}$$

$$= \begin{cases} \left(\frac{h_0 k}{\delta_0}\right)^{1/2} d_0 \left\{1 - \exp\left[-\left(\frac{h_0}{\delta_0^p k}\right)^{1/2} x\right]\right\} & p=1 \\ \left(\frac{h_0 k}{\delta_0^p}\right)^{1/2} \left(\frac{2}{p+1}\right)^{1/2} d_0^{\frac{p+1}{2}} \left\{ 1 - \left[\left(\frac{p-1}{2}\right)\left(\frac{2 h_0}{(p+1) k \delta_0}\right)^{1/2} \delta_0^{\frac{p-1}{2}} x + 1\right]^{-\frac{p+1}{p-1}} \right. & p \end{cases}$$

Boundary layer in case of upward advection



$-\delta_0 v$ = added to column due to advection

$k\frac{\delta_0}{h}$ = lost from column

$$D(x) = k\frac{\delta_0}{h} - \delta_0 v$$

CASE of $p=1$ diffusion & advection

$$k \frac{d^2\delta}{dx^2} - v \frac{d\delta}{dx} - a\delta = 0 \qquad \text{try } \delta = \delta_0 e^{cx}$$

$$c^2 k - cv - a = 0 \qquad c = \frac{v \pm \sqrt{v^2 + 4ka}}{2k}$$

Downward velocity

$$\lim_{v \to 0} c = -\left(\frac{a}{k}\right)^{1/2} \qquad \lim_{k \to 0} c = -\frac{a}{v}$$

$$\delta = \delta_0 e^{-gx} \qquad g = \frac{(v^2 + 4ka)^{1/2} - v}{2k}$$

does $g(v)$ have a minimum or maximum?

$$\frac{dg}{dv} = \frac{\frac{1}{2}(2v)(v^2+4ka)^{-1/2} - 1}{2k} \overset{?}{=} 0$$

$$\frac{v}{(v^2+4ka)^{1/2}} \overset{?}{=} 1$$

$$v^2 \overset{?}{=} v^2 + 4ka \qquad \underline{\text{NEVER}}$$

so no maximum or minimum exists

$$\left.\frac{dg}{dv}\right|_{v=0} = -\frac{1}{2k} \qquad \text{so } g(v) \simeq \left(\frac{a}{k}\right)^{1/2} - \frac{v}{2k}$$

$$\frac{1}{g(v)} \simeq \left(\frac{k}{a}\right)^{1/2} + \left(\frac{v}{2k}\right)\left(\frac{k}{a}\right) \simeq \left(\frac{k}{a}\right)^{1/2} + \frac{v}{2a}$$

$$D(x) = \int_0^x a\delta \, dx = a\int_0^\infty \delta_0 e^{-gx} \, dx$$

$$= \int \frac{a\delta_0}{g}\left[1 - e^{-gx}\right] dx$$

$$D_\infty = \frac{a\delta_0}{g} \approx a\delta_0\left[\left(\frac{k}{a}\right)^{1/2} + \frac{v}{2a}\right]$$

$$= \delta_0 (ak)^{1/2} + \frac{1}{2}\delta_0 v$$

$$\lim_{v \to +\infty} g = \left(\frac{a}{v}\right) \qquad \lim_{v \to -\infty} g = \infty$$