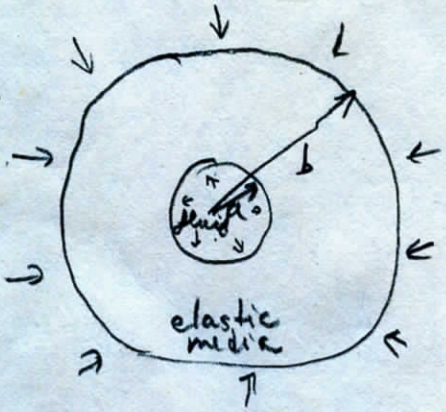
The Displacement and Stress around a Cylindrical hole

from Fung, Y.C., Foundations of Solid Mechanics, Prentice-Hall, Inc, Englewood Cliffs, New Jersey, 1965. pp 243-245.

σ = stress ξ = displacement ϵ = strain

consider



$$\sigma_{rr} = -p \qquad r = b$$

$$\sigma_{rr} = -q \qquad r = a$$

let

$$A = \left[\frac{qa^2}{(1 - a^2/b^2)} - \frac{pb^2}{(b^2/a^2 - 1)} \right]$$

$$C = \left[\frac{2p}{(b^2/a^2 - 1)} - \frac{2q}{(1 - a^2/b^2)} \right]$$

then

$$\xi_r = \frac{1}{E} \left[-\frac{(1+\nu)A}{r} + 2C(1 + \nu - 2\nu^2) r \right]$$

$$\xi_\theta = 0$$

$$\sigma_{rr} = \frac{A}{r^2} + 2C = -p \frac{(b^2/r^2 - 1)}{(b^2/a^2 - 1)} - q \frac{(1 - a^2/r^2)}{(1 - a^2/b^2)}$$

$$\sigma_{\theta\theta} = p \frac{(b^2/r^2 + 1)}{(b^2/a^2 - 1)} - q \frac{(1 + a^2/r^2)}{(1 - a^2/b^2)}$$

ξ = displacement ν = poisson's ratio E = young's modulus now take limit $q \to 0$, $b \to \infty$. we have $A = -pa^2 \quad C = 0$

$$\xi_r = \frac{q(1+\nu)a^2}{E r}$$

$$\xi_\theta = 0$$

$$\sigma_{rr} = \frac{-qa^2}{r^2}$$

$$\sigma_{\theta\theta} = \frac{+qa^2}{r^2}$$

note $\sigma_{rr} + \sigma_{\theta\theta} = 0$ implies no dilatation of material.

$$\sigma_{r\theta} = 0 \quad \text{(by symmetry)}$$

$$\epsilon_{rr} = \frac{\partial \xi_r}{\partial r} = \frac{-q(1+\nu)a^2}{E r^2}$$

$$\epsilon_{\theta\theta} = \left(\frac{1}{r}\frac{\partial \xi_\theta}{\partial \theta} + \frac{\xi_r}{r}\right) = \frac{q(1+\nu)a^2}{E r^2}$$

no net volume strain

$$\epsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial \xi_r}{\partial \theta} + \frac{\partial \xi_\theta}{\partial r} - \frac{\xi_\theta}{r}\right) = 0$$