An attempt to reconcile Saito's and Singh etal's source discontinuities.

saito
$$\delta u = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\,d\omega \int_0^{\infty} k\,dk \sum_{m=0}^{\infty}\left[f_1^L \mathbb{L}_m + \text{etc}\right]$$
$$\delta \tau = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\,d\omega \int_0^{\infty} k\,dk \sum_{m=0}^{\infty}\left[f_2^L \mathbb{L}_m + \text{etc}\right]$$

singh
$$\delta u = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\,d\omega \int_0^{\infty} k\,dk \sum_{m=0}^{\infty}\left[\cancel{2}\pi L_0 C_m \mathbb{C} + \text{etc}\right]$$
$$\delta \tau = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\,d\omega \int_0^{\infty} k\,dk \sum_{m=0}^{\infty}\left[\cancel{2}\pi \mu L_0 k \bar{C}_m \mathbb{C} + \text{etc}\right]$$

so we note:
$$f_1^L = \cancel{2}\pi L_0 C_m$$
$$f_2^L = \cancel{2}\pi \mu L_0 k \bar{C}_m$$

since $L_0 = \dfrac{M_0}{4\pi\mu}$ we have

$$f_1^L = \frac{M_0}{2\mu} C_m$$
$$f_2^L = \frac{M_0}{2} k \bar{C}_m$$

now I will convert Singh's c's into saito's f's. both authors agree that only the following Love modes are non-zero:

m = 1, cosine
m = 1, sin
m = 2, cosine
m = 2, sine

Singh gives the C's as

$$\begin{pmatrix} C_1^c \\ \bar{C}_1^c \end{pmatrix} = \begin{pmatrix} -b_1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=1 \\ c}} = \begin{pmatrix} -\frac{M_0}{2\mu} b_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} C_1^s \\ \bar{C}_1^s \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=1 \\ s}} = \begin{pmatrix} \frac{M_0}{2\mu} a_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} C_2^c \\ \bar{C}_2^c \end{pmatrix} = \begin{pmatrix} 0 \\ 2b_2 \end{pmatrix} \Rightarrow \begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=2 \\ c}} = \begin{pmatrix} 0 \\ M_0 k b_2 \end{pmatrix}$$

$$\begin{pmatrix} C_2^s \\ \bar{C}_2^s \end{pmatrix} = \begin{pmatrix} 0 \\ -2a_2 \end{pmatrix} \Rightarrow \begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=2 \\ s}} = \begin{pmatrix} 0 \\ -M_0 k a_2 \end{pmatrix}$$

$$a_1 = 2 \cos\lambda \cos\delta$$

$$a_2 = \tfrac{1}{2} \sin\lambda \sin 2\delta$$

$$b_1 = 2 \sin\lambda \cos 2\delta$$

$$b_2 = -\cos\lambda \sin\delta$$

now I plug in the definitions of the a's and b's

$$\begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=1 \\ c}} = \begin{pmatrix} -\frac{M_0}{\mu} \sin\lambda \cos 2\delta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=1 \\ s}} = \begin{pmatrix} \frac{M_0}{\mu} \cos\lambda \cos\delta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=2 \\ c}} = \begin{pmatrix} 0 \\ -M_0 K \cos\lambda \sin\delta \end{pmatrix}$$

$$\begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{\substack{m=2 \\ s}} = \begin{pmatrix} 0 \\ -\frac{M_0 K}{2} \sin\lambda \sin 2\delta \end{pmatrix}$$

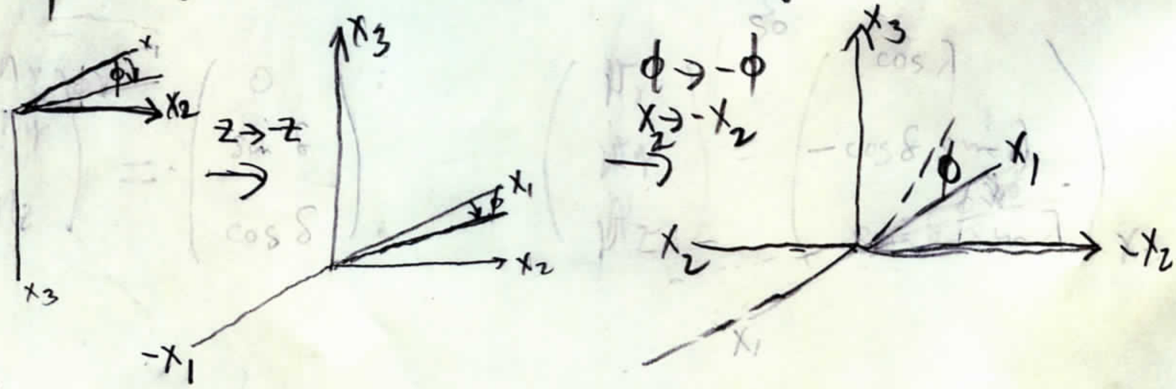
now saito gives these discontinuities in terms of fault normal and fault slip parameters. These are

$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} -\sin\delta \sin\phi_s \\ +\sin\delta \cos\phi_s \\ -\cos\delta \end{pmatrix} \underset{\phi_s=0}{\Longrightarrow} \begin{pmatrix} 0 \\ +\sin\delta \\ -\cos\delta \end{pmatrix}$$

Paul's 7.71 of notes

$$\begin{pmatrix} \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} = \begin{pmatrix} +\cos\lambda\cos\phi_s + \cos\delta\sin\lambda\sin\phi_s \\ +\cos\lambda\sin\phi_s - \cos\delta\sin\lambda\cos\phi_s \\ -\sin\lambda\sin\delta \end{pmatrix} \underset{\phi_s=0}{\Longrightarrow} \begin{pmatrix} +\cos\lambda \\ -\cos\delta\sin\lambda \\ -\sin\delta\sin\lambda \end{pmatrix}$$

but paul's notation is with z^+ downward. Singh's is z^+ upward. so we must change the sign of the z components. But now the coord system is left handed.



so we must change the sign of the x_2 components and chang ϕ to $-\phi$.

This then gives

$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} +\sin\delta \sin\phi_s \\ -\sin\delta \cos\phi_s \\ +\cos\delta \end{pmatrix} \underset{\phi_s = 0}{\Rightarrow} \begin{pmatrix} 0 \\ -\sin\delta \\ +\cos\delta \end{pmatrix}$$

$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} +\cos\lambda\cos\phi_s - \cos\delta\sin\lambda\sin\phi_s \\ \cos\lambda\sin\phi_s + \cos\delta\sin\lambda\cos\phi_s \\ \sin\lambda\sin\delta \end{pmatrix} \underset{\phi_s = 0}{\Rightarrow} \begin{pmatrix} \cos\lambda \\ \cos\delta\sin\lambda \\ \sin\delta\sin\lambda \end{pmatrix}$$

This is now identical to sengh

singh gives

$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} 0 \\ -\sin\delta \\ \cos\delta \end{pmatrix}$$

$$\begin{pmatrix} \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} = \begin{pmatrix} \cos\lambda \\ \sin\lambda\cos\delta \\ \sin\lambda\sin\delta \end{pmatrix}$$

so we can write Saito's f^L's.

$$\begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{m=1,\,c} = \begin{pmatrix} \frac{1}{2\pi\mu}\left(-n_y v_z - n_z v_y\right) \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2\pi\mu}\sin\lambda\,\overbrace{\left(\sin^2\delta - \cos^2\delta\right)}^{-\cos 2\delta} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} f_1^L \\ f_2^L \end{pmatrix}_{m=1,\,s} = \begin{pmatrix} \frac{1}{2\pi\mu}\left(n_z v_x + n_x v_z\right) \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2\pi\mu}\cos\delta\cos\lambda \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} f_2^L \\ f_1^L \end{pmatrix}_{m=2,\,c} = \begin{pmatrix} \frac{k}{2\pi}\left(n_x v_y + n_y v_x\right) \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{k}{2\pi}\sin\delta\cos\lambda \\ 0 \end{pmatrix}$$

note: There is a mistake Saito. This is the corrected term.

$$\begin{pmatrix} f_2^L \\ f_1^L \end{pmatrix}_{m=2,\,s} = \begin{pmatrix} \frac{k}{2\pi}\left(-n_x v_x + n_y v_y\right) \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{k}{2\pi}\overbrace{\sin\delta\cos\delta}^{\frac{1}{2}\sin 2\alpha}\sin\lambda \\ 0 \end{pmatrix}$$

So finally

Singh	Saito
$\frac{M_0}{2\pi}\begin{pmatrix} -\frac{1}{\mu}\sin\lambda\cos 2\delta \\ 0 \end{pmatrix}_{\substack{m=1 \\ \cos}}$	$\frac{M_0}{2\pi}\begin{pmatrix} -\frac{1}{\mu}\sin\lambda\cos 2\delta \\ 0 \end{pmatrix}$
$\frac{M_0}{2\pi}\begin{pmatrix} \frac{1}{\mu}\cos\lambda\cos\delta \\ 0 \end{pmatrix}_{\substack{m=1 \\ s}}$	$\frac{M_0}{2\pi}\begin{pmatrix} \frac{1}{\mu}\cos\delta\cos\lambda \\ 0 \end{pmatrix}$
$\frac{M_0}{2\pi}\begin{pmatrix} 0 \\ -k\cos\lambda\sin\delta \end{pmatrix}_{\substack{m=2 \\ c}}$	$\frac{M_0}{2\pi}\begin{pmatrix} 0 \\ -k\sin\delta\cos\lambda \end{pmatrix}$
$\frac{M_0}{2\pi}\begin{pmatrix} 0 \\ -\frac{k}{2}\sin\lambda\sin 2\delta \end{pmatrix}_{\substack{m=2 \\ s}}$	$\frac{M_0}{2\pi}\begin{pmatrix} 0 \\ -\frac{k}{2}\sin\lambda\sin 2\delta \end{pmatrix}$

Differences :

Factor of M_0 . Saito assumes unit moment. Remedy is to multiply Saito by M_0

factor of $\frac{1}{2\pi}$. I was incorrect in pulling a $(2\pi)^{-1}$ out of Singh's integrand. The $(2\pi)^{-1}$ is implicitly inside the fourier transform.

Remedy is to multiply Singh by 2π .

Conclusions :

$$M_0 f_1^L = L_0 C_m^\sigma = \frac{M_0 C_m^\sigma}{4\pi\mu}$$

$$M_0 f_2^L = k\mu L_0 C_m^\sigma = \frac{k M_0 C_m^\sigma}{4\pi}$$

SAITO = SINGH

so that

$\rho\beta^2 = \mu$

$$\underset{-c}{f_{m=1}} = \begin{pmatrix} -M_0 b_1 / 4\pi\mu \\ 0 \end{pmatrix}$$

$$\underset{-s}{f_{m=1}} = \begin{pmatrix} M_0 a_1 / 4\pi\mu \\ 0 \end{pmatrix}$$

$$\underset{c}{f_{m=1}} = \begin{pmatrix} 0 \\ 2k M_0 b_2 / 4\pi \end{pmatrix}$$

$$\underset{s}{f_{m=1}} = \begin{pmatrix} -2K M_0 a_2 / 4\pi \end{pmatrix}$$