Oldenburg, D. W., One demensional inversion of natural source magneto-telluric observations, Geophysics 44, 1218–1244, 1979.

note resistivity $\rho(z) = \frac{1}{\sigma(z)}$ conductivity

$$\frac{d^2E}{dz^2} + i\omega\mu_0\sigma(z)E = 0$$

$$R = \frac{B}{E} \qquad B = -\frac{1}{i\omega}\frac{dE}{dz}$$

$$\frac{dR}{dz} - i\omega R^2 - \mu_0\sigma(z) = 0$$

homogenous solution $\sigma(z) = \sigma_0$

$$E = E_0\, e^{(i-1)\xi z} \qquad \xi = \left[\tfrac{1}{2}\mu_0\sigma_0\omega_j\right]^{1/2}$$

$$\frac{dE}{dz} = E_0(i-1)\xi\, e^{(i-1)\xi z}$$

$$R = -\frac{1}{i\omega}\frac{dE/dz}{E} = -\frac{1}{i\omega}\frac{(i-1)}{1}\xi$$

$$i-1 = \sqrt{2}\, i\sqrt{i}$$

$$(i-1)^2 = -1-2i+1 = -2i$$

$$(\sqrt{2}\, i\sqrt{i})^2 = -2i$$

so $R = \frac{-\sqrt{2}\, i\sqrt{i}}{i\omega}\xi = \frac{-\sqrt{2}\sqrt{i}}{\omega}\xi$

$$-\left(\frac{1-i}{i}\right) = -\left(\frac{-ii-i}{i}\right) = -(-i-1) = i+1$$

$$R = \frac{i+1}{\omega}\xi = (i+1)\sqrt{\frac{\mu_0\sigma_0}{2\omega}}$$

$$(i+1)^2 = -1+2i+1 = 2i$$

$$R^2 = 2i\,\frac{\mu_0\sigma_0}{2\omega} = \frac{i\mu_0\sigma_0}{\omega}$$

$$+i\omega R^2 = +i\omega\,\frac{i\mu_0\sigma_0}{\omega}$$

$$= -\mu_0\sigma_0$$

$$\frac{dR}{dz} = i\omega R^2 + \mu_0\sigma_0 = 0$$

so given a $\sigma(z)$ over a homogenous halfspace, we can integrate up with $\frac{dR}{dz} = i\omega R^2 + \mu\sigma$

with $R = (i+1)\sqrt{\frac{\mu_0\sigma_0}{2\omega}}$ at ~~the~~ halfspace interface

Numerical Methods for Scientists and Engineers, 2nd Edition, RW Hamming, McGraw Hill, 1973, 721 pp.

p 413 Runga-Kutta Methods, Sec 24.2

$$y' = f(x, y)$$

$$y(x_n) = y_n$$

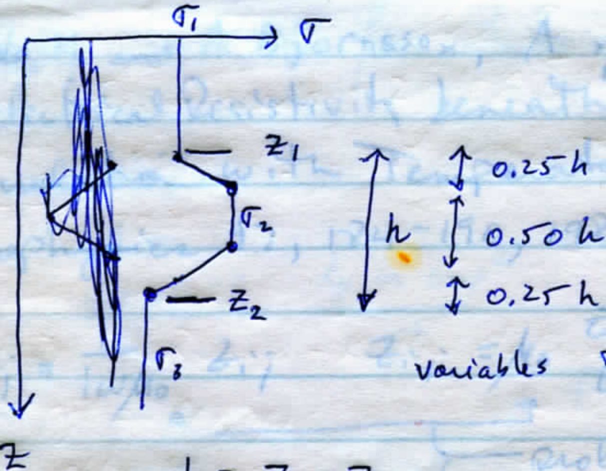
$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$



variables $\sigma_1\ \sigma_2\ \sigma_3$
$z_1,\ z_2$

$$h = z_2 - z_1$$

$z < z_1 \qquad \sigma = \sigma_1$

$z \geq z_1$ and $z < z_1 + 0.25h$

$$\sigma = \sigma_1 + \frac{\sigma_2 - \sigma_1}{0.25h}(z - z_1)$$

$z \geq z_1 + 0.25h$ and $z < z_1 + 0.75h$

$$\sigma = \sigma_2$$

$z \geq z_1 + 0.75h$ and $z < z_2$

$$\sigma = \sigma_2 + \frac{\sigma_3 - \sigma_2}{0.25h}\left(z - (z_1 + 0.75h)\right)$$

$z > z_2 \qquad \sigma = \sigma_3$

Bello, M and A Bjornsson, A model of electrical Resistivity beneath NE iceland, Correlation with Temperature, J. Geophysics 47, 184-190, 1980.

$$C_{ij} = \frac{1}{i\omega\mu_0} Z_{ij} \qquad Z_{ij} = \mu_0 \frac{E_i}{B_j}$$

problem here!

B in nT

E in mV/km

URD

43		resistivity depth
	8	Ωm, km
11		
	12	
25		

period range 20 2000 s

$$\rho_a = \frac{\mu_0 T}{2\pi} |Z_{11}|^2 \qquad \phi_{11} = \arg(Z_{11})$$

$\rho = \Omega m$ ϕ in deg.

units MKSA for Jackson

B tesla

E volts/m

ρ Ω m

μ_0 $4\pi \times 10^{-7}$ $\dfrac{kg\ m\ amp^2}{s^2}$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = \frac{2\pi}{\tau}$$

$$Z = \mu_0 \frac{E}{B} = \mu_0 R^{-1}$$

in homogenous media

$$R^{-1} = \frac{1}{(i+1)} \sqrt{\frac{2\omega}{\mu_0 \sigma_0}} \qquad \sigma_0 = \rho_0^{-1}$$

$$Z = \frac{1}{i+1} \left(\frac{2\mu_0 \omega}{\sigma_0}\right)^{1/2} = \frac{1}{(i+1)} \left(2\mu_0 \rho_0 \omega\right)^{1/2}$$

$$|Z|^2 = \frac{1}{2}\, 2\mu_0 \rho_0 \omega$$

$$\rho = \mu_0 \frac{T}{2\pi} |Z|^2 = \mu_0 \frac{1}{\omega} \left|\frac{1}{i+1}\right|^2 (2\mu_0 \rho_0 \omega)$$

$$= \cancel{\mu_0} \cancel{\frac{1}{\omega}} \cancel{\frac{1}{2}} \cancel{2} \cancel{\mu_0} \rho_0 \cancel{\omega}$$

$$= \mu_0^2$$ This must be wrong.

Handbook of Phys. Prop of R/ p 230

graphite resistivity

36 - 100 $\times 10^{-8}$ Ωm $\parallel$ cleavage

28 - 9900 $\times 10^{-8}$ Ωm $\perp$ cleavage

ohm's Law

$J = \sigma E$

electric field

conductivity

~~[illegible]~~ current density

$\mathcal{E} = R\, I$

current

EMF

resistance

volts Ω amp

$\mathcal{E}/m = \frac{\cancel{\Omega}}{m} m^2 \quad \frac{I}{m^2}$

$\underset{V/m}{E} = (\Omega m) \quad \underset{amp/m^2}{J}$

resistivity Ωm

conductivity $\Omega^{-1} m^{-1}$

Beblo, M and A. Bjornsson, Magnetotelluric investigation of the lower crust and upper mantle beneath Iceland, J. Geophysics 45, 1-16, 1978.

note he define $Z = \frac{E}{B}$

$$\rho_a = \frac{\mu_0 T}{2\pi} |Z|^2 = \frac{\mu_0}{\omega} |Z|^2$$

$$Z = R^{-1} = \frac{1}{(1+i)} \sqrt{\frac{2\omega}{\mu_0 \sigma^2}}$$

$$|Z|^2 = \frac{1}{2} \frac{2\omega}{\mu_0 \sigma_2} = \frac{\omega \rho_0}{\mu_0}$$

$$\rho_a = \frac{\mu_0}{\omega} \frac{\omega \rho_0}{\mu_0} = \rho_0$$

This works, so def of Z in Beblo + Bjornsson (1980) must be off by a factor of μ_0. Looks like he tried patch to make impedance match usual def. + blew

Summary

$$\rho_a = \cancel{\frac{\mu_0}{2\pi}}\ \mu_0 \omega^{-1} |Z|^2$$

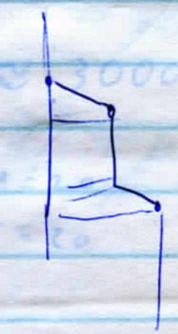
$$\phi = \arg(Z)$$

$$Z = R^{-1}$$

$$\frac{dR}{dz} = i\omega R^2 + \frac{\mu_0}{\rho(z)}$$

ρ = resistivity

$$R_0 = (i+1)\sqrt{\frac{\mu_0}{2\omega\rho_0}}$$



skin depth $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$

$$= \left(\frac{2\rho}{\omega\mu}\right)^{1/2}$$

$$= \left(\frac{4\pi}{T}\frac{\rho}{4\pi\times10^{-7}}\right)^{1/2}$$

$$= \left(\frac{\rho}{T}\right)^{1/2} 10^{7/2}$$

$$= \left(\frac{\rho}{T}\right)^{1/2} 1000\sqrt{10}$$

$$\approx 3000\left(\frac{\rho}{T}\right)^{1/2}$$

$\rho = 20$
$T = 20$ $\qquad \delta = 3000\ m$

$\cancel{y_{n+1} = R y_n}$ -

derivation of Ricotti eqn.

$$R = \frac{B}{E} = -\frac{1}{i\omega}\frac{E_z}{E}$$

$$\frac{d}{dz} E_z + i\omega\mu\sigma E = 0$$

$$E_z = -i\omega E R$$

$$\frac{d}{dz} E_z = -i\omega E R_z - i\omega E_z R$$

$$-i\omega E R_z - i\omega E_z R + i\omega\mu\sigma E = 0$$

$$-i\omega R_z - i\omega \frac{E_z}{E} R + i\omega\mu\sigma = 0$$

$$R_z + \frac{E_z}{E} R - \mu\sigma = 0$$

$$R_z - i\omega R^2 - \mu\sigma = 0$$

$$R_z = i\omega R^2 + \mu\sigma$$

Oldenberg P 1228

$\rho = 10\ \Omega m$: 10

390 240

10

T	R $\times 10^{5}$	my mtz.f program
0.1	0.5359	0.53539
0.178	0.83065	0.83004
0.316	1.2829	1.28361
0.562	1.9563	1.95719
0.100	2.9283	2.92835
0.178	4.2979	4.29577
0.316	6.1806	6.18332
0.562	8.7539	8.75708
1.0	12.234	12.23440
1.78	16.916	16.90856
3.16	23.154	23.16348
5.62	31.517	31.52690
10	42.694	42.69418
17.8	57.625	57.59622
31.6	77.448	77.47714
56.2	103.96	103.99398
100	139.36	139.35899

$$E = e^{(i-1)\xi z} \qquad \xi = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\frac{dE}{dz} = (i-1)\xi\, e^{(i-1)\xi z}$$

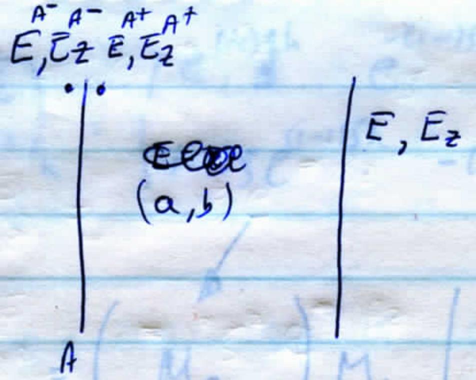
$$\frac{d^2E}{dz^2} = (i-1)^2 \xi^2 e^{(i-1)\xi z} \qquad (i-1)(i-1) = -1 - 2i + 1 = -2i$$

$$\frac{d^2E}{dz^2} + i\omega\mu\sigma E = 0$$

$$-2i\,\frac{\mu\sigma\omega}{2}\, e^{(i-1)\xi z} + i\omega\mu\sigma\, e^{(i-1)\xi z} = 0$$

$$(-i\omega\mu\sigma + i\omega\mu\sigma)\, e^{(i-1)\xi z} = 0$$

yes.



$$E = a e^{+(i-1)\xi z} + b e^{-(i-1)\xi z}$$

$$E_z = (i-1)\xi a e^{(i-1)\xi z} - (i-1)\xi b e^{-(i-1)\xi z}$$

$$E(0) = a + b$$

$$E_z(0) = (i-1)\xi a - (i-1)\xi b$$

$$\begin{pmatrix} E \\ E_z \end{pmatrix}_0 = \begin{pmatrix} 1 & 1 \\ (i-1)\xi & -(i-1)\xi \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \underbrace{\boxed{\frac{1}{-2(i-1)\xi} \begin{pmatrix} -(i-1)\xi & -1 \\ -(i-1)\xi & 1 \end{pmatrix}}}_{M_1} \begin{pmatrix} E \\ E_z \end{pmatrix}_0$$

det

$$-(i-1)\xi - (i-1)\xi$$

$$= 2(i-1)\xi$$

$$\begin{pmatrix} E \\ E_z \end{pmatrix}_h = \begin{pmatrix} e^{(i-1)\xi h} & e^{-(i-1)\xi h} \\ (i-1)\xi e^{(i-1)\xi h} & -(i-1)\xi e^{-(i-1)\xi h} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} E \\ E_z \end{pmatrix}_h = \begin{pmatrix} M_2 \end{pmatrix} M_1 \begin{pmatrix} E \\ E_z \end{pmatrix}_0$$

mt2 program based on this algorithm compares well with Oldenburg's 3-layer result.

28	8	39	18	11	2.6	5.07
26	8	39	18	11	2.5	5.3
26	8	39	15	11	2.1	4.59

26	8	39	15	10	1.97	4.59

URB DATA from Beblo & Bjornson, 1980

T	26.5	ρ_a	ϕ
25	13	30.9	50
35	12.5	29.6	55
50	11	26.0	55
70	10	23.8	52
100	10	23.8	50
150	9.5	22.8	46
220	11	26	45
350	10.5	24.9	43
480	10.3	24.5	46
700	12.5	29.6	45
1000	10.5	24.9	57
1500	11	26	53

2.3025

43 (20-50)

11 (1-15)

25 (10-50)

8 (5-20)

4 (1-20)

initial error $S_a = 2.00$ $\phi = 4.66$

about 80 s for inner 3 loops

43 .8

11 12

25

					ε_{sa}	ε_{ϕ}
~~26~~	~~5~~	~~2~~	~~15~~	~~3~~	~~1.9~~	~~3.93~~
26	5	30	17	4	1.8	4.42
26	6	33	17	6	1.94	4.26
27	5	28	17	5	1.82	4.15
27	5	38	18	5	1.95	4.59
27	6	44	14	7	1.89	4.24

0 0

43

8

27

11

20

18

5

23

25

33

43+11 = 54 / 2 = 27

43 8

11 4

25

using exact data (mt4) ρ_a ϕ

27	10	25	20	5	1.79	3.62
27	11	25	25	5	1.95	4.15

Beblod B. using Mt6

20	10	26	30	3	2.49	6.55