More on the Acceleration of Global Warming

By Bill Menke, October 22, 2023

There is a difference between the questions:

1. "Did the rate of increase in global temperatures increase in 2008?"

and

2. "Did the rate of increase in global temperatures increase sometime around 2008?

Consequently, the statistical tests for significance are different, too. When one inspects at a plot of recent global temperature, one can see what might be an increase in slope in the year 2008.

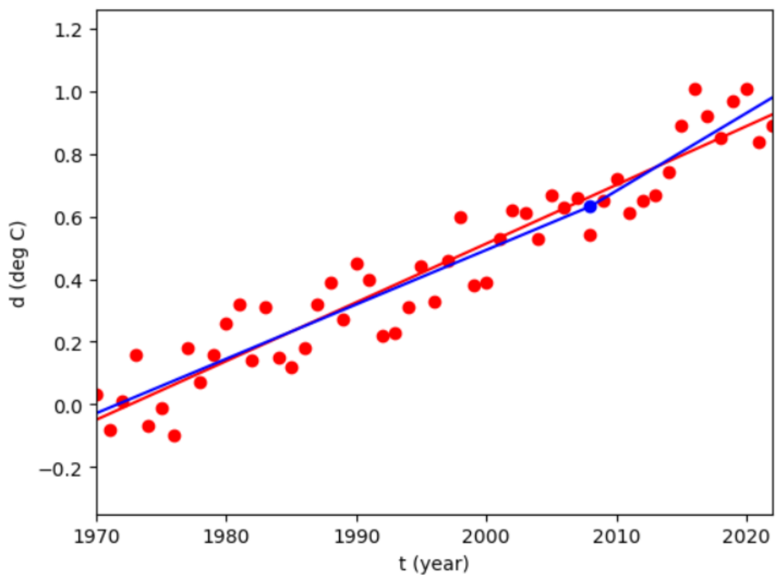


Fig. 1. Global temperature data (red dots), The issue discussed here is whether a fit with a slope break (kink) at the year 2008 (blue) is a statistically better than linear fit (red line)._

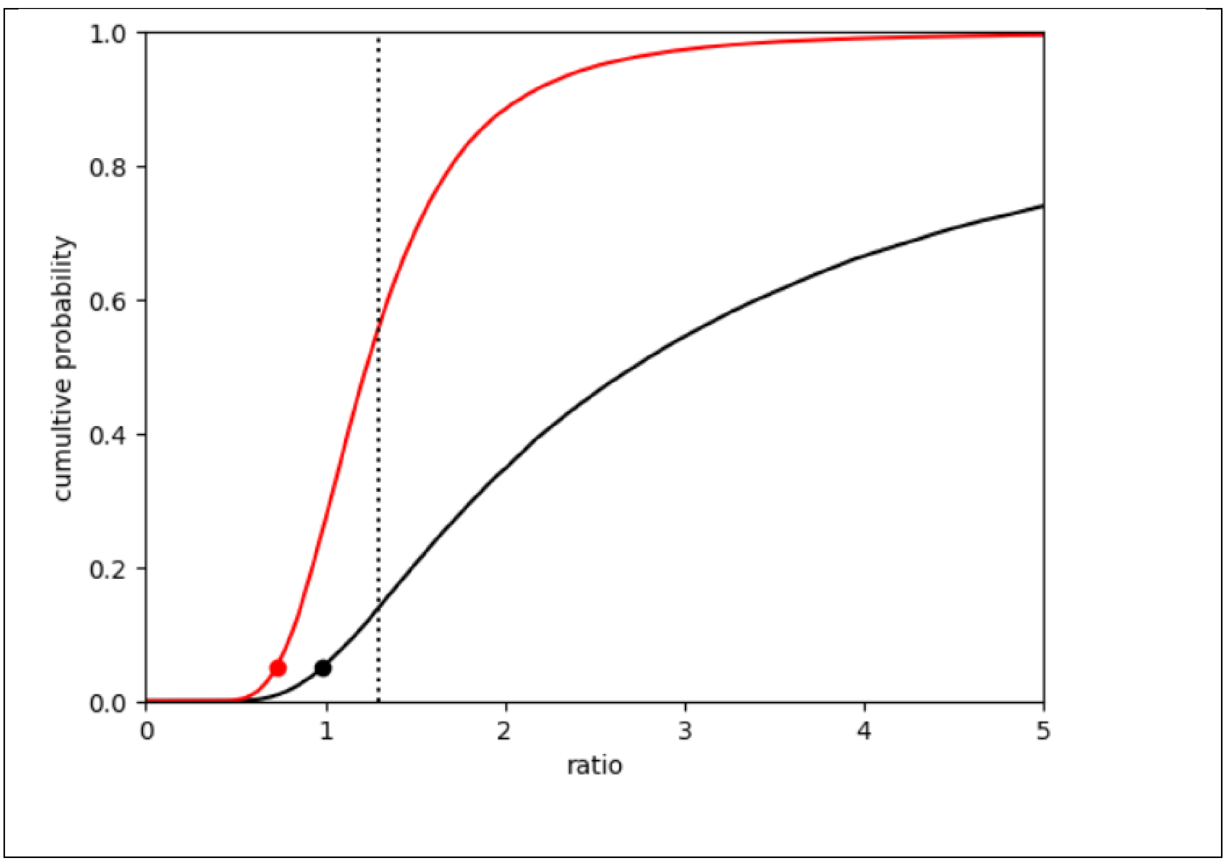
The issue here is how to test the significance of this hypothetical change in slope against the null hypothesis that the change is due to random variation about a linear global warming trend.

Testing the significance of an increase in slope (kink) in the global temperature dataset at a particular year (e.g. 2008, the first question) is straightforward. However, as we have no prior reason to single out the year 2008 as special, this is arguably the wrong test, because, had the dataset appeared to have a kink at a different year, say 2010, we would have focused on that year, instead. Thus, the test really ought to concern the probability of a skink appearing somewhere during a broad time interval (say, 2008 $\pm$ 5 years) near the end of the dataset. The probability of this second scenario is much higher than the first, because there are more ways that random variation can cause it.

The statistic that I use is the ratio, $r = 2\,\sigma_{\Delta m}/\Delta m$, which involves the least square estimate of the change in slope, Δm (for a fixed year of the kink), and the standard error of the slope, $\sigma_{\Delta m}$, as determined by standard error propagation. The latter parameter requires an estimate of the standard error of the data. I use $\sigma_{\Delta m} = 0.0936$ deg C, which is the posterior error of the straight line fit. For the fixed year 2008, the null hypothesis can be rejected with 95% confidence when $r \approx 1$. (I use the approximate sign because $\pm 2\sigma$ does not enclose exactly 95% of the area of a Normal probability density function). The calculated value for the actual dataset (years 1970-2022) is $r = 1.3$, implying that the null hypothesis cannot be rejected with 95% confidence, but only with about 86% probability (a figure arrived at using the Monte Carlo simulation, described below).

However, the 95% threshold for question 2 is about $r = 0.73$, considerably lower than $r \approx 1$, implying that the null hypothesis cannot be rejected to the 95% confidence, but only with 44% confidence (also arrived at using Monte Carlo).

In the Monte Carlo simulation, I generate many (100,000) synthetic global temperature datasets, each of which scatters about a linear trend whose intercept, slope and variance matches the linear fit to the actual data. I then calculate the change in slope, Δm (for a fixed year of the kink), and its standard error, $\sigma_{\Delta m}$, for the years 2003-2013. I tabulate r for 2008, but discard any instances for which $\Delta m < 0$. I also tabulate r', defined as the minimum r for the years 2008 ± 5 years (again discarding any years with negative Δm). Finally, I used histograms to construct empirical cumulative probability distributions, $P(r)$ and $P(r')$.



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Results of Monte Carlo simulation with 100,000 realizations. Cumulative
probability distributions P(r) for r=2*sDd/Dm, where Dm is difference in
slopes and sDm is its standard error. Case A (black): P(r) for the year
2008. Case B (ref): P(r'), where r' is the smallest non-negative ratio for
all kink positions considered. Only positive rs are tabulated. Kinks
between years 2003 to 2013 (inclusive). Posterior standard deviation of
data 0.0936 based on linear fit to 1970 to 2022 data. Estimated r for 5%:
Case A (black dot) 0.9873 Case B (red dot) 0.7274 P(r') for r' at which
P(r)=0.05: 0.2582. For r=1.3, P(r) = 0.1416 and P(r') = 0.5626.
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Although the year 2023 is not yet over, it has been unusually warm through the month of September (the last month for which data currently are available). If one were to use the January-September mean anomaly of 1.096 deg C as a proxy for the whole year (see Note D, below), then repeating the whole calculation, I get $r = 0.9676$ for 1998, and

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Posterior standard deviation of data 0.0949 based on linear fit to 1970 to
2023 data. Estimated r for 5%: Case A (black dot) 0.9773 Case B (red dot)
0.7274. P(r') for r' at which P(r)=0.05: 0.2483 For r=0.9676, For r=0.9676,
P(r) = 0.0490 and P(r') = 0.2387
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So, in this case, the null hypothesis still cannot be rejected for question 2, but can be rejected – but barely – for question 1. As I have been arguing that question 2 is the relevant one, my overall conclusions are unchanged.

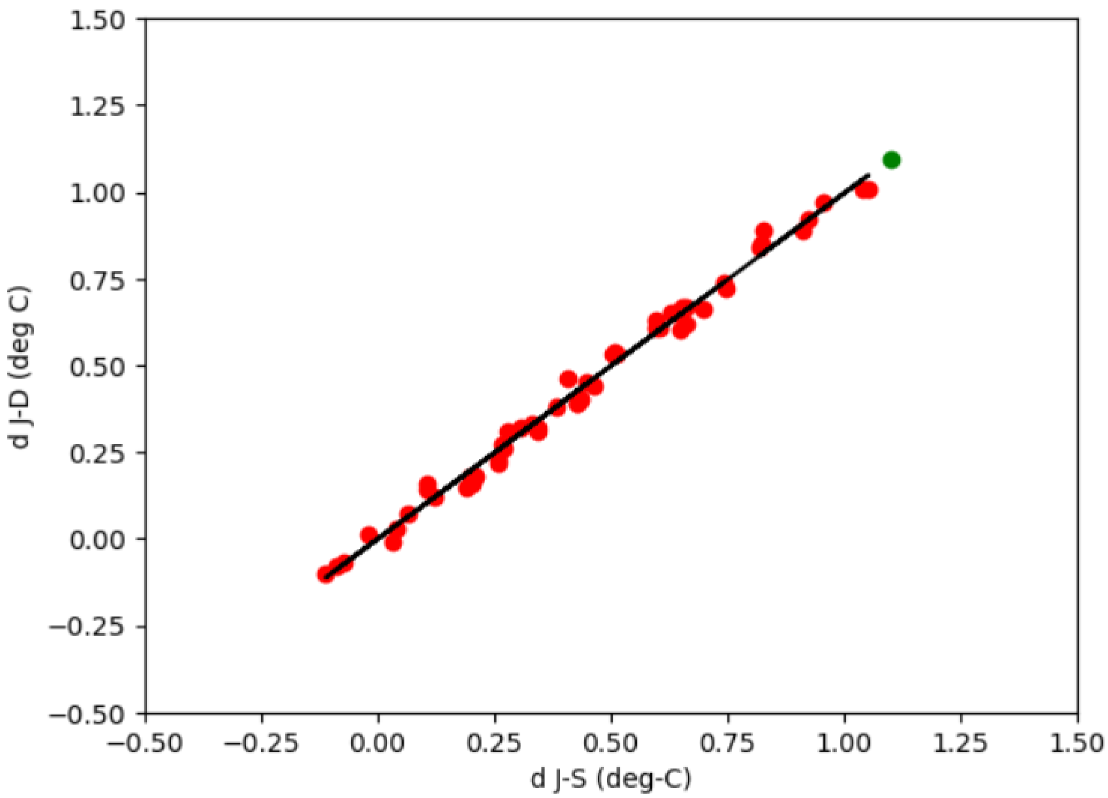
In conclusion, although the actual global temperature data can be fit by a model that has a kink (increase in slope) *in the year 2008*, the null hypothesis that the appearance of a kink *sometime in a broad time interval straddling 20*08 is due to random variation around a purely linear model cannot be excluded at the 95% confidence level. Furthermore, the odds are *close to even* that such a link will appear to be present somewhere towards the end (i.e. 2008 ± 5 years) of a dataset with statistical properties similar to the observed one.

The statistics will improve over the next few years as new data become available. So, we'll just have to wait and see whether our confidence that a change in slope has occurred becomes stronger.

Notes

(A) The 1970-2022 data are from https://data.giss.nasa.gov/gistemp/graphs/graph_data/Global_Mean_Estimates_based_on_Land_and_Ocean_Data/graph.txt
(B) The 2023 data are from https://data.giss.nasa.gov/gistemp/tabledata_v4/NH.Ts+dSST.csv
(C) This work was inspired by my reading: Hausfather, Zeke, I Study Climate Change. The Data Is Telling Us Something New, New York Times, Op-ed Section, Oct. 13, 2023. https://www.nytimes.com/2023/10/13/opinion/climate-change-excessive-heat-2023.htm (who reaches conclusions substantially different from me).

(D) A plot of the yearly mean temperature against the January-September mean is remarkably linear, and with a near-zero intercept and a slope of almost 1:1:



January-December mean temperature vs January-September mean temperature (red dots) for 1970-2023, with simple least squares fit (black) and prediction for 2023 January-December (green), assuming January-September is 1.1 deg C. The trend is remarkably clean, but the statistics of the prediction is complicated and the least squares fit shown here does not correctly capture it.

A simple least squares fit gives

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intercept -0.0017 slope 0.9973
predicted J-D temp for 2023: 1.0952 +/- 0.0096 (95) (deg C) for JS = 1.1000
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However, a simple least-squares fit is not appropriate here, because both axes have error and are inter-correlated. Nevertheless, my guess is that a more correct treatment will give an answer similar to this one and that the true standard error of the predicted annual mean will only be a little bigger than that of “real” annual mean.