THE ROYAL SOCIETY

How to make a rift wide

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Lithospheric necking and magmatic intrusion are thought to be the primary processes responsible for narrow continental rifts. There is much less agreement as to the cause of wide continental rifts. We attempt to shed light on this problem using three approaches. First, we derive approximate relations between the change in force needed for extension and parameters that should control a number of tectonic processes. Processes resulting in a decrease in tectonic force, and so localization of rifting, include: lithospheric necking, magmatic intrusion and loss of brittle layer cohesion. Processes leading to delocalization include viscous flow, local compensation of crustal thickness variations, and regional compensation of extension-related relief. Second, we review the work of others on linear stability analysis applicable to the earliest phase of lithospheric extension. This approach predicts that all conditions lead to wide rift formation and never predicts narrow rift development. Our scaling relations suggest that the pattern of strain for finite extension could be different than that predicted by linear stability theory. Lastly, to test this idea, we carried out a series of numerical experiments for finite-amplitude extension. We find that when strains are quite small the numerical model matches the prediction of linear stability analysis. However, when strains are large a narrow rift can develop. For reasonable model parameters, we get wide rifting only when local crustal buoyancy effects dominate localizing processes. Consistent with our simple scaling relations, this only occurs when the crust is much thicker than the brittle lithosphere. This agrees with observations that wide rifts only occur in regions of higher than normal heat flow with relatively thick crust.

Keywords: lithosphere; viscosity; buoyancy; necking; boudinage

1. Introduction

The more closely we look at areas of continental extension the more we see that all rifts are not created equal. Some form slowly, some quickly. There are wide rifts, narrow rifts, volcanic rifts, asymmetric rifts and failed rifts. A variety of approaches is used to analyse how different forms of continental extension occur. One fruitful method involves specifying a kinematic description of the strain field. Crustal thickness and the thermal field change in response to the proscribed strain. This results in predictions of isostatic subsidence and uplift (e.g. McKenzie 1978). These models have been used to test different assumed strain fields against the observed pattern of subsidence, uplift and heat flow in and around rifts (e.g. Steckler & Watts 1981; Kusznir *et al*. 1987; Buck *et al*. 1988; Driscol & Karner 1998). This method provides constraints on the geometry and history of extension in different areas, but does not address the causes that underlie these differences.

A simple way to study the mechanics of rifting builds on the kinematic models. The initial pattern of lithospheric extension is specified and the changes in the force needed to continue extension are estimated. If the force decreases, then extension should stay localized where it initiated. If the force increases, then it should be easier for the locus of extension to migrate laterally, resulting in a wide rift. The advantage of this approach is that many parameter combinations can be tested in a short time. The disadvantage is that underlying assumptions and approximations may not always be valid. England (1983) used this approach to investigate the cause of wide rifts. He considered the evolution of continental yield strength during extension. England (1983) noted that if the mantle is much stronger than the crust, then very slow extension could lead to a strength increase in the extending area.

Buck (1991) used a similar approach, but included other effects, such as the change in stresses due to localized thinning of buoyant crust. He concluded that unrealistically slow extension rates were needed to give wide rifts for average values of crustal thickness and heat flow. Extension of average lithosphere at a moderate rate gives a narrow rift according to this model. The wide rift mode occurs for moderate to high extension rates when the continental crust was thicker and hotter than normal. However, if the lower crust were extremely hot and weak, then this model predicts a different pattern of extension. In this 'core complex mode', the upper crustal extension remains fixed in one place while the lower crust thins over a broad region.

A different kind of model approximation is used for the perturbation, or linear stability, analysis of lithospheric extension. This approach also permits testing of many parameter combinations. The linearization of the model equations, implicit in these calculations, may mean that these results are strictly applicable only to the earliest phase of extension when strains are small. As discussed below, these models predict only two forms of extension of an initially uniform lithosphere, both of which could be termed a wide rift. The lithosphere extends either in a uniform pure shear mode, or with laterally periodic variation in the rate of extension. This second type of extension, often called 'lithospheric boudinage', is suggested to explain the development of a series of basins and ranges in a broad region of continental extension (e.g. Fletcher & Hallet 1983; Martinod & Davy 1992).

Thus there is a contradiction between the results of the approximate finite-amplitude extension models, which predict a wide rift only for a restricted range of conditions, and the linear stability analyses, which predict wide rifts essentially for any initial conditions.

To reconcile the predictions of these different model approaches we consider a small set of model calculations which involve fewer approximations than either the simple finite-amplitude models or the linear stability models. These 'full' numerical models of lithospheric extension have been developed by a number of workers (e.g. Braun & Beaumont 1989; Dunbar & Sawyer 1989; Bassi 1991; Christensen 1992). The main limitation in these numerical experiments is that only a few parameters can be treated in a reasonable amount of computer time. Our numerical experiments were set up to mimic the rheological conditions assumed by one of the early linear stability analyses. Thus, we can do two things at once. We can check the small strain behaviour of our numerical model against the predictions of previous work. Then we can show whether the finite-amplitude behaviour of the models is consistent with the linear stability models, with the approximate finite-amplitude models, or with neither.

In the first part of the paper, we give an overview of processes affecting the pattern of surface deformation, subsidence and uplift in areas of continental extension. In the next section we describe linear stability models for 'lithospheric boudinage'. In the last section we compare the results of linear stability analysis with finite-amplitude numerical models for extension.

2. Processes affecting continental extension

Several factors can affect where extensional strain is concentrated in a continental region. Extension may, and probably does, nucleate in a site of a previous weakness. The question for us is whether extensional strain remains concentrated in that initial site or migrates to other places. We first discuss three processes that may allow deformation to remain concentrated and then look at three things that can lead to delocalization of strain. Here we will not consider propagation of rifting orthogonal to the direction of extension, but we will concentrate on simple two-dimensional models concerned with the vertical plane parallel to the direction of extension.

The first three processes may lead to localization of deformation either in a region that could be described as narrow (here narrow means about as wide as the lithosphere is thick) or to strain localization on a fault. These localizing processes are illustrated in figure 1. The other mechanisms may lead to a wide region of extension and they are illustrated in figure 2. Below we derive approximate relations between physical parameters, like the thickness and strength of the lithosphere and changes in the force needed for continued extension in the light of the different processes.

(*a*) *Necking*

The most obvious thing that may keep extension focused in one area is the localized thinning of lithosphere as it extends. Extension implies that material points horizontally offset from each other move apart. Conservation of mass requires vertical motion in response to this lateral extension. This implies some upward movement of the material within the lithosphere and therefore advection of heat. If this heat advection is significantly faster than thermal diffusion, then isotherms at the base of the lithosphere should move up. The lithosphere is thinned and so it is weakened. This process of 'necking' is self-reinforcing. The weaker the thinned or necked area the more concentrated, and perhaps more rapid, the extension. The more concentrated the extensional strain the faster the rift becomes weaker.

To quantify the weakening for a given amount of lithospheric thinning we must estimate the lithospheric strength. The stress needed to extend cold, brittle lithosphere may vary mainly with depth while the stress needed to extend hot, flowing crust and mantle depends on temperature and composition. Formally, the difference between the maximum and minimum principal stresses required to cause rock to deform at a given depth, the yield stress, is taken often to be the lesser of two stresses: the stress to produce brittle failure or the stress to cause ductile flow at a specified strain rate. Following Brace & Kohlstedt (1980), the brittle stress is taken to increase linearly with depth at a rate that depends on friction, rock density and pressure in any pore fluid present in the rock. Byerlee (1978) showed that many different rock types have

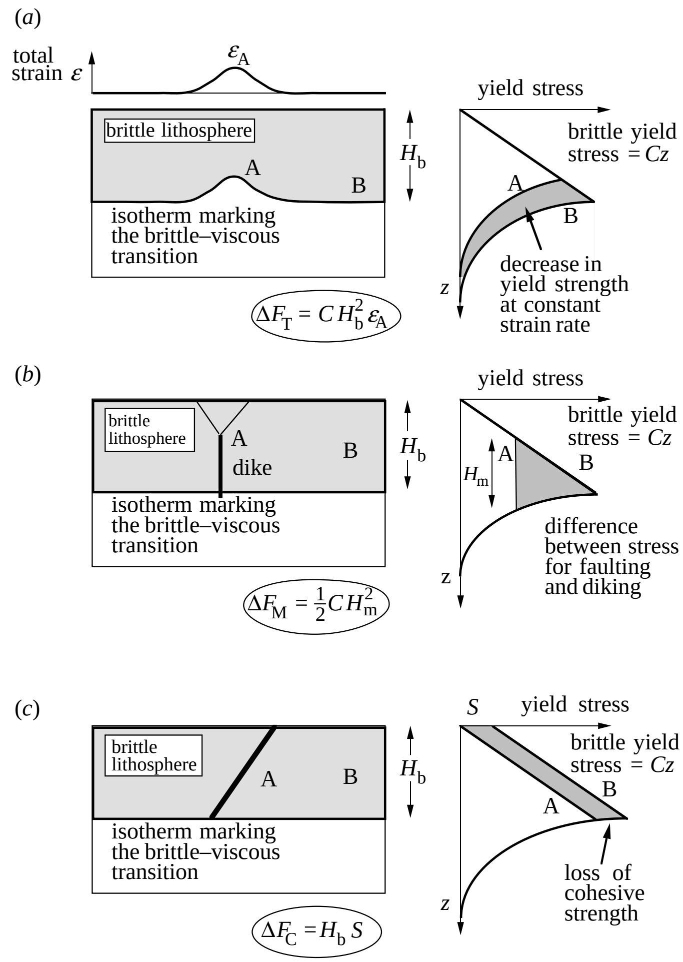

Figure 1. Schematic illustrations of three processes that may lead to localization of strain during lithospheric extension. Plots to the right show the approximate distribution of yield stress with depth both at the centre of a rift (A) and at an area unaffected by the rifting (B). The difference in the two curves is marked with vertical hatchers and that area is proportional to the change, here reduction, in the tectonic force needed for continued rifting. The scaling of these force changes with parameters defined in table 1 are given within ovals. See text for further explanation.

nearly the same coefficient of friction, μ; many in the range between 0.6 and 0.8. Using this result, Brace & Kohlstedt (1980) estimate that for zero pore pressure the brittle yield for extension increases at *ca.* 22 MPa km^{-1}, while for pore water at hydrostatic pressure it increases at just over 12 MPa km^{-1}. Implicit in their analysis is the assumption that one of the principal stresses is vertical (Anderson 1951).

At high temperature, rocks can flow in response to deviatoric stress without forming macroscopic fractures. For such ductile flow the stress σ_{d} and strain rate, $\dot{\varepsilon}$, are found to be related through a flow law of the form:

$$\sigma_{\mathrm{d}} = (\dot{\varepsilon}/A)^{1/n} \exp(E/nRT), \tag{2.1}$$

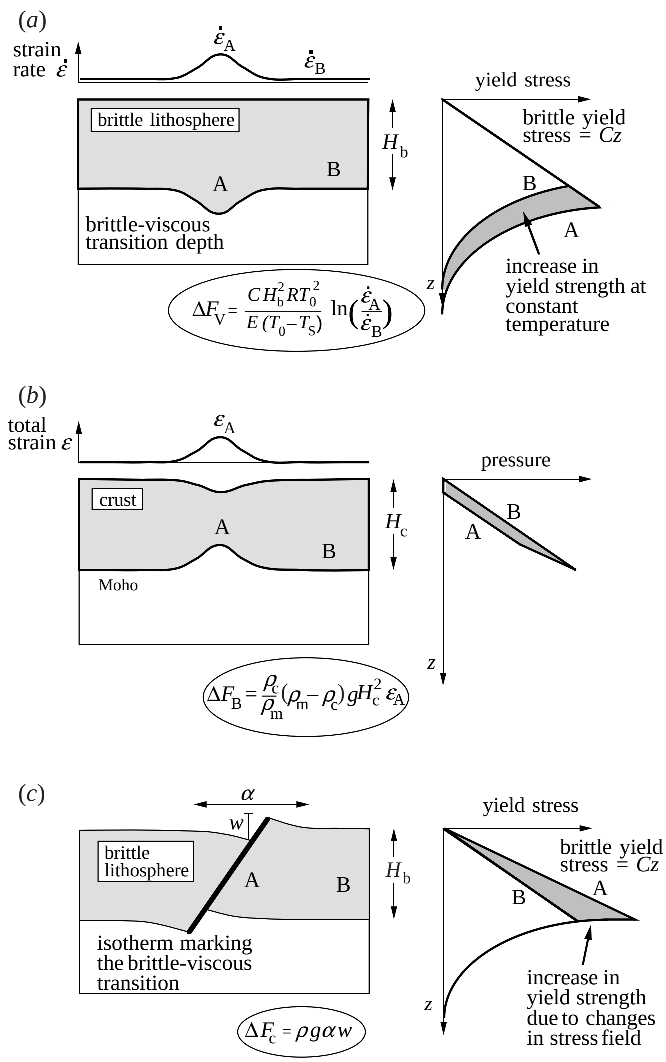

Figure 2. Same as figure 1 except that three processes causing delocalization of extensional strain are pictured.

where T is the absolute temperature, R is the universal gas constant, and E is the activation energy (e.g. Goetze & Evans 1979). Table 2 gives the constants E, A and n for some minerals that may control the rheology of the crust and mantle.

The horizontal force required to cause extensional yielding of the entire lithosphere is estimated by integrating yield stress over depth. The integrated yield stress, or lithospheric strength, depends strongly on the temperature profile and the crustal thickness (if the crust is weaker than the mantle; see Kohlstedt *et al.* 1995; Mackwell *et al.* 1998). To extend lithosphere with a typical heat flow, of *ca.* 50 mW m^{-2} (Pollock & Chapman 1977), may require as much as 3×10^{13} N m^{-1} of force to be applied, while to deform lithosphere with twice that heat flow can take less than 10^{12} N m^{-1} of force.

In figure 1*a* we estimate the amount of reduction in the yield strength for a given amount of concentrated extensional strain. In doing this we ignore the effect of strain rate on yield stress, which is treated in the section below on viscous stresses. The key result is that the first-order reduction in strength depends on the strain times the initial strength of the lithosphere.

Thermal diffusion works against necking. Cooling tends to return isotherms perturbed by advection to their pre-extensional configuration. Naturally, the importance of diffusion to advection of heat depends strongly on the rate of deformation (e.g. Cochran 1983). England (1983) suggested that slow extension could in fact lead to a strengthening of the lithosphere. This strengthening requires very low extension rates and that the yield strength for the lower crust is much lower than that for the mantle. White (this issue) suggests that England's (1983) mechanism might explain the abandonment of many intracratonic rifts that seem to have extended at very low strain rates. Sawyer (1985) has questioned whether the crust–mantle strength contrast is likely to be as large as that assumed by England (1983). Specifically, Sawyer noted that the brittle failure stress may be independent of the rock type so that the contrast in crust and mantle strength is limited. Also, if the crust and mantle are dry, then the viscous yield strength of the crust may not be significantly different from that of the mantle (Kohlstedt *et al*. 1995; Mackwell *et al*. 1998).

(*b*) *Magmatic intrusion*

Separation of lithospheric plates requires extensional stresses at a rift. Besides fault slip and viscous flow, those stresses may allow dike intrusion (figure 1*b*). Dikes may intrude at a lower stress difference than needed for brittle and viscous deformation. Dikes are magma intrusions with a thickness much smaller than their width or length. Dikes should form in planes perpendicular to the least principal stress, σ_3; for a rift, this should be in vertical planes parallel to the rift (Anderson 1951).

To estimate the stresses needed for dikes to intrude all or part of the lithosphere we make several simplifying assumptions. We assume pre-existing vertical fractures to avoid the complications of fracture mechanisms (e.g. Rubin & Pollard 1987). We also neglect the viscous stresses associated with the flow of magma in a dike, since the goal is to estimate the minimum stress difference required to have magma stop and freeze at a given depth in a dike.

Before freezing, magma in a dike can cease moving up or down when the static pressure in the dike equals the horizontal stress at the dike wall (Lister & Kerr 1991). The vertical pressure variation in a static column of magma is related to its specific weight, so for magma emplacement $\partial\sigma_3/\partial z = \rho_{\mathrm{l}} g$, where ρ_{l} is the liquid magma density. For an extending area, a reasonable assumption is that the stress in the vertical or z-direction is the largest principal stress and equals the lithostatic stress (Anderson 1951). Then the stress difference, or magmatic yield stress, varies vertically with a gradient $\partial(\sigma_1-\sigma_3)/\partial z = g(\rho_{\mathrm{r}}-\rho_{\mathrm{l}})$, where ρ_{r} is the rock density. For the case pictured in figure 1*b* the rock density and magma density are the same. Then the stress difference for crustal diking does not change with depth. The reduction in tectonic force is then dependent on the thickness, H_{m}, of dikes that are intruded. This should depend on the flux of magma into a given area compared to the rate of extensional separation. Excess magma (i.e. $H_{\mathrm{m}} > H_{\mathrm{b}}$) should permit extension at very low tectonic force levels and may produce extrusion of magma.

Many simple rifting models (e.g. McKenzie 1978; Buck 1991) and complex ones (e.g. Braun & Beaumont 1989; Bassi 1991; Dunbar & Sawyer 1989) assume that the average stress or tectonic force required to initiate rifting is available. This may not be true, however, for rifting of thick, strong lithosphere in the absence of magmatism. Several authors have estimated that the tectonic forces likely to be available for rifting are in the range of 3–5 $\times 10^{12}$ N m^{-1} (Forsyth & Uyeda 1975; Solomon *et al*. 1975). The tectonic force needed for amagmatic extension of initially thick lithosphere may be up to an order of magnitude greater (Kusznir & Park 1987; Hopper & Buck 1996).

Areas of initially thin lithosphere should rift at very low levels of tectonic force. Further, some areas of high heat flow and initially thick crust, such as the North American Basin and Range Province, started extending with little or no basaltic volcanism. Models neglecting magmatism do predict the general patterns of observed extensional strain inferred for such areas (Buck 1991). It should be noted that these 'hot' weak areas are not typical, but may require some kind of preheating. The effect of orogenesis, especially thickening of radiogenic crust, has been implicated as a way of heating regions such as the Basin and Range and the Aegean Sea extensional provinces (e.g. Sonder *et al*. 1987). However, in areas of low to normal heat flow, the earliest phase of rifting is often accompanied by basaltic magmatism.

The association of magmatism and rifting in areas of low to normal heat flow is probably not coincidental. Magmatic intrusion may allow rifting to proceed given available tectonic forces.

(*c*) *Cohesion loss*

Within a rift, extension is not smoothly distributed, at least close to the near surface, but is concentrated on normal faults. For faults to accommodate so much strain, they must be weaker than the surrounding less deformed rock. One possibility is that faults have lost some or all of the cohesion of the surrounding rocks.

The failure of brittle materials, such as cold rock, has been described by a number of criteria. We assume the Coulomb–Navier criterion, which states that failure occurs when the shear stress exceeds cohesion, S, plus a friction coefficient, μ, multiplied by the normal stress. In deriving the brittle yield stress above, we followed Brace & Kohlstedt (1980) and others, who adopt this criterion with $S = 0$.

When the yield criterion is reached, the rock may break as cohesion is lost. The optimum orientation of faults is controlled by the friction coefficient. For total cohesion loss, the yield stress on optimally orientated faults is reduced by an amount equal to $2S/[(1+\mu^2)^{1/2}/2+\mu]$. For μ close to 0.75 the reduction in yield stress is approximately equal to S. Laboratory measured values of rock cohesion, also known as inherent shear strength, range from almost zero for weak sediments to nearly 50 MPa for some igneous rocks (Handin 1966). The existence of large vertical cliffs requires cohesion in large volumes of rock that are of order 10 MPa.

If strain weakening results in non-cohesive faults, then their yield stress would be weaker than the surrounding rocks by approximately S. Multiplying this cohesion drop by the thickness of the brittle layer gives the amount of reduction in the yield strength of a fault compared to unfaulted rock. One of the main effects of cohesion loss is that faults may be able to slip a finite amount before being replaced by new faults (Forsyth 1992). As we will see below, the offset of a dip-slip fault produces changes in the stress field that make it harder for the fault to move further.

(*d*) *Viscous flow*

By definition, the strength of viscous material is proportional to strain rate, or strain rate raised to a power. This means that when viscous material strains faster its flow stress is larger. This results in stress being delocalized in a viscous layer. A thought experiment can help illustrate this effect. Assume that the strain rate is greater in one region, as shown in figure 2*a*. Since the viscous flow stresses are greater in the region of high strain rate, the transition between brittle and viscous behaviour (sometimes called the brittle–ductile transition) is deeper there. Clearly, where the transition depth is deeper the total yield strength is greater. Thus, the area straining the fastest will be the strongest.

The amount of deepening of this transition and the lithospheric strengthening depend on two things: the contrast in strain rate and the length-scale for changes in viscosity. At constant strain rate the vertical distance Z_e over which the viscosity changes by a factor e (2.72) is related to the temperature gradient in the region of the transition, $\mathrm{d}T/\mathrm{d}z$, and to the temperature there, T_0, as $Z_e = nRT_0^2/(E\mathrm{d}T/\mathrm{d}z)$. To get the expression given in figure 2*a* we assume constant stress at the transition depth and then estimate change in that depth for a given contrast in strain rate. Also we assume a linear temperature gradient through the lithosphere so that $\mathrm{d}T/\mathrm{d}z = (T_0 - T_\mathrm{s})/H_\mathrm{b}$.

It is the viscous stress effect that can lead to both folding and boudinage structures in layered rocks (Ramberg 1955; Biot 1961; Smith 1977). It is this viscous delocalization that has been suggested to contribute to the boudinage-like structure of the wide Basin and Range Province of the western United States (Fletcher & Hallet 1983), as discussed below.

(*e*) *Local (crustal) isostasy*

Local isostasy is an idealized description of how lithosphere floats on underlying fluid asthenosphere. The term 'local' implies that the surface elevation at a point depends only on the average density of the column of lithosphere below that point. Essentially, the shear stress on vertical planes is taken to be zero. However, horizontal stresses should be continuous across vertical planes. Vertical stresses at a given depth should depend on the topography and average density of material in a column. Thus, where there is topographic relief in local equilibrium, there will be non-zero stress differences $(\sigma_1 - \sigma_3)$. For example, an elevated area will be in relative tension. Conversely, a low area will be in relative compression as material tries to flow into it.

Localized extension results in crustal thinning. Because crust is less dense than underlying mantle, local crustal thinning should produce lowered elevations in the centre of a rift (e.g. McKenzie 1978). This puts the centre of the rift into relative compression, and this makes continued extension harder.

To estimate the magnitude of this effect we follow previous workers (e.g. Artemjev & Artyushkov 1971; Fleitout & Froidevaux 1983) and assume that the wavelength of crustal thickness variations is large compared to crustal thickness. Then, the increase in the tectonic force for extension due to crustal thinning, ΔF_c, equals the integral over depth of the difference in lithostatic pressure, ΔP. This pressure difference equals $\Delta w \rho_0 g$, where Δw is the change in surface elevation between the centre of the rift and the adjacent, unrifted area. Here ρ_0 is the density of crust and g is the acceleration due to gravity. Topographic relief Δw in local isostatic equilibrium has an amplitude of $\varepsilon_a H_0(\rho_m - \rho_0)/\rho_m$, where ρ_m is the density of the mantle and $\varepsilon_a H_0$ is the amount of crustal thinning at the centre of a rift. As long as the strains are not too large then the crustal buoyancy force, ΔF_c approximately equals $\Delta P H_0$, as expressed in figure 2*b* in terms of primary parameters.

Just as thermal diffusion may act to diminish the effect of thermal advection during lithospheric necking, lower crustal flow can act to even out crustal thickness variations and so reduce the crustal buoyancy effect. The idea that lateral crustal flow may be important in some areas is discussed in Block & Royden (1990), Bird (1991), and the effect of lower crustal flow on crustal buoyancy is extensively discussed in Buck (1991).

Isostatic topography in a rift can be caused by temperature differences. However, the thermal buoyancy force change is usually much smaller than that due to crustal thinning (Buck 1991). This occurs because density changes due to a temperature change of 500 °C are 10 times smaller than the density difference between crust and mantle. However, when the lithosphere is thick, thermal buoyancy effects can be of larger magnitude than the crustal buoyancy force change because the depth extent of temperature anomalies can be much greater than the depth to Moho, as noted by Turcotte & Emmerman (1983) and LePichon & Alvarez (1984). Thermal and crustal buoyancy are of opposite sign for a rift.

(*f*) *Regional isostasy*

The deflection of an elastic plate changes stresses within that plate. Note that Fletcher & Hallet (1983) considered this regional buoyancy effect in their model of lithospheric boudinage. Deflection of the lithosphere by offset along a fault is one of the easiest ways to illustrate this effect. Fault offset results in stress changes around and on the fault. Forsyth (1992) argues that those stress changes should inhibit continued offset on the fault. Eventually it could be easier to break a new fault in adjacent lithosphere than for continued slip on an original fault.

Figure 2*c* illustrates this effect. The scaling between the force and the vertical deflection of the lithosphere is based on Forsyth's (1992) analysis of a thin perfectly elastic layer. This force is directly proportional to the horizontal wavelength of the response of the lithosphere to vertical loads, represented in terms of the flexural parameter, α. It also depends on the vertical offset, w, across the fault.

By including the finite yield stress of lithosphere, Buck (1993) showed that finite offset of a fault would reduce the wavelength of flexural response. This resulted in a large decrease in force resisting continued fault offset as compared to the purely elastic prediction. Based on this result, Buck (1993) suggested that very large offset normal faults could result from extension of a thin brittle layer. This thin layer also had to have a high value of cohesion, S, away from the fault. Extension of a thicker or less cohesive layer should result in normal faults with offsets that are smaller compared to the lithospheric thickness.

Recently, Lavier *et al*. (1998), have used a finite-element approach to simulate extension of a brittle layer overlying an inviscid substrate. They have confirmed the prediction of Buck (1993) that the ratio of cohesion to layer thickness controls whether small or large offset normal faults develop. However, they show that another parameter, the rate of reduction of cohesion with strain, can also strongly affect the pattern of faulting developed in an extended layer.

Other factors may have a considerable effect on the geometry of extension, but they are even harder to quantify than the factors discussed here. For example, erosion and sedimentation may reduce both the crustal buoyancy process and the effect of resistance to strong layer offset. This could therefore diminish those delocalizing effects. However, it can be very difficult to quantify the rate of erosion or to specify either the distribution of sediment or its density.

3. Wide rifting and lithospheric boudinage

There has been considerable discussion of the physical processes that lead to a delocalized mode of rifting, which is sometimes called 'wide rifting'. As noted above, England (1983) suggested that slow extension could locally increase the yield strength. According to him, as well as other workers (Kusznir & Park 1987; Sonder & England 1989), this strengthening effect explains the occurrence of wide regions of continental extension like the Aegean Sea and the Basin and Range Province. However, observations do not seem to confirm this prediction since some wide rifts did not extend at a slower velocity than did some narrow rifts (see Buck 1991). Other workers suggested that viscous effects may lead to wide rifting (Bassi 1991), or that the buoyancy forces resulting from local (crustal) isostasy (Buck 1991) could produce wide regions of extension.

In the remainder of the paper, we investigate the possible causes of wide rifting. We do this by comparing the results of new finite-amplitude numerical models of extension with the predictions of the linear stability analysis of Fletcher & Hallet (1983). In doing so, we will focus on the competition between lithospheric necking and the three delocalizing processes described above: viscous stresses, local (crustal) isostasy and regional isostasy. One goal is to determine whether the predictions of the stability analysis, which are rigorously valid for only small deformation, continue to hold when deformation is large. Another specific prediction of interest concerns the conditions under which lithosphere undergoing extension would deform with a pattern of periodically varying high and low strains, sometimes called 'lithospheric boudinage'.

(*a*) *Linear stability analysis*

Fletcher & Hallet (1983) and Zuber *et al*. (1986) propose that such large-scale boudinage is seen in the fairly regular distribution of basins and ranges seen in the Basin and Range Province of the western United States. Further, they suggest that viscous stresses play a large role in producing lithospheric boudinage. Fletcher & Hallet (1983) base their suggestion on a linear stability analysis of the deformation of an effectively brittle layer overlying a half-space with a viscosity that decreases exponentially with depth. Viscous stresses and regional isostasy are considered as two different mechanisms leading to delocalization. Linear stability analysis allows the study of the competition between those delocalizing effects and the localizing effect of lithospheric necking.

Fletcher & Hallet (1983) predict only two possible outcomes of extension, as illustrated in figure 3. Either the deformation is uniform and homogeneous or it results in a boudinage structure. The deformation style depends on two parameters. One is the ratio of the brittle layer thickness, H_{b}, to the characteristic distance for viscosity

Table 1. *Parameter definitions and representative values used*

symbols and parameters		units and values used for numerical modelling
ε	strain	—
$\dot{\varepsilon}$	strain rate	s^{-1}
τ	shear stress	Pa
μ	friction coefficient	0.6
S	cohesion on brittle layer	20 MPa
H_b	thickness of brittle layer	km
H_c	crustal thickness	30 km
H_m	thickness of lithosphere intruded by dikes	km
g	acceleration due to gravity	$9.8\ m\ s^{-1}$
α	flexural parameter	km
ρ_m	mantle density	$3300\ kg\ m^{-3}$
ρ_c	crustal density	$2700\ kg\ m^{-3}$
Z_e	depth scale for changes in viscosity by a factor of e	km
τ_b	shear stress at yield for the base of the brittle layer	MPa
T_s	temperature at the surface of the brittle layer	283 K
T_0	temperature at the brittle–ductile transition	K
dT/dz	initial temperature gradient with depth	$40\ K\ km^{-1}$
n_c, n_m	power-law exponent for creep in crust, mantle	3, 3.6
E_c, E_m	activation energy for creep in crust, mantle	see table 2
A_c, A_m	pre-exponent for creep in crust, mantle	see table 2

change with depth, Z_e (see table 1). The other is the ratio of $\rho_c g H_b$ to the average shear strength of the layer, τ_b.

The scaling we derived above may give some insight into why these parameters control the model behaviour. The localizing tectonic force reduction due to necking should be proportional to $H_b \tau_b$. Note that this is different from the expression in figure 1*a* because we assume that the average shear strength of the brittle layer depends on its thickness (i.e. that $\tau_b = C H_b$). The delocalizing viscous stress effect scales with $\tau_b Z_e$. To make a comparison with the expression given in figure 2*a* recall that $Z_e \approx H_b (R T_0^2)/E(T_0 - T_s)$. The parameter H_b/Z_e is proportional to the ratio of the necking force decrease to the viscous force increase. Likewise, $\rho_c g H_b/\tau_b$ can be thought of as a ratio of the local isostatic effect to the necking effect.

Small values of H_b/Z_e indicate large viscous stresses relative to the necking effect and so promote delocalization. Large values of $\rho_c g H_b/\tau_b$ reflect large regional isostatic resistance to localized deformation of the brittle layer and also would be delocalizing. Here we assume that the wavelength of regional deformation, μ, scales with the brittle layer thickness, H_b. Also, strain, ε, scales with vertical fault offset, w, divided by H_b. When the brittle layer is strong (small $\rho_c g H_b/\tau_b$), and thick (large H_b/Z_e), localizing processes are larger and the necking effect should become dominant.

Fletcher & Hallet (1983) predict uniform extension when weakening due to necking is relatively small compared to viscous stress plus regional isostatic effects. When the necking effect is relatively larger, linear stability analysis predicts boudinage. The

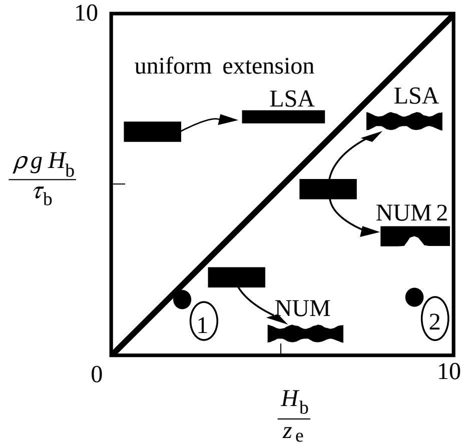

Figure 3. Predictions of the linear stability analysis of Fletcher & Hallet (1983) as functions of two parameters. In the upper left region they predict that lithospheric stretching should produce uniformly distributed strain while in the lower right region stretching should result in alternating zones of high and low strain which they term lithospheric boudinage. The vertical scale is a ratio of factors that affect buoyancy (regional isostatic) force changes over the shear stress at yield for the base of the brittle layer. The horizontal scale is the ratio of the brittle layer thickness to the length-scale for changes in the viscosity defined at a constant strain rate. This ratio depends on the activation energy for viscous flow, as described in the text. The points are for the parameters used in two of our numerical models of finite-amplitude extension. Case 1 gave a boudinage type of strain pattern for both small and large strains. Case 2 gave boudinage for small strains, but the deformation eventually concentrated in one place, forming a 'narrow rift'.

rate of growth of the boudinage instability depends on how far into the boudinage field a case plots. For cases lying far to the right of the heavy line in figure 3, the boudinage would grow quickly, while for cases closer to the uniform extension field the instability would grow slowly.

Linear stability analysis does not predict a narrow rift mode of extension. Since many rifts, like the Rhine graben and the Red Sea, are apparently narrow, one wonders what situations lead to their development as compared to wide rifts like the Basin and Range. Zuber & Parmentier (1986) and Bassi (1991) show that a large initial perturbation in lithospheric thickness or strength might cause localization in a single narrow rift, as opposed to a sequence of basins and ranges. However, we would like to investigate the conditions under which a narrow rift developed even without a pre-existing large perturbation. A complementary motivation for the numerical experiments described below is to determine the conditions likely to lead to a wide rift like the Basin and Range, given laboratory and geological estimates of important parameters.

(*b*) *Numerical method and rheology*

We approach this problem by using a numerical method that allows mechanically consistent interactions among elastic, plastic, and viscous deformation. This method is based on an explicit Lagrangian finite-element scheme similar to the fast Lagrangian analysis of continua (FLAC) technique (Cundall 1989). It permits us to

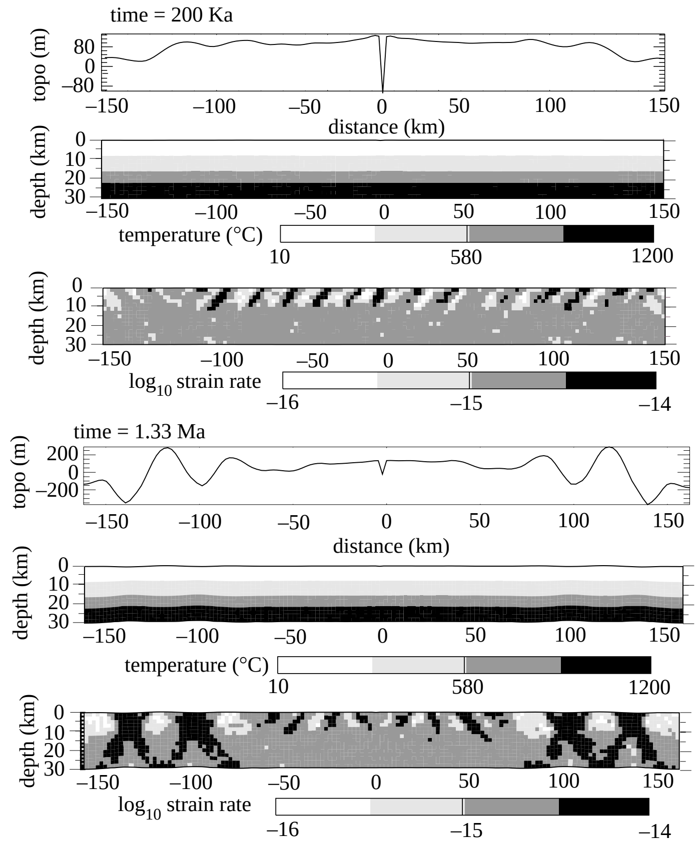

Figure 4. Results of numerical case 1 for two model times. From top to bottom the panels show local isostatic topography, temperatures and the instantaneous strain field for each time. The spike in the centre of the topography plot results from an initial perturbation in the initial strength field. A grid element in the middle of the model was set to have no cohesion, while all other elements have 20 MPa of cohesion. Note that the first 2 km of extension (occurring in 200 ka) result in small topographic undulations that grow in amplitude after more than 25 km of extension.

obtain a direct solution of Newton's second law for every grid point including the effects of inertia. However, inertial stresses are kept low compared to characteristic tectonic stresses in order to approximate quasi-static processes. Inertial effects are reduced by limiting the boundary velocities relative to sound velocities. This method has been used to simulate localized deformation (approximating faults) in elasto-plastic materials (Hobbs & Ord 1989; Cundall 1989; Poliakov & Herrmann 1994; Poliakov *et al*. 1994; Hassani & Chery 1996), visco-elastic flow (Poliakov *et al*. 1993) and visco-elasto-plastic diapirism (Poliakov *et al*. 1996).

In the models described below, the shallow, cold part of the lithosphere is treated as an elastic material as long as stresses are below those required to cause faulting. Elastic effects are not treated in the linear stability studies of extension. In our model elastic behaviour is described in terms of Lamé's parameters: which are both set equal to 3×10^{10} Pa. Faults, or regions of concentrated plastic deformation, develop spontaneously wherever stresses reach a brittle yield criterion. Warm regions behave in a viscous manner with no localization of strain. The thermal field is coupled to the deformation field. We do not prescribe where the transition between brittle faulting and viscous flow takes place. Instead, the brittle–ductile transition zone arises from the calculation of the stress state in the model. It is located where the brittle yield stress equals the viscous flow stress. This position depends on material properties such as the activation energy, which controls the temperature dependence of viscous stresses, and the friction coefficient and cohesion which control the magnitude of brittle stresses (see tables 1 and 2).

Formally, these rheologies are implemented by allowing either elasto-plastic or visco-elastic deformation. The word 'plasticity' as used here describes the Mohr–Coulomb plasticity theory (e.g. Vermeer 1990; Rudnicki & Rice 1975). Briefly, this theory is based on two assumptions: one is that a brittle yield stress governs when a material fails (here it is assumed to be linearly proportional to the pressure) and the second is a flow rule for material at the yield point (we assume that it is incompressible, i.e. material does not change volume during plastic deformation).

(*c*) *Boundary and initial conditions*

At the surface, shear and normal stresses are assumed to be zero. At the side boundaries we apply constant horizontal velocities and zero shear stress. Due to the decrease of the viscosity with depth, a perfect fluid is a good approximation of a material with very low viscosity. Thus, we use hydrostatic boundary conditions at the bottom of our model region (i.e. a Winkler foundation): we apply normal stress equal to the hydrostatic pressure at the bottom boundary and zero shear stress. The position of the boundary changes during a simulation as the layer is stretched and thinned. When any element has been deformed by over 200%, we generate a new grid and interpolate properties and stresses from the old to the new grid. The bottom boundary is moved back to the original position with each regridding.

The temperature is fixed at 10 °C at the top surface, the conductive heat flux is zero at the side boundaries. On the bottom boundary the temperature is maintained at the initial value set there. The initial temperature profile is linear with depth with a gradient of 40 °C km^{-1}.

The numerical domain is either 30 or 40 km thick in the vertical dimension and 300 km wide. The grid elements are always 2 km by 2 km. One grid element at the top of the middle (horizontal direction) of the numerical domain is set to have zero cohesion. This small perturbation is meant to promote localized extension far from the boundaries.

(*d*) *Results*

Here, we report results of three illustrative model calculations. The first two are done for comparison with the results of linear stability analysis. We use a characteristic distance for viscosity change with depth, Z_e, a brittle layer thickness, H_b, and an average shear strength of the layer, τ_b, that are comparable to those considered by Fletcher & Hallet (1983). The parameters defining cases 1 and 2 are given in tables 1 and 2 and in figure 3. Figures 4 and 5 show results for two time-steps during the evolution of the model cases. For each time-step, they include the topography, the temperature and strain rate fields.

Both cases 1 and 2 have a value of $\rho_c g H_b/\tau_b$ that is close to that predicted from laboratory estimates of lithospheric strength (e.g. Brace & Kohlstedt 1980). Here we assume that τ_b is the shear stress at yield at the base of the brittle layer. The cases differ only in the value of H_b/Z_e. The value of H_b/Z_e for case 2 is based on laboratory estimates of typical crustal and mantle rocks (e. g. Kirby & Kronenberg 1987), while for case 1 the value of this parameter is set to an artificially small value. It should be noted that neither of these non-dimensional parameters depends strongly on brittle layer thickness or on temperature gradient.

As shown in figure 3, case 1 is in the parameter range which should produce slowly growing boudinage according to linear stability analysis, while case 2 is in the range predicted to give rapidly growing boudinage. For cases 1 and 2 (figures 4 and 5), the early stages of the calculations (i.e. after 200 ka and 4 km of extension) are consistent with the predictions of the linear stability analysis. At that time the pinches and swells for case 1 have developed a small amount of topography compared to case 2. For large strains (i.e. 25 km of extension) the early-formed pinches and swells in case 1 have grown in amplitude without a major change in shape. We would expect the pinches and swells to develop uniformly across the model domain, but they are not well developed in the centre of the region. This may be due to the initial perturbation causing some localized deformation in the centre of the domain, which inhibits the growth of boudins.

For case 2, the small strain deformation pattern for 4 km of extension (figure 5) shows horizontally periodic variations in the topography and the strain rate field, again consistent with the linear stability analysis. However, the results at large strains are markedly different from the predictions of the linear theory. In this situation strain is concentrated in one narrow region, which progressively narrows as the lithosphere necks.

The viscosity parameters (table 2) for case 2 are consistent with laboratory estimates. Thus, these results for stretching of a compositionally uniform lithospheric layer suggest that regional isostatic and viscous stress effects are dominated by the necking effect. A single lithospheric layer should extend to form a narrow rift. Then how do we get a wide rift?

In case 3, we analyse the effect of a density contrast between crust and mantle in a model with a reasonable rheological structure. Gravitational stresses related to the thinning of the crust should now contribute to the delocalization of strain. We find that adding the effect of crustal isostasy leads to a different pattern of deformation than for case 2 (figure 6). Extension never concentrates in one place; rather it is distributed in a boudinage pattern for small and large strains.

4. Conclusions

We have discussed different processes that may affect rifting and looked at models that include some of these effects. We have been able to show that a finite-amplitude numerical model, which treats all of these processes, gives results that are consistent with linear stability analyses for small strains. This is useful since it gives us more confidence that the numerical models are correctly implemented. Also, we show that the linear stability analysis may not predict the pattern of extension developed with finite strain models. For extension of a single strong lithospheric layer, with properties consistent with laboratory estimates, we get a narrow rift. This suggests that narrow

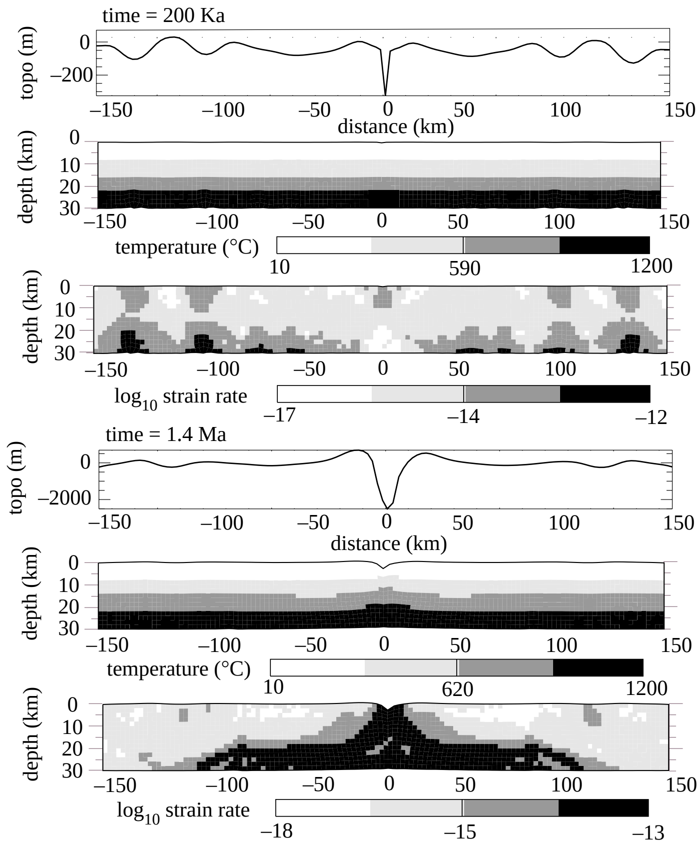

Figure 5. Similar to figure 4 except that case 2 with a larger value of H_b/Z_e is shown. This value of H_b/Z_e is more consistent with expected values of these parameters. In this case the early development (at 4 km offset) shows more of a tendency for discrete zones of high strain spaced at fairly regular intervals, as compared to case 1. Recall that for these parameters the linear stability analysis of Fletcher & Hallet (1983) predicts boudinage development. However, further offset led to localization of deformation in one place (as the site of the initial small weakness) and a narrow rift with flanking highs develops.

Table 2. *Numerical model parameters*

parameters used in the models	case 1	case 2	case 3
E_c (kJ mol^{-1})	50	276	276
E_m (kJ mol^{-1})	—	—	535
A_c (Pa s)$^{1/n}$	2.9×10^{-37}	3×10^{-20}	3×10^{-20}
A_m (Pa s)$^{1/n}$	—	—	7.2×10^{-18}
H_b/Z_e	1.7	9.4	9.4[a]
$\Delta\rho g H_b/\tau_b$	1.5	1.5	1.5

[a]Note there is brittle behaviour in crust and mantle, but the given value applies only for the upper crustal brittle layer and crustal viscosity profile.

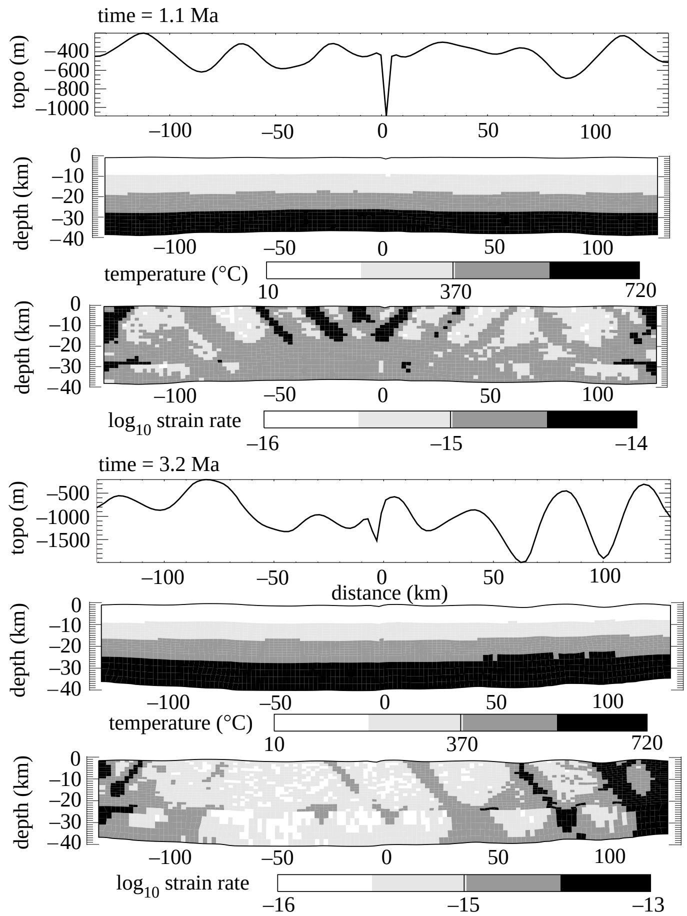

Figure 6. Illustration of results for case 3. The main difference between this and case 2 is that a crustal density contrast is assumed between 30 km thick crust and underlying mantle. The ration $H_{\rm b}/Z_e$ for the near-surface brittle layer is similar to that for case 2. However, the inclusion of the crust/mantle density contrast leads to stabilization of the early-formed boudinage even for large amounts of extension.

zones of extension, such as narrow rifts and core complexes, may form without large pre-existing perturbations in strength.

Our numerical results should be considered preliminary, since we have not yet thoroughly tested the effect of initial perturbation size, location of the bottom boundary, or looked at a wide range of parameters. However, the fact that the small-amplitude results are similar to that predicted by linear stability theory and the large-amplitude results are consistent with approximate scaling relations, gives us some confidence in our calculations.

It appears that estimates of the size of various localizing and delocalizing effects, based on approximations discussed in the first part of the paper, can be useful in predicting the behaviour of finite-amplitude numerical models. Thus, approximate models such as Buck (1991) and Hopper & Buck (1996) may be a valid, though rough guide to expected lithospheric behaviour.

Considering these results we conclude that wide rifting and lithospheric boudinage require that the local isostatic stress effect, sometimes called crustal buoyancy, dominates over the lithospheric necking effect. Unless our estimates of rheological parameters are grossly incorrect, viscous stress and regional isostatic delocalizing effects are too small to control wide-rift evolution. For rifting in our models requires brittle lithosphere that is thinner than the crust. This only occurs when the regional heat flow is greater than normal. This is consistent with observations of the conditions under which wide rifts develop.

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Discussion

K. Rohr (*Stable Geophysics, North Saanich*). Over what sort of width are these abyssal hills predicted to be still active?

W. R. Buck. This question refers to a previously published model of faulting at mid-ocean ridges (Buck & Poliakov 1998). The width of the zone of active faulting depends on two things: the regional isostatic topography and the viscosity structure below the brittle lithosphere. We always initially get inward (toward the axis) faults that cut the thinnest lithosphere. As axial valley relief builds, we get both inward and outward dipping faults, which are active to the edge of the axial valley. We ascribe these temporal variations in faulting to changes in the gravitational stresses related to flexure of the lithosphere. Viscous flow stresses also may affect the width of the region of faulting. The model viscosity depends on temperature with the relationship controlled by an activation energy. The viscous effect causes a wider region of faulting only if the rate of deepening of isotherms with distance was very small or the activation energy was set to a value lower than that estimated from laboratory measurements.

J. Jackson (*Bullard Laboratories, University of Cambridge, UK*). In Professor Buck's simulations of fault evolution, do new faults preferentially form in the footwalls or hanging walls of the older faults, and is there any reason for this?

W. R. Buck. We see both jumping into the footwall and into the hanging wall of the first formed fault, depending on the rate of strain weakening. When the rate of strain weakening is small, we see that second faults form in a variety of locations, often in the footwall. When the weakening is fast we always get second faults in the hanging wall. We think about it like this. The first fault is weakened progressively as it is offset. The weakening reduces the regional stress difference needed for continued fault offset. However, fault offset also bends the lithosphere and changes the stress field around the fault in a way that requires increased regional stress for continued fault slip. When the rate of strain weakening is very small then the regional stresses increase as the fault is offset and a new fault can occur almost anywhere. When the rate of strain weakening is very large, the first fault loses all its strength before it has been offset very far. Regional stresses go down below the level needed to break through the cohesive brittle layer. The deflection of the layer results in bending stresses, which on the top of the hanging wall add to the extensional regional stresses to reach the yield stress. If the cohesion is lost at very small strain, then an incipient fault can propagate down to cut through the hanging wall. The location of the second fault seems to be at the position of maximum bending of the hanging wall, much as Vening Meinesz predicted for the formation of grabens.

J. Jackson. All Professor Buck's strength profiles show an increase in strength with depth within the (seismogenic) upper crust. Is there any evidence that this is, in fact, true? The increases predicted by Byerlee's law (or its variants) are substantial, and not, I think, seen in earthquake stress drops.

W. R. Buck. For faulting, I think what's important is really just a ratio of a strength drop to whatever residual strength there is. We have never tried anything with uniform background strength, but my sense is that with different magnitudes of weakening we could still get small- or large-offset normal faults. I do not think we understand the factors that affect fault patterns well enough to claim to constrain the depth dependence of brittle strength based on fault observations. We are in better shape in terms of making inferences based on the large-scale pattern deformation. In effect, it's only when you can compare to something like crustal buoyancy, which we can constrain to about a factor of two, that we will have a hold on the magnitude of lithospheric strength. The observation of narrow rifts argues that necking-related strength variations dominate over crustal buoyancy effects. These strength variations depend on the average strength of the lithosphere. Our models would not give narrow rifts in continental regions without a mean yield stress that increases with depth. The strength may not have to increase as fast as has been implied for typical rock friction coefficients.

A. Chadwick (*British Geological Survey, UK*). At what initial dip does Professor Buck's modelled faults form in unstrained crust, i.e. in intact crystalline rock? Does he believe that new normal faults can form with low dips (*ca.* 30°) in intact crystalline rock?

W. R. Buck. They form about the way we would expect, given the model parameters, with initial dips of about 60°. In some cases we see higher near-surface fault dips. However, we have yet to sort out what is physical and what is numerical about the near-surface deformation. Many people suggest that normal faults may become steeper near the surface. We tend to see steeper faults in numerically difficult cases where significant strength is lost for very small strains. We have never obtained initially low-angle faults.

A. Chadwick. Professor Buck's model assumes that cohesion decreases with strain, which is essentially saying that faults are weaker than unfaulted rock. Clearly, this would be an important factor in subsequent fault reactivation. In terms of the Mohr–Coulomb failure criterion, does he think that the weakness of faulted rock is primarily due to a low 'cohesive strength' or to a low coefficient of internal friction?

W. R. Buck. Some people have suggested that cohesion should be lost at very low strain and that friction may be reduced gradually with larger strain. We have not studied strain-dependent friction. When we started this work we did not foresee the importance of the rate of weakening with strain. Since we started to look at cohesion loss we decided to do a thorough study of this case before moving onto more general cases.

M. Osmaston (*Woking, UK*). I have a little problem with the necking and multiple boudinage story. My original training was as an engineer and you could make a steel test specimen as uniform as you could possibly do and yet at a certain point in the strain rate it would suddenly start to neck. So my feeling is that since we all know that the mantle is essentially non-uniform, there must be some places in it at which sudden necking like that would occur. When that happens it is going to be self-concentrating because of the temperature dependence of the viscosity. Under those circumstances I cannot see that multiple boudinage will happen; rather the whole strain is going to be concentrated into the place that has necked.

W. R. Buck. Well, actually I think I must have gone through our results too quickly because that's indeed what the finite-amplitude calculations show. Consider what we see for numerical extension of lithosphere with properties which, according to linear stability theory, should produce boudinage. We did initially see boudin-like structures form, but as deformation proceeded all the extension concentrated in one location. This is due to exactly the reason you describe about thermally dependent strength and necking. When we put in crustal buoyancy we still get delocalized extension for a geologically significant length of time.

N. Kusznir (*University of Liverpool, UK*). Has Professor Buck examined the effect of strength heterogeneity in the first set of models that he described? One can imagine that if you put in a random distribution of strength within certain limits, then the heterogeneity would start to dominate the system. The question is, what would we expect those limits to be? One might expect the strength heterogeneity to eventually 'kill' the boudinage.

W. R. Buck. In a way that is what is happening. The initially small-strength variations grow with strain. When one area gets weak enough the initial phase of distributed, boudinage-type deformation stops and a narrow rift forms. We have yet to analyse the amplitude of strength variations needed to do this.

J.-P. Brun (*Université de Rennes, France*). I have a problem with your definition of necking because boudinage can also result from necking instability. I agree, however, with Professor Buck's analysis when he says that boudinage and multiple boudinage are a function of the strain rate increasing the viscous stresses. But, returning to the point about wide rifts versus narrow rifts, I do not understand how the balance between necking and boudinage controls the difference between the two types of extension. This difference could simply be due to the absence or presence of a high-strength sub-Moho mantle that 'localizes' the deformation. In the former case, the highest strength is at the base of the upper crust.

W. R. Buck. First, I think of boudinage as arising from a competition between viscous flow stresses and advective weakening (necking). In that case the viscous flow stresses are the main thing leading to a wide rift. We found that such finite-amplitude lithospheric boudinage only occurred when our activation energy for viscous flow was much smaller than laboratory estimates. We concluded that the effect of crustal thickness variations was important to produce a wide rift. Second, what you say about wide versus narrow rifting is true, given the way we parametrize rheology. Namely, strong mantle below weaker crust can cause localized deformation. However, it is possible that dry crust may be as strong as dry mantle over a range of temperatures. Dry, strong crust does not preclude a transition from narrow to wide rifting, but it would change the conditions of crustal thickness and thermal structure that would mark the transition.

A. Roberts (*Badley Earth Sciences Ltd, Lincs, UK*). One of Professor Buck's successful models showed a 'rolling hinge' mechanism for generating core complexes. If this model is applied to the Basin and Range, why did the rolling hinge stop moving, to be replaced by the 'typical' steeper faults active at the present day?

W. R. Buck. A possible reason is that the hot lower crust is thinned and perhaps cooled during extension. This would lessen the ability of the lower crust to flow into the region of core complex extension. Therefore, the crust would thin locally and the surface would subside in the area of the rolling-hinge fault. Crustal thinning produces a buoyancy force that resists continued local extension. It may then be easier to break the upper crust in another area. This is the mechanism we think leads to a wide rift in our numerical model calculations.

L. Géli (*IFREMER, France*). I would like to point out that we need to consider not only the forces that cause the extension but also the context within which the extension takes place. So, for instance, when the Red Sea is compared with the Basin and Range, the first thing that is striking is that in the two cases there is a very different kinematic situation in terms of plate tectonics and in terms of how the stresses are probably distributed. So, an important factor is that when you neglect magmatism, you can neglect it of course locally, but you cannot neglect accretionary phenomena in general, because as soon as there is extension new crust is accreted.

W. R. Buck. Great statement! I imagine that basaltic and granitic magmatism might have very different effects. Basalts might intrude as shallow-level dikes that preclude faulting while granites might intrude at depth and facilitate faulting at shallow depths. I'd love to be able to say more, but I do not understand enough about magmatism to do so.

Additional reference

Buck, W. R. & Poliakov, A. N. B. 1998 Abyssal hills formed by stretching oceanic lithosphere. *Nature* **392**, 272–275.