Factors controlling normal fault offset in an ideal brittle layer

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Abstract. We study the physical processes controlling the development and evolution of normal faults by analyzing numerical experiments of extension of an ideal two-dimensional elastic-plastic (brittle) layer floating on an inviscid fluid. The yield stress of the layer is the sum of the layer cohesion and its frictional stress. Faults are initiated by a small plastic flaw in the layer. We get finite fault offset when we make fault cohesion decrease with strain. Even in this highly idealized system we vary six physical parameters: the initial cohesion of the layer, the thickness of the layer, the rate of cohesion reduction with plastic strain, the friction coefficient, the flaw size and the fault width. We obtain two main types of faulting behavior: (1) multiple major faults with small offset and (2) single major fault that can develop very large offset. We show that only two parameters control these different types of faulting patterns: (1) the brittle layer thickness for a given cohesion and (2) the rate of cohesion reduction with strain. For a large brittle layer thickness (>22 km with 44 MPa of cohesion), extension always leads to multiple faults distributed over the width of the layer. For a smaller brittle layer thickness the fault pattern is dependent on the rate of fault weakening: a very slow rate of weakening leads to a very large offset fault and a fast rate of weakening leads to an asymmetric graben and eventually to a very large offset fault. When the offset is very large, the model produces major features of the pattern of topography and faulting seen in some metamorphic core complexes.

1. Introduction

The natural faults that either bound or cut an asymmetric or symmetric graben in rifts show a wide range of offsets [e.g., Roberts et al., 1991; Morley, 1995]. The offset varies from microscopic slip on incipient faults to ~5 km slip on rift basin bounding faults [e.g., Vening Meinesz, 1950; Ebinger et al., 1987] up to several tens of kilometers on metamorphic core complexes structures [e.g., Coney, 1980; Karson et al., 1987; Davis and Lister, 1988; Wernicke, 1985; Lister et al., 1986, 1991; Cann et al., 1997; Tucholke et al., 1998]. Fault with offsets smaller than the thickness of the brittle layer generally dip at high-angle (~45°) [Anderson, 1951; Jackson, 1987]. These faults are the most common features encountered in rift settings [Jackson, 1987]. They usually define a series of grabens structured by major or secondary high-angle normal faults. Such features are consistently observed in most extensional settings such as the Basin and Range, the Gulf of Suez and the East African Rift [Hamilton, 1988; Patton et al., 1994, Morley et al., 1992, Bosworth and Morley, 1994] or at mid-oceanic ridges [e.g., MacDonald and Luyendyk, 1977, Karson et al., 1987].

Recent observations in both continental and oceanic settings show that large offset normal faults with dip < 30° or even with negative dips [e.g., Coney, 1980; Wernicke, 1985; Karson et al., 1987; Axen and Bartley, 1997; Cann et al., 1997; Tucholke et al., 1998] are major features of Earth's topography. Some authors [e.g., Davis and Lister, 1988; Wernicke, 1995] believe that these faults formed and slipped at a low dip. However, this mechanism violates Andersonian faulting theory [Anderson, 1951; Wills and Buck, 1997] which states that normal faults form and slip at a high dip angle. An alternative to this model is that normal faults originate at high angle and, as fault offset increases, are rotated flexurally to an inactive low-angle configuration [Hamilton, 1988; Wernicke and Axen, 1988; Buck, 1988]. Field observations in favor of or against either of these models are numerous (see review from Axen and Bartley [1997]) so that it is often difficult to determine whether the fault rotated to its present configuration after large offset or if it initiated at low angle.

We would like (1) to understand the physics controlling the different kinds of normal faulting obtained in a set of idealized numerical experiments and (2) to define the possible range of behaviors that are relevant to geologic environments. It is likely that many processes not treated here, such as thermal advection, magmatism, and crustal thinning affect the pattern of faulting in a brittle layer. However, our goal is to start to understand one of the simplest systems that leads to the formation and offset of normal faults. We therefore use a dynamic model of faulting to study the evolution of fault offset during the extension of a uniform thickness brittle layer.

2. Previous Work

2.1. Modeling of Normal Faults

A great deal of work has been done to understand the mechanics of normal faulting using different approaches. The
main difference between these methods is the simplifying assumption made in order to study the evolution of normal faulting. A first approach considers the initiation of normal faults and ignores the long-term development of faulting. Anderson [1951] shows that in a uniform brittle layer subject to side-driven extension, faults are produced with dips ranging from about -70° down to 45°, depending on the coefficient of friction. On the basis of laboratory estimates of rock friction [e.g., Byerlee, 1978] normal faults should form with dips close to 60°.

Most continuum models realistically approximate the overall deformation of the lithosphere during rifting but generally do not examine the evolution of primary versus secondary faults. They either do not treat the localization of deformation associated with faults or must explicitly specify the initial fault properties. Also, a systematic analysis of the influence of each physical parameters on the faulting behavior is lacking in most of these studies.

Several authors modeled topography caused by normal fault offset by treating the lithosphere as an elastic plate [e.g., Vening Meinesz, 1950; Kuszniir et al., 1987; Weissel and Karner, 1989]. They assumed slip on one or more high-angle normal faults and look at the topography resulting from the bending of the lithosphere around the fault. Vening Meinesz [1950] proposed that grabens were produced by flexure of the hanging wall of a major normal fault bounding a rift when secondary faults antithetic to the first fault form in the part of the hanging wall that undergoes the most bending.

In order to study the potential formation of antithetic or synthetic faults in the vicinity of a high-angle normal fault, Melosh and Williams [1989] used a finite element method to model the lithosphere as a thick elastic plate. They inserted a normal fault in the lithosphere and predict the initiation of new faults by assuming a Mohr-Coulomb criterion for brittle failure in the elastic plate. However, they could not model large fault offsets. In a further improvement of these types of studies, Hassani and Chery [1996] were able to simulate the localization and formation of new faults around an initial prescribed weak fault by assuming plastic behavior in a thick elastic-plastic plate to allow for the localization of deformation in shear zones.

Stein et al. [1988] showed that, to accurately model the observed pattern of deformation around high-angle normal faults assuming an elastic lithosphere, the effective elastic thickness of the lithosphere must be much smaller than the seismogenic thickness. By treating the lithosphere as a thin elastic-plastic plate, Buck [1988] confirmed that result by showing that finite offset on a normal fault could greatly reduce the wavelength of the flexural response in the area surrounding the fault.

Several authors have studied loading of elastic layers in different ways that might cause initiation of normal faults with low-angle dip (<30°) [Yin, 1989; Parsons and Thompson, 1993]. However, these studies looked only at stress orientation and not at magnitude of stresses needed for slip on faults. Wills and Buck [1997] looked at stress magnitude and showed that the loading described by Yin and Parsons and Thompson would not lead to slip on low-angle faults cutting an entire layer. Indeed, they show that in the areas where the stress orientations suggest low-angle fault initiation the stress differences (and so shear stresses) approach zero, in contrast to adjacent areas where the stress orientations and magnitudes allow for high-angle fault initiation. Further, Gerbault et al. [1998] showed that elastic solutions for faulting can predict only the initiation of fault patterns when the stress field is slightly above yield. Thus these predictions are only useful for small deformations, and give erroneous results for large deformation encountered in natural faults.

2.2. Modeling of Normal Faults With Finite Offset

Forsyth [1992] and Buck [1993] have attempted to define the parameters controlling finite offset on normal faults by using very simplified models of lithospheric bending. Forsyth [1992] treated the brittle layer as a thin perfectly elastic beam, and Buck [1993] chose to treat it as a thin elastic plate having a finite yield strength (elastic-plastic). Both assumed the lower crust to be an inviscid fluid and prescribed an initial cohesionless fault in the model domain. Both studies attempt to show that a normal fault will continue slip as long as the material around it is stronger than the fault itself. A weak frictional fault surrounded by a stronger frictional and cohesive material fits that criterion. To slip on this fault, there needs to be a force increase to compensate for the resistance of the layer to bending and the resistance of gravity to the buildup of topography. If the force increase is sufficient to cause the material around the fault to reach yield, a new fault will form. If the force increase is too small to cause a significant change in stress around the fault, the fault may slip indefinitely. Therefore both of these authors studied the effect of this "gravitational" force increase on the strength of the layer surrounding the fault.

Using the simpler approximation, Forsyth [1992] inferred that the maximum fault offset is controlled by the dip angle of the fault and the layer cohesion. A very large offset fault can only develop on a fault with an initial low dip angle, lower than that predicted for the formation of a normal fault in a homogenous layer [Anderson, 1951; Wills and Buck, 1997]. By using a more complete description of the strength of the layer, Buck [1993] found that for a given cohesion, fault offset is controlled by the thickness of the elastic-plastic layer. For a brittle layer thickness >10 km and for a reasonably low cohesion value the fault can build up only a few kilometers of offset before being replaced by a new one. For a thinner brittle layer the fault offset may be unlimited.

It can be seen (Figure 1), that in a frictional brittle layer with a reasonably low value of cohesion (20 to 40 MPa [e.g., Handin, 1966]), the remaining strength on an active fault (proportional to the area in light shaded area on the yield stress profiles) represents a much smaller proportion of the total strength (proportional to the area in dark shaded area on the yield stress profiles) in a thin layer than in a thick layer. Moreover, Buck [1993] showed that the gravitational force increase is proportional the square of the layer thickness. Therefore a fault will remain relatively much weaker than the surrounding material in a thin layer (Figure 1a) than in a thick layer (Figure 1b), and the gravitational force increase will be smaller in a thin layer than in a thick layer. For these reasons, normal faults can accumulate large offset in a thin layer. In a thick layer, fault offset should be small and multiple faults should form.

3. Model Formulation

Forsyth's [1992] and Buck's [1993] studies are limited because they assumed thin plate behavior for the lithosphere and that faults are set as preexisting surfaces in the lithosphere. The thin plate approximation does not render
Figure 1. Strength differences between (a) a thin and (b) a thick layer. The strength is the integration of the yield stress over the thickness of the layer. Yield stress $\sigma_y$ is proportional to the shear stress assuming principal stresses are horizontal or vertical. The different strengths are therefore equal to the shaded areas. The layer is frictional and cohesive. Therefore, when a fault has formed, its strength is proportional to the remaining frictional strength of the layer. The strength of the material surrounding the fault is proportional to the cohesive and frictional strength of the layer. One can see that in a thin layer the strength on the fault is much smaller than that of the surrounding material. In a thick layer the fault is still strong compared to the surrounding layer.

accurately the dynamic state of strain and stress in an elastic-plastic layer. More importantly, the assumption that faults are preexisting surfaces in the layer obliterates the processes of weakening leading to the formation of the fault.

In the following experiments we use a numerical model that allows for the calculation of the dynamic state of strain and stress in an ideal brittle (elastic-plastic) layer in which plastic strain is given by a non-associated plastic flow rule (Poliakov and Buck, 1998). Non-associated plasticity allows for the determination of the yield surface according to the Mohr-Coulomb yield condition and for localization in shear zones or "faults." Moreover, we can parametrize the weakening processes that lead to the formation of fault by assuming that, when in the layer the yield stress is reached, the layer locally loses its cohesion with plastic strain (Buck and Poliakov, 1998; Poliakov and Buck, 1998). This approach is a major advance since it provides a tool that allows us accurately to take into account the effect of the gravitational force increase and the effect of the decrease in force caused by the weakening processes occurring in the fault zone. Because of the complexity of the numerical method and to reduce the number of input parameters we choose the following setup (Figure 2).

3.1. Model Setup

The width of the model domain is taken as 10-15 times the layer thickness $H$. The brittle layer is assumed to overlie an inviscid substrate and has a constant thickness. Each time we remesh, new material is added to the bottom of the grid, keeping the interface between brittle and inviscid materials flat, assuming that material added to the bottom has the same properties as the material in the layer. We take the density $\rho$ of the brittle layer and the inviscid substrate equal to 2700 kg m$^{-3}$ and the acceleration of gravity $g$ equal to 10 m s$^{-2}$. At the upper surface, shear and normal stresses are assumed to be zero. The right and left side of the box are pulled steadily apart (Figure 2a). At the bottom we apply normal stress equal to the lithostatic pressure in the brittle layer and zero shear stress (Winkler foundation).

The cohesion is reduced with increasing strain after yielding (Figure 2b). The shear stress at yield is given by Mohr-Coulomb theory:

$$\tau = \mu \sigma_n + C(\varepsilon_p),$$

where $\tau$ is the shear stress at yield, $\mu$ is the coefficient of friction, $\sigma_n$ is the normal stress, and $C$ is the cohesion, which is defined to depend on the total plastic strain $\varepsilon_p$. The plastic strain is the non recoverable strain accumulated by plastic deformation when the stress in the layer is locally greater than the yield stress. Up to the point where material loses all cohesion, the reduction of cohesion with strain is linear (Figure 2b):

$$C(\varepsilon_p) = C(0)[1-(\varepsilon_p/\varepsilon_c)],$$

where $C(0)$ is the initial cohesion of the layer. We define $\varepsilon_c$ as a characteristic value of plastic strain. When the plastic strain reaches $\varepsilon_c$, the fault is cohesionless. However, as seen in all models allowing for the localization of deformation [e.g., Cundall, 1989], the width of the fault $\Delta w$ is consistently 2 to 4 times the grid size. For this reason, for a given displacement $\Delta L$, the strain is dependent on the grid size. In order to scale the characteristic strain and the rate of cohesion weakening between two models with different grid sizes we use characteristic offset, $\Delta x_c = \varepsilon_c \Delta w$, rather than $\varepsilon_c$ as a measure of
Figure 2. (a) Model setup for extension of a brittle layer overlying an inviscid fluid. The layer has the finite yield stress of a Mohr-Coulomb type material. Yield stress $\tau_y$ is proportional to the shear stress assuming principal stresses are horizontal or vertical. (b) The reduction of cohesion $C$ is linear with plastic strain. The maximum cohesion loss is given by $C(0)$, and the rate of strength reduction is given by $C(0)/\varepsilon_c$, $\varepsilon_c$ being the characteristic plastic strain for which a fault has lost all its cohesion.

3.2. Numerical Method

The numerical method used for the experiments is based on an explicit finite element method similar to fast Lagrangian analysis of continua (FLAC) technique [Cundall, 1989]. It has been used to simulate localized deformation (approximating faults) in elastic-plastic materials in a variety of problems [Hobbs and Ord, 1989; Poliakov et al., 1993; Poliakov and Herrmann, 1994; Hassani and Chery, 1996; Poliakov and Buck, 1998]. For each numerical time step the modeling involves direct solution of Newton's second law for every grid point. In order to approximate quasi-static processes the effects of inertia must be damped in a way akin to oscillations in a damped oscillator. Starting from a non-equilibrium state, the forces present at each grid point are summed to determine the new out-of-balance forces. This dynamic response is then damped to approach a quasi-static equilibrium.

FLAC is a very powerful technique for simulating non-linear rheological behavior at very high resolution because the explicit time-marching scheme does not require storage of large matrices which are needed for implicit methods. The time step of the calculation scales with the elastic-plastic property of our model. If the problem is purely elastic the time step of the dynamic response scales with the velocity of the elastic wave propagating through the elements. This time step is of the order of a few hundredths of a second. Therefore the resolution of the domains studied and the timescale needed for our numerical experiments would yield very long running times. In order to decrease the CPU time needed to perform the numerical experiments we increase the speed of calculation by setting the boundary displacement as a fraction of grid spacing per time step. To set the boundary displacement, we chose a ratio of boundary velocity to sound velocity of 5x10^{-5}. We find that this ratio allows for fast enough runs and at the same time minimizes the error on the strain calculation.

When the layer cohesion reaches zero at the surface of the model, the material at the surface behaves as a cohesionless pile of sand flowing on the slope of the topography. The resolution of the model does not resolve such flow, and therefore numerical instabilities arise from that phenomenon. To solve that problem, we set the cohesion of the layer to a negligible value of 4 MPa in all experiments. As a result, when the process of cohesion reduction has taken place, 4 MPa of cohesion remain on the fault.

The initial mesh of the model is made of quadrilaterals subdivided into two pairs of superimposed constant-strain triangular zones. The use of triangular zones eliminates the problem of “hourglassing” deformation sometimes experienced in finite differences [Cundall, 1989]. Since this method is Lagrangian (i.e., the numerical grid follows the deformations), the simulation of very large deformations (locally more than 50%) involves remeshing to overcome the
problem of degradation of numerical precision when elements are distorted. We trigger remeshing when one of the triangles in the grid elements is distorted enough that one of its angles becomes smaller than a given value. Every time remeshing occurs, strains at each grid point are interpolated between the old deformed mesh and the new undeformed mesh using a nearest-neighbor algorithm. The new state of strain is then used with the rheological laws to calculate the stress and resulting out-of-balance forces to start the time step cycle again. Also, every time we remesh, errors in the interpolation of the strains result in an increase in the out-of-balance forces, and artificial accelerations and oscillations occur. For this reason, the solution may not be in equilibrium immediately after remeshing. This results in transient variations in stresses of order 10% from the average values as we will see in section 4. After about a hundred numerical time steps, damping brings the solution back to equilibrium. We have tested different criteria to trigger remeshing in order to reduce the time to obtain a reproducible result and chose to use a minimum angle before remeshing of 10°.

4. Results

We ran 44 models (Table 1), varying the layer thickness, the cohesion of the layer, the characteristic plastic strain, the grid size, and the flaw size and location. The model results in the spontaneous development of an evolving system of faults. For all models a fault or faults start where the plastic flaw is the cohesion of the layer, the characteristic plastic strain, the thickness). For all models a fault or faults start where the plastic flaw is the cohesion of the layer, the characteristic plastic strain, the thickness).

We find two fundamentally different regimes of faulting with some additional regimes included within each domain (Plate 1). In the “large offset” or “unlimited mode” the initial fault accommodates nearly all the extension for as long as the numerical experiment continues (Plate 1a). For the “small offset” mode, two or more faults form and accommodate significant offset (Plates 1b and 1c) and no fault is offset by a large amount (i.e., with an offset greater than the layer thickness).

Within the unlimited mode we find that in some cases, secondary faults can break the layer in the hanging wall and/or in the footwall of the initial fault. For “hanging wall snapping”, both the hanging wall and the footwall of the fault break after a very small offset (i.e., a few hundred meters) on the initial fault (Plate 1d). For “footwall snapping” the initial fault develops a limited amount of offset (a few kilometers) before a secondary fault breaks the footwall (Plate 1e). We believe that this is due to a local reduction of the bending moment in the elastic-plastic layer [Buck, 1997]. When a beam is bent sufficiently, it can break in a distributed manner over a broad area or it can break in one place (the way a cracker snaps). Buck [1997] showed that an elastic-plastic layer should break in a distributed manner or “crunch” when its bending moment increases with increasing bending of the plate. It should snap when the bending moment decreases locally with increasing bending. We believe that hanging wall snapping and footwall snapping around the major fault occur when the bending of the plate is large enough that the bending moment starts to decrease. This qualitatively explains why this behavior depends on the rate of strain weakening. Note that snapping generates secondary faults for the unlimited

<p>| Table 1. Parameters Used for Each of the Numerical Experiments |
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Because running a numerical experiment with the adequate resolution takes between 3 CPU days to 3 CPU weeks, the investigation of the parameter space was limited. $H$ is the thickness of the layer; $C(0)$ is the initial cohesion of the layer or the maximum cohesion loss; $\Delta x$ is approximately the width of the fault in the models; $\varepsilon_c$ is the characteristic strain for maximum strength reduction, $\Delta x$, the characteristic offset for maximum strength reduction, $\Delta L$ is the maximum fault offset before a new fault forms. The offset is taken as the maximum offset before a second fault forms. We estimate the offset by measuring the dip of the fault and the horizontal and vertical offsets of the fault. When the fault offset is large and the fault surface curved, we measure each straight portion of the fault footwall and add them up. The perturbation size indicates in number and position of elements. The perturbation is a plastic strain flaw of strain equal to the characteristic strain $\varepsilon_c$ of every individual models. The perturbation alters the initial strength of the layer and allows for the initiation of faulting in the middle of the layer.
(A) UNLIMITED, \( H = 10 \text{ km}, C(0) = 44 \text{ MPa}, \Delta x_a = 1.5 \text{km} \)

(B) MULTIPLE, \( H = 10 \text{ km}, C(0) = 44 \text{ MPa}, \Delta x_a = 3 \text{km} \)

(C) MULTIPLE, \( H = 20 \text{ km}, C(0) = 44 \text{ MPa}, \Delta x_a = 4.5 \text{km} \)

(D) UNLIMITED HANGING WALL SNAPING, \( H = 10 \text{ km}, C(0) = 44 \text{ MPa}, \Delta x_a = 0.015 \text{km} \)
mode but that the primary fault still can potentially develop very large offset.

5. Analysis of Results

We expect that two processes will affect the development of fault offset in the layer: (1) the increase in gravitational force due to the resistance of the layer to bending and the resistance of gravity to the buildup of topography and (2) the reduction in force due to the weakening processes on the fault. We know from our parametrization (equation (2)) that the reduction in force $\Delta F_w$ due to cohesion loss is linear with fault offset $\Delta x$ and can be described as

$$\Delta F_w = C(0)H(\gamma + (1-\gamma) \Delta x / \Delta x_e) \quad \Delta x < \Delta x_e$$

$$\Delta F_w = C(0)H(\gamma - 1) \quad \Delta x > \Delta x_e$$

where $\gamma$ is the size of the initial perturbation divided by the layer thickness, the subscript $w$ denotes weakening.

The form of the increase in bending related force as a function of fault offset is not specified; therefore, we ran a set of numerical experiments that allows us to estimate it. We performed six numerical experiments where a fault zone with negligible cohesion (4 MPa) and zero friction is prescribed in the middle of 5-, 10-, and 20-km-thick layers with two different values of cohesion, $C(0) = 4$ MPa, $C(0) = 24$ MPa and $\mu = 0.6$ (Figure 3). The number of grid elements is 100 in the $x$ direction and 20 in the $z$ direction for each experiments, the 20-km-thick layer having the lowest resolution (1000 m) and the 5-km-thick layer having the highest resolution (250 m). The result of each of these experiments is the bending component of the force change $\Delta F_b$ as a function of horizontal offset (Figure 3).

As mentioned previously, for each remeshing, the out-of-balance forces in the model are reinterpolated far away from the static equilibrium. This results in a discontinuity in the calculation of forces that occurs about every 300 to 500 m of horizontal offset for a grid size of 250 m and every 700 to 1500 m for a grid size of 1000 m. The forces reach a maximum for each case after an amount of horizontal offset that scales approximately with layer thickness (700 to 800 m for $H = 5$ km, 1700 to 2100 m for $H = 10$ km and 6000 to 6400 m for $H = 20$ km).

Figure 3. Plot of the bending force needed to stretch layers (5, 10 and 20 km thick) of increasing strength against the horizontal offset. The layers have a friction coefficient $\mu = 0.6$ and different initial cohesions $C(0) = 4$ MPa and 24 MPa. There is no loss of strength with plastic strain in the layer. A strengthless fault is initially set in the layer. The forces increase until they reach their yield point. The thicker layer, the larger the force needs to be. After reaching the yield point the forces decrease to a constant value.
Plate 2. (a) Multibeam bathymetry data gridded at 200 m spacing (RIDGEx Multibeam Synthesis, available online at http://imager.ideo.columbia.edu). Topographic profile along estimated direction of spreading. (b) Satellite imagery (Landsat RGB 4/5/6) of Whipple Mountains area. Outlined in red is the mylonite front, and outlined in black is the Whipple detachment [Axen and Selverstone, 1994]. Topographic profiles across mountains are shown along estimated direction of fault motion. (c) The modeled topography and plastic strain plotted for the last step of the model in which $H = 10$ km, $\Delta x = 1.5$ km and the total offset is 27 km. The grid spacing is 500 m. White dashed contour on plastic strain profile bounds uplifted lower crustal material. Inactive footwall rotates to horizontal as extension increases.
Figure 4. Schematic representation of the two components of the regional force needed to extend and fault the brittle layer for (a) a thin and (b) a thick layer. Initially, the transition between very large offset faults and small offset faults is controlled by the competition between the rate of increase in bending force and the rate of decrease in force due to the weakening on the fault. In the long term the transition is controlled by the thickness of the layer H.

20 km). From there, the regional force needed to pull on the layer decreases, finally reaching a steady state value (Figure 3).

Our results confirm the analysis of Buck [1993] suggesting that the maximum increase in force is proportional to the square of the layer thickness $H^2$. There is also some dependence on the layer cohesion. On the basis of the results of the present experiments we choose to approximate the bending component of the force change $\Delta F_b$ by an exponential function:

$$\Delta F_b = AH^2 \left[1 - \exp(-B\Delta x/H)\right],$$

where $A$ defines the maximum force change and $B$ controls the rate of initial increase in force change with horizontal offset. $A$ and $B$ are obtained from our numerical experiments (Figure 3). $A$ is estimated at 1200 Pa m$^{-1}$ and $B$ is estimated to be equal to 50.

When added, the two components give the total force change necessary to extend the layer. In Figure 4 we sketch each component of the average force change and their sum for a thin and a thick brittle layer. This suggests that two mechanisms may control whether extension of the brittle layer leads to the large offset or the small offset mode. The first is related to the maximum possible changes in $\Delta F_b$ and $\Delta F_w$, while the second is related to the rates of change of these force components.

Pertinent to the first mechanism, the maximum in bending force $\Delta F_b$ is proportional to the square of the thickness $H^2$ of the layer and the maximum decrease in force $\Delta F_w$ is proportional to the thickness $H$ of the layer. Thus, the maximum of $\Delta F_b$ will exceed the maximum of $\Delta F_w$ only for a thick enough layer (Figure 4b). In this case the bending changes are larger than the weakening and a second fault forms, limiting the offset on the initial fault (small offset mode) (Plate 1b). For a thin layer (Figure 4a) the bending component is small enough and the weakening component is large enough that the weakening processes dominate the total force change needed to offset the fault. In that case the initial fault can accumulate very large offset (Plate 1a) (large offset mode).

For either a thick or a thin layer the rate of change of forces could control whether the force change is positive and so a new fault would form. When the rate of cohesion reduction is very slow, the reduction in force is so small that the initial increase in bending force dominates the force change in the layer. In our experiments after a few ten’s of meters of offset on the fault another fault forms (Plate 1c) (small offset mode). If the rate of cohesion reduction is moderate to fast, for a thin enough layer the weakening processes dominate the force change. In our experiments we were able to obtain large offset faults (Plates 1a, 1d and 1e).

As we have seen, the modes of faulting in our numerical experiments are controlled by the thickness of the layer $H$ relating to the amount of weakening possible on a fault and the maximum increase in bending force and the characteristic fault offset $\Delta x_c$ relating to the rate of weakening on a fault. We plotted our model cases using axes of $H$ and $\Delta x_c$ (Figure 5a) (as defined in Table 1) for a cohesion $C(0) = 44$ MPa. Using this parameter space to plot the results of our model experiments yields two well-defined domains of fault behavior confirming the theory developed previously: (1) a large offset mode domain and (2) a small offset mode domain. Also confirming our theory, we find two main transitions between large offset and small offset domains: (1) One is transition that depends on the maximum changes in cohesion and bending related forces, which here occur at about $H = 22$ km. Below 22 km the layer is thin enough for the given cohesion to allow for large or unlimited offset. Above 22 km the layer is so thick that the bending force needed to offset the fault is such that the stresses around the fault exceed yield and another fault forms. (2) A second transition corresponds to the point at which the
that maximum equals zero, we predict a transition from small infinitely small perturbations exist, the transition occurs for large offset fault. For a given value of layer thickness $H$ the layer (Figure 5b). However, in an ideal system where only is for an initial perturbation of 10% of the thickness of the layer (Figure 5b). We find that this transition depends on the initial characteristic fault offset. The plot shows the two domains of very large offset and small. The very large offset faults domain is separated into an unlimited and a snapping domain where secondary faults form in the footwall and hanging wall of the initial fault. (b) Same plot as in Figure 5a but highlighted in dark and light shading the domains for small offset and large offset predicted by theory for a layer with a very small initial perturbation (PER = 1%). The best fitting theoretical curve to our experiments is the one for a layer with a larger perturbation (PER = 10%).

![Diagram](attachment:image.png)

**Figure 5.** Domain plot for snapping and faulting by regional stretching. (a) Thickness of the layer as a function of the characteristic fault offset $\Delta x_c$. The plot shows the two domains of very large offset and small. The very large offset faults domain is separated into an unlimited and a snapping domain where secondary faults form in the footwall and hanging wall of the initial fault. (b) Same plot as in Figure 5a but highlighted in dark and light shading the domains for small offset and large offset predicted by theory for a layer with a very small initial perturbation (PER = 1%). The best fitting theoretical curve to our experiments is the one for a layer with a larger perturbation (PER = 10%).

6. Discussion and Conclusion

We have developed a simple theory for understanding the processes and the factors controlling normal fault offset in an ideal brittle layer. We find that not only the thickness of the layer [Buck, 1993] but also the rate of reduction in cohesion in the fault controls whether a fault can develop large offset or small offset. In this idealized system we were able to show that for a given space of geologically reasonable values of layer thickness, cohesion loss, and rate of cohesion loss a broad range of behaviors is possible. In order to test the validity of our model we compare our results to some geological features observed geological faults.

6.1. Core Complex

We predict that a primary fault can slip by an unlimited amount only when a layer with 40 MPa of cohesion is thinner than 22 km and when the fault forms for a moderate rate of cohesion reduction corresponding to a characteristic fault offset ($\Delta x_c = 1$ to 4 km depending on the layer thickness) (Figure 5b). In that case, the inactive footwall of the fault rotates to become nearly horizontal. These are the first self-consistent numerical models that produce this kind of behavior. It is possible, though highly controversial, that such fault rotation may occur in some continental core complexes and along some parts of slow spreading ridges [Lavier et al., 1999]. We compare our unlimited offset model results (Plate 1a and Plate 2) to some of the main geologic features of known core complexes. These are (1) a large massif located at the inside corner of the intersection between the Mid-Atlantic Ridge and the Atlantis transform fault and (2) the Whipple Mountains (Plate 2). Plate 2a is the sea-floor topography in the vicinity of the Atlantis transform fault [Tucholke et al., 1998; Blackman et al., 1998]. Plate 2b shows a satellite image of the area of the Whipple Mountains [e.g., Lister and Davis, 1989].

The Whipples and other terrestrial core complexes have been extensively mapped and sampled. However, the severe erosion and sedimentation affecting these continental areas and the fact that they have been inactive for several million years complicate the observations and their interpretation. In fact, no real consensus exists as to whether some normal faults observed in core complexes formed at high angle or originated with a low dip angle. The detachment fault of the Mineral Mountains of southwestern Utah is thought to be a clear example of a normal fault that initially dipped at 60° in the brittle crust [Coleman and Walker, 1994] and then rotated to a low angle at the surface. In contrast, most workers conclude that the Whipple detachment may have had an original dip of <35° [e.g., Davis and Lister, 1988; Axen and Bartley, 1997]. However, even the Whipples remain a controversial area as some authors contend that the rolling-hinge model explains some of the major structural features [Hamilton and Howard, 1991] whereas other authors believe that it cannot [Beratan and Nielson, 1996].
Work over the past 10-20 years on large-offset low-angle normal faults along the slow spreading Mid-Atlantic Ridge sheds new light on this problem. These structures may be currently active and are far less affected by erosion and sedimentation. As a result, observations and interpretation of the first-order topographic features are easier than in the continental domain. However, detailed mapping is more difficult than for terrestrial core complexes: it is undertaken with a combination of sonar imagery, near-bottom photography, sampling from submersibles, and drilling.

In the oceanic case illustrated in Plate 1a the dip of the exposed detachment can vary from horizontal to ~30° or greater near the contact between the footwall and the hanging wall of these faults. The fault offset inferred for this oceanic massif is roughly 30 km [Tucholke et al., 1998; Blackman et al., 1998]. Thus, we show our model results for about 30 km of horizontal extension (Plate 2c). The shape and the amplitude (2600 m) (Plate 2a) of the topography is readily comparable to the modeled topography (2100 m) (Plate 2c). As in the model, inferred high-density lower-crustal material was uplifted during exposure of the inactive footwall of the fault [Blackman et al., 1998] (Plate 2c).

The topographic features at the Whipple Mountains are at first order different from the modeled topography. However, the modeled inactive-footwall dip and offset and the position of the mylonitic front (Plate 2c) (15 km away from the detachment) are all similar to what is observed on the real topographic profiles (Plate 2b). Also, filling the topographic low with sediments would make the amplitude of the modeled relief similar to the topographic trends of the Whippies.

6.2. Implications for Material Parameters
To model the evolution of low-angle normal faults, an important requirement beside having a thin layer, is that the rate of cohesion reduction with plastic strain be moderate (Δτp = 1 to 4 km). There is some geologic and laboratory evidence for rapid loss of cohesion with strain (i.e., that Eɕ = 2% [Scholz, 1990]. For such behavior our results suggest that while a large or unlimited offset fault develops and slips, secondary faults form in the hanging wall of the major fault and accumulate slip in a way similar to an asymmetric graben. We believe that in our model such a rapid loss of cohesion with strain (i.e., that Eɕ = 2%) of a weak natural faults occurs only after a considerable amount of damage has taken place over its width and length. This complex problem clearly requires further study. However, we believe that in our model a moderate rate of cohesion reduction is the one that scales best with natural faults.

6.3. Continental Rifts
Studies of rifts and continental margins have shown the importance of half grabens as a basic unit in accommodating extension. In addition, the asymmetry of faulting in both continental rifts and passive margins has been documented. Extension at rifts results in subsidence within the rift and uplift of the rift flanks. The factors that control these basic components of rift expression are poorly understood at present. We believe that the partitioning of strain and the pattern of faulting in a rift depend on a small number of parameters that control either localizing or delocalizing processes. Heat advection occurring during necking of the lithosphere reduces the strength of the lithosphere and tends to narrow the width of the necking area. The corresponding diffusion of heat cools and strengthens the lithosphere and tends to delocalize the deformation in the upper crust. The delocalizing effect of buoyancy forces generated by the density differences from surface topography and Moho relief is countered by lower crustal flow which decreases relief and buoyancy forces. The localizing process of cohesion reduction of faults [Buck and Poliakov, 1998] is countered by viscous strengthening in the lower crust, due to its strain-rate dependent rheology. Since our model does not take into account all of these processes, we do not compare the results of our model to current data from rifts. However, we believe that the small offset behavior of our model (Plates 1b and 1c) is the closest to the behavior observed in continental rifts. What controls the maximum offset in this regime when the effect of the viscous lower crust and the effect of crustal thinning are included is still an open question. Ebbing et al. [1987, 1991] and Scholz and Contreras [1998] have shown that in places such as the East African Rift, there is a relationship between the maximum possible offset on a major normal fault and the effective elastic thickness of the lithosphere. They find that the effective elastic thickness of the lithosphere is larger where the observed offset is the largest. In the future, we will explore the effect of viscous stresses, heat transfer, crustal thinning and erosion and sedimentation in order to determine the natural condition that should lead to the formation of multiple faults. Also we will attempt to define the factors controlling normal fault offset in setting such as narrow and wide rifts.

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References
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