Ice Profiling Sonars: a Comparison of Error Budgets

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***Abstract*- An accurate understanding of the volume of ice in the Arctic is a critical component of understanding the Earth's heat budget. Estimation of this volume is a complex problem involving both spatial coverage issues as well as accuracy of the ice thickness measurements themselves. Much of the data from which such estimations are made has been taken from 637-class US Navy submarines. Unfortunately, there will be little or no data from this class of submarine in the future as they are all being decommissioned. New observations will be made with new sonars and the ability to accurately and robustly compare observations between the new and old measurement systems is necessary to minimize the confusion due to differences between them.**

This paper describes the sonar system used to collect the existing data sets from the 637-class submarines, a new sonar for making similar observations from autonomous underwater vehicles (AUVs) in the future and establishes a reference framework for evaluating error budgets in this type of sonar system. A careful intercomparison between the old and the new sonars to establish a robust basis for extending the observational time series should be done.

I. Introduction

Accurate estimation of the volume of sea ice in the arctic Ocean is a sensitive indicator of the thermal budget of the Arctic Ocean and hence important clues in understanding recent changes in ocean temperature [1, 2]. The existing record of ice draft measurements taken from submarines operating under the pack ice goes back to the first submarine transpolar crossing by the USS Nautilus, in 1958. All the U.S. submarine ice profile measurements were made with systems with the same angular and range resolution. Since 1975 all have been made with the same instrument (OD-161) from the same class of submarine (SSN637).

The era of this class of submarine and this sonar system is over and new measurements will necessarily be made with new instrumentation. Accurate comparison of the old data and any new data to be collected will be crucial in order to usefully extend the climate record.

In this paper we define a frame of reference as a tool to aid in comparing ice draft observations from different platforms, document the OD-161 based observations, attempt to identify the magnitude of possible errors in the historical data and attempt to outline important issues in the design, deployment and processing of future ice draft measurement systems.

The following sections will describe the OD-161 ice profiler and a new sonar from ASL Environmental Sciences Inc that is being adapted for use on an autonomous underwater vehicle (AUV) and which will be deployed in the near future.

II. Instrumentation

In the past, the majority of ice draft observations were made from nuclear powered military submarines. Most of the available data was collected with a modified version of the standard US Navy sounder. In the future, we expect that the majority of ice draft data will be collected with purpose-build sonars mounted on autonomous underwater vehicles (AUVs) such as Autosub [3,4] and the ALTEX [6] vehicles.

A. Digital Ice Profiling Sonar (DIPS) History and Description

1. Introduction

The Digital Ice Profiler System (DIPS) is a modification to the ice profiling function of the OD-161 sounder installed on the Navy's SSN637 Class submarine, to enable automatic detection, digitization, and recording of ice draft when the submarine is operating beneath the sea ice canopy in the Arctic. Timing functions in DIPS are derived from the OD-161 itself, so the DIPS cannot be functionally described separately from the OD-161. DIPS has been used aboard submarines for archival recording of ice draft since the late 1970's. The recording medium for DIPS has evolved from magnetic tape, to various generations (and densities) of floppy disks, to zip disks. There is currently underway an effort to recover and digitally archive older ice draft data which (since about 1958) was recorded only on electrographic strip-chart recorders.

In 2000, the last of the Navy's SSN637 Class submarines was decommissioned. The newer "Improved 688", or 688I Class, is now used for arctic operations. The 688I Class submarine is not equipped with the OD-161 sounder (and hence, does not have DIPS). The ice profiling function on this newer class is part of the ship's integrated combat system. Although ice draft is available in real-time within the combat system in digital form, no system has yet been developed to record such information.

The following sections describe the historical system (OD-161 and DIPS) used to collect ice draft observations from US submarines.

2. History

Since the development of nuclear submarines in the 1950's, each US Navy submarine has been equipped with a sonar system that acoustically detects ice keels, and presents the ice draft information graphically, on a strip chart recorder. In all cases, the ice profilers employed sensors with half-power beam widths of about three degrees. Earlier systems employed a recorder with a rectilinear format; beginning in about 1975, the OD-161 sounder, which employs a recorder with a curvilinear format, was introduced. The DIPS was designed to work with the OD-161 sounder.

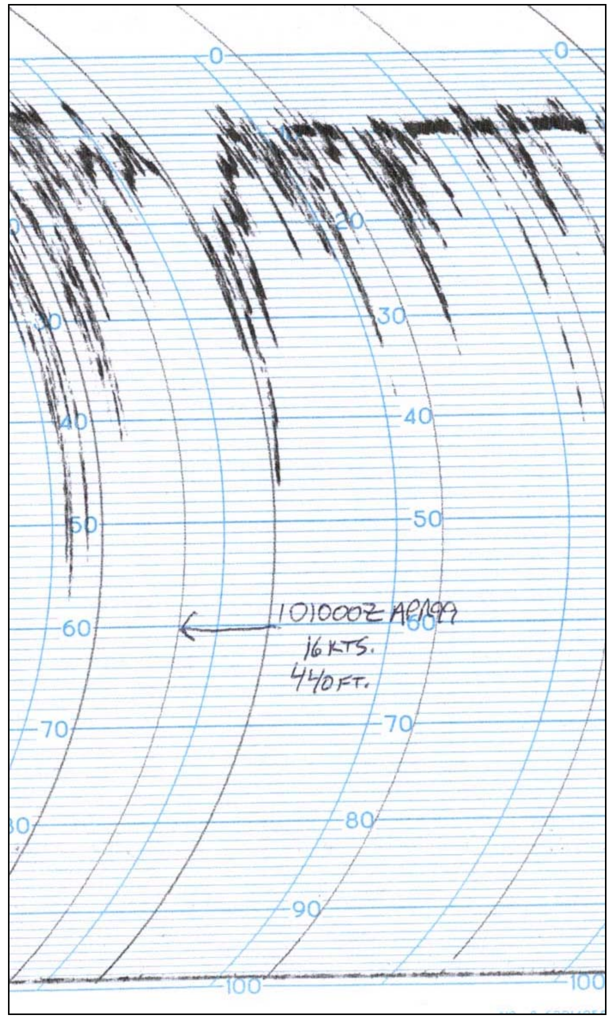


Figure 1: A scanned image of a hardcopy record produced by an OD-161 sounder in real-time during SCICEX-99. The sea-surface is at the zero line.

3. System Description:

The OD-161 ice profiler is essentially a depth sounder which has been inverted to look upward, and with the displayed data compensated for the transducer depth, such that zero depth (the sea surface, when there is no ice) appears at the top of the recorder page and ice keels extending below the surface appear below the zero depth line. Real-time depth compensation data is provided by a mechanical pressure gage.

The ice profiler's recorder mechanism employs a stylus mounted on a rotating arm. The stylus passes across the recording paper during one quadrant of its revolution. The recorder's stylus arm makes one revolution for each 4000 cycles of the clock, hence there are 1000 cycles of the clock during the time the stylus is over the recording paper. If the operator has selected the 100 foot range scale, this means there are ten clock cycles per foot, and so the clock is referred to as the "0.1 foot clock". If the 200-foot scale is selected, each clock cycle corresponds to 0.2 feet.

DIPS determines ice draft by sensing the returning echo from the ice profiler to start a counter, which is clocked by the 0.1 foot clock of the OD-161. The counter is stopped by the receipt of a "surface sync" signal from the OD-161. (The surface sync signal synchronizes the phase of the rotating stylus arm with the position of the chart paper.) When stopped, the counter contains the ice draft value, to a precision of 0.1 feet.

DIPS also reads ship's parameters of speed, depth, and heading from either synchro or digital sources (depending upon signal availability in a particular installation) with a periodicity of once per six samples of ice draft. This corresponds to an approximate sampling rate of once per second.

Records of ice draft data from the submarine-installed ice profiler – both data recorded by DIPS, and older data recovered from strip chart recordings – are being made public on the web site of the National Snow & Ice Data center. The released data are ice draft at one meter spacing along the ship's track, obtained by fitting a cubic spline to the raw data, and sampling the spline function at one-meter periodicity.

The half-power beamwidth of the OD-161 ice profiler transducer is approximately three degrees. The spatial resolution of this sensor on the underside of the ice is, of course, dependent upon the distance of the transducer from the ice. In the example shown in the figure above, the ship was operating at a keel depth of 440 feet, which puts the transducer at a depth of 388 feet. At this depth, the sonar is acoustically illuminating a spot 6.2 meters in diameter on the surface (or on shallow ice). The ship's speed of 16 knots equates to approximately 8 meters per second. At the ping rate of approximately six pings per second, the sonar is only traversing 1.3 meters between pings.

B. Ice Profiling Sonar (ASL Ice Profiler): History and Description

1. Introduction:

Upward looking sonars have been used to monitor ice coverage for at least the last 20 years. A basic system employs a single beam transducer mounted either on a mooring or directly placed on the sea floor in shallow regions which records return times and/or full echoes from the air-water or ice-water interface, depending on whether ice cover was present. While these measurements have proved valuable, their spatial coverage is limited by the drift characteristics of the ice floating above. In an effort to improve on this limitation and complement data collected by submarines, the Monterey Bay Aquarium Research Institute (MBARI) has undertaken a project to mount an upward looking sonar on an AUV capable of operating under the ice.

The sonar being used for this work is a commercial upward looking sonar traditionally used for moored applications. It has recently been mounted and used aboard an Arctic capable AUV during test missions in the Monterey Bay where it successfully detected the range to the air-water interface. The more difficult test of detecting the ice-water interface, and thus, ice draft, is scheduled for testing in the Fall of 2001 in the waters north of Svalbard, Norway.

2. History

A basic design for an ice profiling sonar (IPS) was developed by Dr. Humfrey Melling of the Canadian Department of Fisheries and Oceans at the Institute of Ocean Sciences (IOS). It was redesigned and tested in 1985 by IOS with the help of an industry partner, ASL Environmental Sciences Inc., who were subsequently awarded license to manufacture the fourth generation model. This model, known then as the IPS-4, now called the Ice Profiler, began production in 1996, and since then has been deployed in such regions as the Arctic Ocean, in shelf waters off Antarctica, in the Sea of Okhotsk, and off Northern Japan, to name a few.

3. System Description

The Ice Profiler, like the OD-161, is fundamentally an inverted depth sounder. It also has an integrated tilt sensor and pressure gauge. The system not only records the data it collects internally, but also generates a real-time serial data stream that is sampled by the AUV main vehicle computer. The instrument also comes with software allowing the user to modify various aspects of the acoustic and processing system.

An estimate of the ice draft is produced using the time of travel, tilt, and pressure measured by the Ice Profiler, ASL provided post-processing software, and additional knowledge of the sampled environment (such as atmospheric pressure and average sound speed velocity in the water column). The time of flight of the returned acoustic ping is determined using amplitude and persistence parameters that the user can adjust.

The Ice Profiler operates at 420 kHz with a 1.8 degree beamwidth (at –3 dB). The system will typically be operated at about 50 meters below the sea surface while the AUV is swimming a straight and level flight path. At this operating depth the diameter of the acoustically illuminated region of water (or thin ice) will be approximately 3.2 meters. As with the OD-161 system, this is dependant on both the sensor depth as well as the ice thickness. The velocity of the AUV is approximately 1.5 meters per second, and the ping rate of the Ice Profiler is one ping per second, resulting in a distance of 1.5 meters between pings.

In addition to the Ice Profiler data, all of the positional and other scientific data gathered on the AUV is collected and logged concurrently.

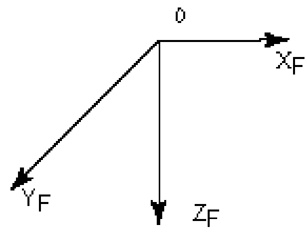
III. Reference Frame

A. Approach

The following section defines a reference frame based upon [6].

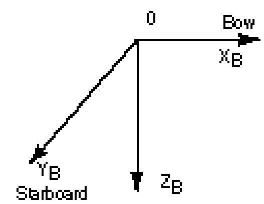
Consider an initial “fixed” reference frame with an origin that moves with the body, but that does not rotate its axes relative to the earth:

$$"Fixed"\,Frame \Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.1)$$



and consider a body reference frame with X positive toward the bow, Y positive to starboard, and Z positive pointing toward the center of the earth:

$$BodyFrame \Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B \qquad (0.2)$$



Now, taking the following angular definitions based on the “fixed” frame:

Rotation about: $Y_F\Theta =$ angle of pitch (0.3)

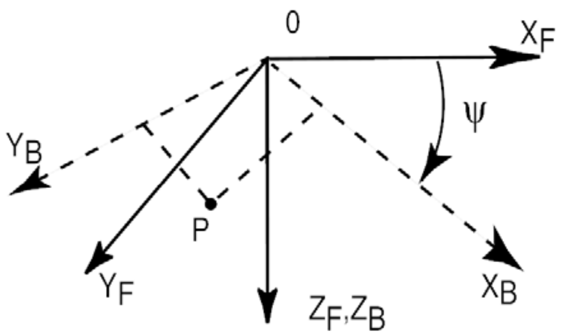
Rotation about: $Z_F\Psi =$ angle of yaw (0.4)

Rotation about: $X_F\theta =$ angle of roll (0.5)

Pitch, yaw and roll (in the equations above) are positive in the sense of rotation of a right-hand screw advancing in the positive direction of the axis of rotation [6]. Platform rotation angles used must be conventional Euler angles (as opposed to space fixed angles).

Where, if yaw, pitch, and roll = 0, then the two coordinate reference frames are coincident.

Then for a positive yaw of ψ from the fixed reference frame to the body reference frame.



The coordinates of point P in the body frame are:

$$X_B = X_F \cos\psi + Y_F \sin\psi$$
$$Y_B = -X_F \sin\psi + Y_F \cos\psi \qquad (0.6)$$

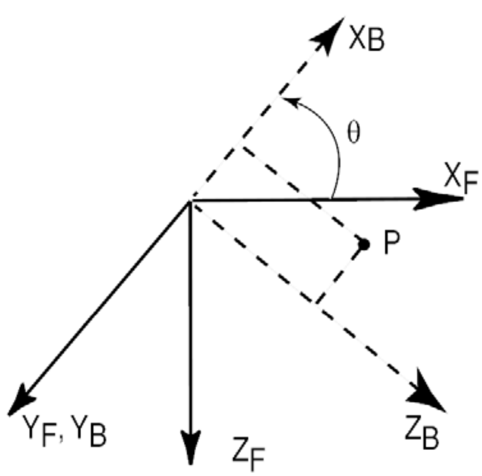
or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.7)$$

or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = R_{B/F}^{YAW} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.8)$$

Likewise, for a positive pitch rotation of θ from the fixed frame to the body frame, the coordinates of point P in the body frame are:



$$X_B = X_F \cos\theta - Z_F \sin\theta$$
$$Z_B = X_F \sin\theta + Z_F \cos\theta \qquad (0.9)$$

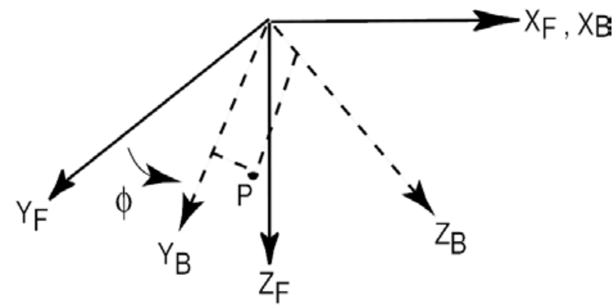
or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.10)$$

or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = R_{B/F}^{PITCH} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.11)$$

and finally, for a positive roll of ϕ from the fixed frame to the body frame, the coordinates of point P in the body frame are:



$$Y_B = Y_F \cos\phi + Z_F \sin\phi$$
$$Z_B = -Y_F \sin\phi + Z_F \cos\phi \qquad (0.12)$$

or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.13)$$

or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = R_{B/F}^{Roll} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.14)$$

Now, combining them all (recalling that Euler angles are being used):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = R_{B/F}^{Roll} R_{B/F}^{Pitch} R_{B/F}^{Yaw} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_F \qquad (0.15)$$

where:

$$R_{B/F}^{Roll} \quad R_{B/F}^{Pitch} \quad R_{B/F}^{Yaw} = R_{B/F} \qquad (0.16)$$

so:

$$R_{B/F}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & \sin\phi\\ 0 & -\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix} \quad (0.17)$$

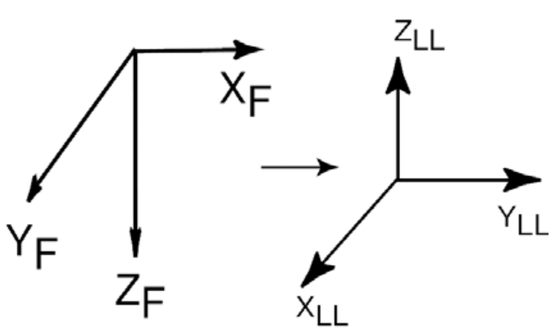
Multiplying this out gives:

$$R_{B/F}=\begin{bmatrix}\cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta\\ \begin{matrix}-\cos\theta\sin\psi\\ +\sin\theta\sin\phi\cos\psi\end{matrix} & \begin{matrix}\cos\phi\cos\psi\\ +\sin\theta\sin\phi\sin\psi\end{matrix} & \cos\theta\sin\phi\\ \begin{matrix}\sin\phi\sin\psi\\ +\sin\theta\cos\phi\cos\psi\end{matrix} & \begin{matrix}-\sin\theta\cos\psi\\ +\sin\theta\cos\phi\sin\psi\end{matrix} & \cos\theta\cos\phi\end{bmatrix} \quad (0.18)$$

Now, what we really want is the latitude and longitude of each ice draft measurement. The sounding positioning coordinates can be calculated by adding offset coordinates of each sounding from the positioning system reference point latitude and longitude observations (as the acoustic transducer and the positioning system are generally not coincident.) These offsets will be addressed presently.

First, the coordinate reference frame will be transformed one more time into what we will call the local-level reference frame to facilitate the use of the output coordinates in a geo or global referenced frame using a globally based positional reference. The local-level (LL) frame is a right-handed coordinate system with Z axis positive up, Y axis positive north and X axis positive east.

So, to relate the "fixed" frame to this new local level frame whose origin is coincident with the "fixed" frame:



In mathematical terms, this transformation takes the form:

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_B=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{LL} \quad (0.19)$$

or

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_F=R_{F/LL}\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{LL} \quad (0.20)$$

However, we really don't care about the "fixed" frame, what we want is a relationship between the body coordinates and the local-level coordinates which can be found such that:

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_Z=R_{B/F}R_{F/LL}\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{LL} \quad (0.21)$$

Where:

$$R_{B/F}R_{F/LL}=R_{B/LL} \quad (0.22)$$

But we would further like to start with the body coordinates and end up with local-level coordinates so:

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{LL}=R_{B/LL}^{-1}\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_B \quad (0.23)$$

Where:

$$R_{B/LL}=R_{B/F}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta\sin\psi & \cos\theta\cos\psi & \sin\theta\\ \begin{matrix}\cos\phi\cos\psi\\ +\sin\theta\sin\phi\sin\psi\end{matrix} & \begin{matrix}-\cos\phi\sin\psi\\ +\sin\theta\sin\phi\cos\psi\end{matrix} & -\cos\theta\sin\phi\\ \begin{matrix}-\sin\phi\cos\psi\\ +\sin\theta\cos\phi\sin\psi\end{matrix} & \begin{matrix}\sin\phi\sin\psi\\ +\sin\theta\cos\phi\cos\psi\end{matrix} & -\cos\theta\cos\phi\end{bmatrix} \quad (0.24)$$

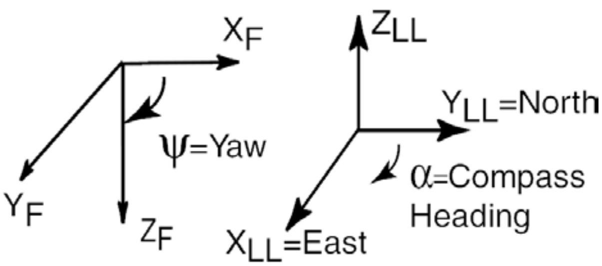
And so,

$$R_{LL/B} = R_{B/LL}^{-1}$$

$$= \begin{bmatrix} \cos\theta \sin\psi & \cos\phi \cos\psi + \sin\theta \sin\phi \sin\psi & -\sin\phi \cos\psi + \sin\theta \cos\phi \sin\psi \\ \cos\theta \cos\psi & -\cos\phi \sin\psi + \sin\theta \sin\phi \cos\psi & \sin\phi \sin\psi + \sin\theta \cos\phi \cos\psi \\ \sin\theta & -\cos\theta \sin\phi & -\cos\theta \cos\phi \end{bmatrix}$$

(0.25)

To simplify the terminology slightly we would propose to replace the yaw measurement with the more commonly collected measurement of compass direction α. Recalling the "fixed" frame from which all angles are defined and the local-level frame:



In the local-level frame, compass heading measured clockwise from North is the same angle as yaw in the "fixed" frame. Given that this is true and that the "fixed" and local-level frame do not rotate with respect to each other, then yaw in the previous equations can be replaced with compass heading (assuming the remaining angles are still applied as Euler angles.)

Moving on to the measurement of ice draft:

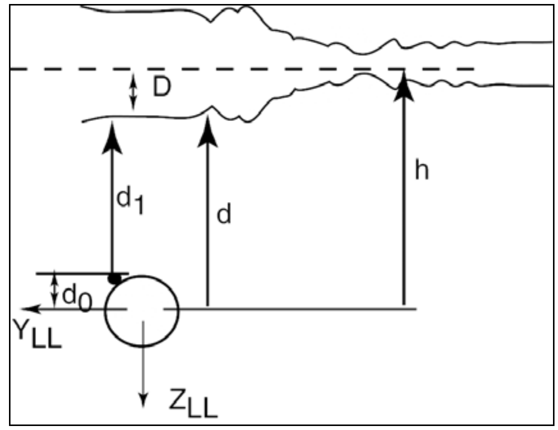


Figure 2: Schematic representation of the measurement of ice draft (D) from observations of distance from the transducer and depth of the vehicle (h).

D	ice draft
d	the distance to the bottom of the ice from the body and local level origin
h	the distance to mean water level from the body and local level origin
d_1	distance from the transducer to the bottom of the ice in local level coordinates
d_0	the distance from the transducer to the local level origin in local level coordinates.

Relationships:

$$d = d_1 + d_0 \qquad (0.26)$$

$$D = h - d = h - (d_1 + d_0) \qquad (0.27)$$

Now if we apply the coordinate transformation to get from a body frame measurement of range, r, to the local-level frame:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{LL} = R_{LL/B} \begin{bmatrix} 0 \\ 0 \\ -r \end{bmatrix}_B \qquad (0.28)$$

Where r equals the range from the transducer to the bottom of the ice in body reference frame coordinates.

Then the coordinates of that measurement are:

$$X_{LL} = r(\sin\theta \cos\alpha - \sin\theta \cos\phi \sin\alpha) \qquad (0.29)$$

$$Y_{LL} = -r(\sin\phi \sin\alpha + \sin\theta \cos\phi \cos\alpha) \qquad (0.30)$$

$$Z_{LL} = -r(-\cos\theta \cos\phi) \qquad (0.31)$$

Which simplifies to:

$$Z_{LL} = r(\cos\theta \cos\phi) \quad (0.32)$$

Now, given the definition of Z_{LL} as being the rotational transformation of the distance from the transducer to the bottom of the ice from the body to the local reference frame, then:

$$Z_{LL} = d_1 \quad (0.33)$$

So:

$$D = h - (r\cos\theta \cos\phi + d_0) \quad (0.34)$$

Note that D is given in local-level coordinates and, thus, is measured perpendicular to a level surface that we can assume is the mean water level (perpendicular to the gravity-vertical.)

Note also that r is the true straight-line distance from the transducer to the bottom of the ice and as such, the actual time-of-flight must be corrected for ray-bending and propagation effects caused by density variations in the water column.

Also, this equation does not take into account errors caused by misalignment of the sensor relative to the attitude measurement system.

The following section describes an attempt to treat these errors along with other error sources using the approach of Hare et. al. [7].

1. Range measurement errors

Errors in the measurement of the actual range to the ice (r):

$$r = r_{meas}\left(\frac{v}{v_{meas}}\right) \quad (0.35)$$

Where ν is the true speed of sound.

The total variance in range is:

$$\sigma_r^2 \approx \sigma_{r_{meas}}^2 + \left(\frac{r_{meas}}{v_{meas}}\right)^2 \sigma_{v_{meas}}^2 \quad (0.36)$$

As discussed in section 3.1.1 of Hare et. al. [7], the standard deviation of the measured range encompasses numerous errors associated with the echo sounder including the accuracy of the time of flight measurement and the target detection implementation.

Travel time acoustic measurements in the upper portions of the Arctic are highly problematic due to substantial variability in temperature and salinity spatially as well as vertically. In order to reduce the magnitude of errors in estimating ranges (and hence draft), it is common practice to establish a local baseline calibration by identifying "open water" in the observed data. This method is described in [8]. This method results in the range error being proportional to the ice draft rather than the distance to the ice.

2. Misalignment errors

$$\phi = \phi_{meas} + \Delta\phi_{align} \quad \text{[Roll]} \quad (0.37)$$

$$\theta = \theta_{meas} + \Delta\theta_{align} \quad \text{[Pitch]} \quad (0.38)$$

$$\alpha = \alpha_{meas} + \Delta\alpha_{align} \quad \text{[Heading]} \quad (0.39)$$

From section 3.1.5 of Hare et. al. [6]:

$$\sigma_\phi = \sqrt{\sigma_{\phi_{meas}}^2 + \sigma_{\Delta\phi_{align}}^2} \quad (0.40)$$

$$\sigma_\theta = \sqrt{\sigma_{\theta_{meas}}^2 + \sigma_{\Delta\theta_{align}}^2} \quad (0.41)$$

$$\sigma_\alpha = \sqrt{\sigma_{\alpha_{meas}}^2 + \sigma_{\Delta\alpha_{align}}^2} \quad (0.42)$$

3. Errors in mapping sonar or echosounder errors into measured distance to the ice

$$\sigma_{d_1}^2 = \left(\frac{\partial d_1}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial d_1}{\partial \theta}\right)^2 \sigma_\theta^2 + \left(\frac{\partial d_1}{\partial \phi}\right)^2 \sigma_\phi^2 \quad (0.43)$$

$$\left(\frac{\partial d_1}{\partial r}\right)^2 \sigma_r^2 = \left(\cos\theta \cos\phi\right)^2 \sigma_r^2 \quad (0.44)$$

$$\left(\frac{\partial d_1}{\partial \theta}\right)^2 \sigma_\theta^2 = \left(r\sin\theta \cos\phi\right)^2 \sigma_\theta^2 \quad (0.45)$$

$$\left(\frac{\partial d_1}{\partial \phi}\right)^2 \sigma_\phi^2 = \left(r\cos\theta \sin\phi\right)^2 \sigma_\phi^2 \quad (0.46)$$

4. Beam width

The effect of beamwidth on accuracy is discussed in sections 3.1.7.1 and 3.1.7.2 of Hare et. al. [7] and in Wadhams [2,9].

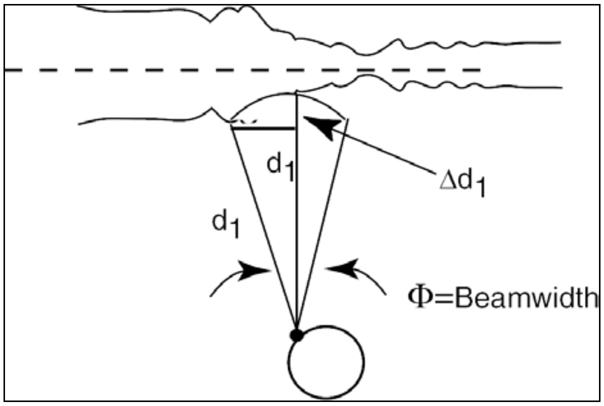


Figure 3: Schematic representation of the effect of beamwidth. Note that the sonar system records the first return that may not be directly above the vehicle.

$$\Delta d_1 = d_1 - d_1 \cos\left[\frac{\Phi}{2}\right] \tag{0.47}$$

$$\sigma_\Phi^2 \approx \left\{ d_1 \left[1 - \cos\left(\frac{\Phi}{2}\right)\right]\right\}^2 \tag{0.48}$$

The total measured error in d_1 is:

$$\sigma_{d_1} = \sqrt{\begin{aligned}&(\cos\theta\ \cos\phi)^2 \sigma_r^2 \\ &+ (r\sin\theta\ \cos\phi)^2 \sigma_r^2 \\ &+ (r\cos\theta\ \sin\phi)^2 \sigma_\phi^2 + \sigma_\Phi^2\end{aligned}} \tag{0.49}$$

5. Dynamic bias errors

The only errors left to calculate from the equation for ice draft, D, are those for h and d_0 which will be wrapped together in this section.

Let:

$$h = h_{calc} + h_1 \tag{0.50}$$

Where:

- h_1 = the vertical distance from the pressure sensor to the local-level origin
- h_{calc}= he depth calculated at the location of the pressure sensor

The pressure sensor offset is:

$$\begin{bmatrix} 0 \\ 0 \\ Z = h \end{bmatrix}_{LL} = R_{LL/B} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B \tag{0.51}$$

$$h_{1_{LL}} = \begin{pmatrix} X_B^P \sin\theta \\ -Y_B^P \cos\theta \sin\phi \\ -Z_B^P \cos\theta \cos\phi \end{pmatrix} \tag{0.52}$$

Where:

$$\begin{bmatrix} X^P \\ Y^P \\ Z^P \end{bmatrix}_B \tag{0.53}$$

are the coordinate offsets between the pressure sensor and the body frame origin in body coordinates.

Likewise, for d_0:

$$d_{0_{LL}} = \begin{pmatrix} X_B^X \sin\theta \\ -Y_B^X \cos\theta \sin\phi \\ -Z_B^X \cos\theta \cos\phi \end{pmatrix} \tag{0.54}$$

Where:

$$\begin{bmatrix} X^X \\ Y^X \\ Z^X \end{bmatrix}_B \tag{0.55}$$

are the coordinate offsets between the transducer and the body frame origin in body coordinates.

Now, looking at the errors (using the method of propagation of errors):

$$\sigma_h = \sqrt{\sigma_{h_{calc}}^2 + \sigma_{h_1}^2} \tag{0.56}$$

Where:

$$\sigma_{h_1}^2 = \begin{pmatrix} \left(\dfrac{\partial h_1}{\partial X^P}\right)^2 \sigma_{X^P}^2 \\ +\left(\dfrac{\partial h_1}{\partial Y^P}\right)^2 \sigma_{Y^P}^2 \\ +\left(\dfrac{\partial h_1}{\partial Z^P}\right)^2 \sigma_{Z^P}^2 \\ +\left(\dfrac{\partial h_1}{\partial \theta^P}\right)^2 \sigma_{\theta^P}^2 \\ +\left(\dfrac{\partial h_1}{\partial \phi^P}\right)^2 \sigma_{\phi^P}^2 \end{pmatrix} \quad (0.57)$$

$$= (\sin\theta)^2 \sigma_{X^P}^2 + (\cos\theta \sin\phi)^2 \sigma_{Y^P}^2 + (\cos\theta \cos\phi)^2 \sigma_{Z^P}^2$$
$$+ \left[X_B^P \cos\theta + Y_B^P \sin\theta \sin\phi + Z_B^P \sin\theta \cos\phi \right]^2 \sigma_\theta^2$$
$$+ \left[-Y_B^P \cos\theta \cos\phi + Z_P^B \cos\theta \sin\phi \right]^2 \sigma_\phi^2$$
$$(0.58)$$

$\sigma_{h_{hcalc}}^2$ = The error in absolute pressure measurement plus the error in atmospheric pressure measurement plus the error in calculation of depth from the pressure measurement.

$\sigma_{d_0}^2$ is the same as $\sigma_{h_1}^2$ with:

$$\begin{bmatrix} X^P \\ Y^P \\ Z^P \end{bmatrix} \text{ replaced by } \begin{bmatrix} X^X \\ Y^X \\ Z^X \end{bmatrix}$$

Note that these errors are analogous to the heave errors described in section 3.2.1 of Hare et. al. [7].

At this point the variances in D are expressed entirely as a function of Roll, Pitch, Heading, alignment errors and sensor variances. So:

$$\sigma_D = \sqrt{\sigma_h^2 + \sigma_{d_0}^2 + \sigma_{d_1}^2} \quad (0.59)$$

which is equal to the total ice draft estimation error.

IV. Conclusion

In order effectively extend the ice draft time series into the future where new sonar systems will be used, a careful intercomparision should be done between an OD-161 and more modern sonars.

A set of robust estimates of the errors in the existing data sets should be developed and the existing processing methods should be reviewed to insure recovery of the best possible draft estimates.

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