Tracer Study of Mixing and Transport in the Upper Hudson River with Multiple Dams

Theodore Caplow1; Peter Schlosser2; and David T. Ho3

Abstract: In October 2001, ~0.2 mol of SF6 was injected into the upper Hudson River, a modified natural channel with multiple dams, at Ft. Edward, N.Y. The tracer was monitored for 7 days as it moved ~50 km downriver. The longitudinal evolution of the tracer distribution was used to estimate one-dimensional advection (9.0±0.2 km d−1) and dispersion (17.3±4.0 m2 s−1) along the river axis. Comparison of these results to tracer studies on channels without dams suggests that dams reduce longitudinal dispersion below the value expected in a natural channel with the same discharge. SF6 loss through air–water gas exchange along the river and at two dams (10.7 m combined height) was estimated by observing decay in peak concentration. Losses at dams (approximately 50% per dam) were dominant. The estimated gas exchange at dams was compared to a simple model adapted from those available in literature. Small amounts of tracer were trapped in a canal segment (~5 km long) that parallels the river, where advection and dispersion were sharply reduced.

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Introduction

The evolution of a soluble contaminant or nutrient introduced into a river can be divided into three successive stages, corresponding to periods of vertical, transverse, and longitudinal mixing, respectively (Fischer et al. 1979). The third stage, sometimes referred to as the far field, begins many river widths downstream of the solute’s source, when vertical and transverse mixing are essentially complete. The focus of this contribution is the behavior of solutes in the far field, a subject of primary concern for estimation of time-of-travel and/or dilution capacity at the watershed or catchment scale (typically tens or hundreds of km).

Predicting the far-field behavior of contaminants in a river requires flow models, and these models, whether analytical or numerical, must be calibrated to accurate observations of stream transport processes. As they affect solutes in the far field, these processes can be divided into advection (bulk downstream motion), longitudinal dispersion (mixing), and sources/sinks (e.g., air–water gas exchange). Here, we describe the measurement of advection, longitudinal dispersion, and gas exchange in a river by means of a deliberately released tracer, sulfur hexafluoride (SF6).

The experiment covered a 65 km reach of the upper, nontidal Hudson River between Fort Edward, N.Y. and Troy, N.Y. and was conducted between October 17 and 23, 2001. This reach has been heavily altered from its natural state by the addition of a dredged channel, dams, locks, and canals. A number of industrial facilities and power plants are located along this part of the Hudson, and a public water supply intake is located near the bottom of the study area (at Waterford, N.Y.), underscoring the need for an accurate transport model in the event of a chronic or sudden release of a toxic contaminant. The experiment described here was the first application of SF6 tracer to the study area, and among the first large-scale applications of SF6 to a dammed channel. The goal was to provide basic empirical data on transport in the study area while verifying the SF6 tracer methodology for applications to other similar riverine environments.

Previous tracer work in rivers has been performed with fluorescent dyes such as rhodamine (e.g., Wright and Collings 1964; Lowham and Wilson, Jr. 1971; Atkinson and Davis 2000), although there is a growing body of literature for the application of SF6 to river studies (e.g., Clark et al. 1994, 1996; Hibbs et al. 1998; Chapra and Wilcock 2000; Ho et al. 2002). SF6 has several properties that make it preferable to fluorescent dyes for studies of advection and mixing in rivers on large space and time scales. It is nontoxic, inert, inexpensive, and detectable over a large concentration range (the technique used here is sensitive over a concentration range of at least four orders of magnitude from about 10 fmol L−1 to about 100 pmol L−1). SF6 is conservative in the environment and is only lost from the river through gas exchange with the atmosphere.

Study Conditions

The Hudson River flows southward over 500 km from its source in the Adirondack Mountains to New York Harbor, draining an area of 35,000 km2 (Limburg et al. 1986). The river is navigable...
3.7 m downstream of Fort Edward, N.Y. at kilometer point (kmp) 313, measured axially upstream from the southern tip of Manhattan. Below the Federal Dam at Troy (kmp 248), the river is tidal, with bidirectional flow. Above the Federal Dam, the river changes character (Fig. 1), becoming a narrower shallower stream, altered from its natural course by the construction of the Champlain Canal system. There are six locks between kmp 248 and kmp 313, for a total change in elevation of 31 m (Erie 2002). Each lock is 13 m wide and 91 m long and is accompanied by a dam and spillway. Many of the locks are placed at the southern end of man-made canals dug parallel to, but separate from, the natural river. There are four canals in the study area, all located upstream of a lock. These canals, which are approximately 30 m wide, are found above Lock No. 6 (≈5 km), Lock No. 5 (≈1.5 km), and Lock No. 4 (≈0.9 km), and below Lock No. 1 (≈0.7 km). The controlling depth for the canals, locks, and navigable river is 3.7 m (12 ft).

The 25-year mean flow of the Hudson River is 147 m³ s⁻¹ at Ft. Edward and 385 m³ s⁻¹ at Troy (USGS 2002; USEPA 2000). Major tributaries within this reach include the Batten Kill (kmp 293, average flow 17 m³ s⁻¹), the Hoosic River (kmp 270, average flow 38 m³ s⁻¹), and the Mohawk River (kmp 250, average flow 160 m³ s⁻¹). The mean flow at Ft. Edward during October 2001 was 53 m³ s⁻¹. This volume was the lowest recorded in the past 25 years and is equivalent to 36% and 45% of the annual and October means, respectively. This anomaly is expected to have

had a significant effect on rates of advection, dispersion, and gas exchange.

Bathymetry from USEPA (2000) was used to estimate the channel width and mean depth at the injection point, as well as the mean properties for the study reach as a whole. The bathymetry consists of mean depth and mean width for a grid that is one cell wide in the transverse direction, with a 2-km longitudinal resolution, except between kmp 304 and kmp 313, where the transverse resolution is 3 boxes and the longitudinal resolution is 1 km. The mean width and depth for the upper Hudson River are 268 m and 3.1 m, respectively.

Field Methods

Measurements of SF₆ conducted before the tracer injection indicated a background concentration of less than 10 fmol L⁻¹ in the upper Hudson River. On October 17, 2001 at 08:30 EST (“day 0.0”); time is measured in elapsed days since injection), approximately 0.2 moles of SF₆ were bubbled into the river midstream at kmp 311 through a porous rubber/polyethylene tubing (7 m long, 0.6 cm inner diameter, 150 μm pore size). The tubing was wrapped around a metal cylinder attached to a weight and its depth was controlled at approximately 3 m. Flow was maintained for 1 min. Shortly thereafter, survey of the SF₆ patch began, using a boat-mounted sampling and measurement system (Ho et al. 2002). Water from the river was pumped continuously through a counterflow membrane contactor, where gases were extracted and analyzed for SF₆ by electron-capture gas chromatography. This equipment takes a concentration measurement every two minutes. Boat speed during surveys ranged from 3 to 8 knots (5 to 15 km h⁻¹), resulting in a spatial data resolution of 0.2 to 0.5 km along the axis of the river.

The survey was conducted for seven consecutive days, between 8 am and 6 pm. The SF₆ patch was surveyed either as the boat traversed the patch in a longitudinal direction upriver or downriver, or from fixed positions as the tracer-tagged water flowed past. In addition to the main patch, a slug of SF₆ trapped in a 5 km canal above Lock No. 6 (kmp 299) was observed for the duration of the experiment.

Analytical Methods

The far-field study of advection and longitudinal dispersion in rivers is often approached via a one-dimensional (1D) approximation, in which the width and depth are treated as constants, and lateral and vertical mixing are neglected (Fischer et al. 1979; Rutherford 1994). This approximation is justified if the time scale is sufficiently large such that horizontal and vertical mixing are essentially complete before measurements of longitudinal dispersion are made. If a tracer injection is used, analysis is greatly simplified if the injection can be regarded as nearly “instantaneous” in comparison with the time scale of observation.

The transverse scale is much larger than the vertical scale in most rivers, including the upper Hudson. A test of transverse mixing time against longitudinal measurement interval, if successful, suffices to justify the 1D approximation in nonsaline rivers (where vertical water column stratification is assumed to be transient and/or weak). Rutherford (1994) performs this test by assuming a constant dispersion coefficient and estimating transverse mixing time $T_v$.
\[ T_r = \frac{\alpha B^2}{K_x} \]  

where \( B \) = mean width of the river (225 m at the injection site); \( K_x \) = transverse dispersion coefficient; and \( \alpha \) = coefficient defining the well mixed state. According to Fischer et al. (1979), if \( \alpha \) is set to 0.075 then the tracer concentration will be within 10% of its mean anywhere on the cross section after an elapsed time of \( T_r \) (since injection of a center-channel point source). The analytical prediction of \( T_r \) remains unsatisfactory to many investigators (e.g., Heard et al. 2001; Boxall and Guymer 2003) but \( K_x = 0.6Hu^* \) is an expression currently used for slightly meandering channels (Fischer et al. 1979), where \( H \) is mean channel depth (2.6 m at the injection site) and \( u^* \) is the shear velocity. The analytical determination of \( u^*/u \) is beyond the scope of this discussion, but for the present purpose a value of 0.10 is appropriate (Fischer et al. 1979; also supported by empirical data in Deng et al. 2001). Using \( u = 0.1 \text{ m s}^{-1} \) (9 km d\(^{-1}\), derived from this study), \( T_r \) was found to be 65 h. The first survey of SF\(_6\) concentration used in the calculation of longitudinal dispersion took place 54 h after injection. Although this survey took place slightly before \( T_r \), several factors indicate that this survey (and later surveys) can be subjected to 1D analysis, without correction for transverse effects: (1) The flow passed over two dams during this period, accelerating mixing; (2) the injection was not in fact a point source, but instead was made along a traverse of half the river’s width, substantially accelerating transverse mixing; and (3) as transverse mixing approaches completion, transverse effects on longitudinal dispersion become much smaller than other sources of error (e.g., limits of equipment precision and consequent scatter of the concentration measurements). For these reasons, transverse mixing was assumed essentially complete by the time of the first survey.

All measurements used for the calculation of advection, longitudinal dispersion, and gas exchange were made in reaches of the river that were free from sudden changes in width, depth, or cross-sectional area. Analysis of the measured SF\(_6\) distributions was approached via the idealized 1D advection–dispersion equation (Taylor 1954; Levenspiel and Smith 1957; Fischer et al. 1979; Rutherford 1994):

\[ \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = K_x \frac{\partial^2 c}{\partial x^2} - \lambda c \]  

where \( u \) = mean advection; \( K_x \) = longitudinal dispersion coefficient; and \( \lambda \) = first-order loss term due to air–water gas exchange. Solving Eq. (2) yields:

\[ c(x,t) = \frac{vM}{A} \exp \left[ -\frac{(x-ut)^2}{4K_x t} - \lambda t \right] \]  

where \( v \) = specific volume of the gas tracer; \( M \) = mass of trace gas injected; and \( A \) = mean cross-sectional area of the river (all three of these values are constant for the purposes of this analysis). One method for determining \( K_x \) from experimental data is via the “change of moment” (Fischer et al. 1979; Rutherford 1994):

\[ K_x = \frac{1}{2} \left( \frac{d\sigma^2}{dt} \right) = \frac{1}{2} \left( \frac{\sigma_x^2 - \sigma_y^2}{t_2 - t_1} \right) \]  

where \( \sigma_x^2 \) and \( \sigma_y^2 \) = variances (second moments) of the tracer distributions at times \( t_1 \) and \( t_2 \), respectively. Eq. (4) is applied between SF\(_6\) surveys, taken on different days, to calculate a single mean value of \( K_x \) for the temporal and spatial extent of the study.

The value of \( \sigma^2 \) for the SF\(_6\) patch from a single survey was estimated by fitting Gaussian curves (minimizing chi square) to two longitudinal profiles of the patch, taken very close together in time but in opposite directions, and averaging \( \sigma^2 \) from both curve fits. Bidirectional profiles were used to reduce uncertainty and to remove any possible bias from lag or memory effects in the sampling equipment. The maximum discrepancy between observed values for \( \sigma^2 \) within a single pair of bidirectional profiles was 10%. Four profiles suitable for the fitting of Gaussian curves were collected (a bidirectional pair on both days 3 and 4). In addition, three pairs of bidirectional profiles were collected in the canal above Lock No. 6 (on days 2, 3, and 4) and analyzed for dispersion by the same method. This approach corresponds to the simplest analytical model, which is symmetrical and constant (i.e., Fickian) dispersion. More complex models are also in use, in particular the “dead zone model,” attributed to Hays (1966), which accounts for an asymmetrical tracer patch (with longer upstream tail) by proposing that portions of the river, or its bed, retard pockets of tracer while the rest is advected downstream. The SF\(_6\) profiles from the present study display a slight asymmetry, but the upstream tails were truncated by dams, as it was not possible to survey continuously past these structures.

Eq. (4) requires a set of observations of \( \sigma^2 \), each of which is assumed to be synoptic. The data were normalized to a synoptic form by shifting each data point a small distance, determined by the mean advection \( (u) \) and the elapsed time between data collection at that point and the time when the peak SF\(_6\) concentration was observed. The maximum shift was \(<0.1 \sigma\), where \( \sigma \) was the variance of the profile. Dispersion during the profile itself was neglected.

Loss of SF\(_6\) via gas exchange at the air–water interface was investigated by examining the decline, between successive observations, of the peak SF\(_6\) concentration in the tracer patch. (This approach was chosen because the collection of complete SF\(_6\) inventories, from which losses could be directly calculated, was only possible on one occasion due to interference from locks and dams.) Fischer et al. (1979) observed that in a 1D system with symmetrical Fickian dispersion and no losses the evolution of the peak concentration \( (c_p) \) is proportional to the inverse of the square root of the elapsed time since injection: \( c_p \propto t^{-0.5} \). This conclusion is evident from a consideration of Eq. (3) at the peak \( (x=ut) \) with no losses \( (\lambda=0) \). If such a tracer peak is measured at two times \( t_1 \) and \( t_2 \), then it follows directly that

\[ \frac{c_{p1}^2}{c_{p2}^2} = \left( \frac{t_2}{t_1} \right)^{0.5} \]  

where \( c_{p1} \) and \( c_{p2} \) = peak tracer concentrations at times \( t_1 \) and \( t_2 \), respectively. For the purpose of evaluating net tracer loss from peak concentration data, it is useful to introduce \( f_{12} \), the fraction of tracer which is removed from the water between times \( t_1 \) and \( t_2 \):

\[ f_{12} = 1 - \frac{c_{p1}^2}{c_{p2}^2} \left( \frac{t_2}{t_1} \right)^{0.5} \]  

The metric \( f_{12} \) is used instead of \( \lambda \) because the latter implies a steady, first-order, exponential loss or decay process, whereas \( f_{12} \) refers to the integrated loss between two points in time, whether steady, unsteady, or effectively instantaneous (e.g., at a dam). Eq. (6) does not depend upon the actual form or temporal distribution of the losses, provided that the tracer profiles at both \( t_1 \) and \( t_2 \) are well represented by the Gaussian curves described by Eq. (3), and that Eq. (5) is valid. Jobson (1997) performed regression analysis on a heterogeneous data set of 422 cross sections from 60 rivers in the United States and found that \( c_{p1} \propto t^{-0.89} \), whereas various
other investigators have found values for the exponent ranging from −0.7 to −1.0. This increase in longitudinal dispersion above the intensity suggested by the Fickian model is usually ascribed to additional modes of mixing (e.g., dead zones). The simpler relationship $c_p \times t^{0.5}$ was adopted here for two reasons. First, this relationship maintains consistency with the Fickian model used for the dispersion calculations, which appears the most generic model for observing first-order transport effects under these conditions. Second, this relationship matches the observed decay of the SF$_6$ peak in an interdam reach of the study area about 35 km downstream of the injection point (see Results section), indicating that the impact of patch asymmetry was minor for the experiment described. Considering the potential for gas exchange losses over this reach, it appears the appropriate exponent relating $c_p$ and $t$ may have been even higher than −0.5, a result at odds with the aforementioned regressions from literature.

Eq. (6) was applied to data from three surveys where $c_p$ was well defined (one each on days 2, 3, and 5) to evaluate the effect of dams upon SF$_6$ losses to the atmosphere. A large increase in gas exchange is expected at dams, due to the extreme turbulence, bubble invasion, and vertical mixing that occur downstream (e.g., Cirpka et al. 1993; Asher et al. 1996). The reach between the survey locations for days 2 and 3 is regular in form and unobstructed. Two dams (4.8 m and 5.9 m; Erie 2002) are located between the survey locations for days 3 and 5.

A minor correction was made to $f_{1S}$ to account for dilution from tributaries (20% over this reach; USEPA 2000). This dilution was assumed to cause a linear reduction in $c_p$ as a function of distance downstream from the injection point.

Results

SF$_6$ was measured in the river until day 6, at which time the signal of the main patch had decreased to nearly undetectable levels. The downstream motion and longitudinal spreading of the SF$_6$ peak is well represented in the data, despite irregularities in the river (locks and dams). Results from a longitudinal profile taken on day 3 (Fig. 2) clearly indicate small slugs of tracer trapped in canals at Lock Nos. 5 and 6, as well as the main tracer patch further downstream. The SF$_6$ mass inventory in the main channel on day 3 was estimated as 1.5±0.15 mmol, approximately 1% of the estimated amount injected. Based on prior experience (Ho et al. 2002) and the relatively shallow depth of the river, it is expected that the majority of the SF$_6$ injected (more than 90%) escaped from the water as bubbles during the injection process. Additional reduction in the SF$_6$ inventory is attributed to air–water–gas exchange, including losses at dams.

Advection

Observations of the peak concentrations within the tracer patch enabled estimates of the mean advection rate (Fig. 3). Mean advection over a 42 km reach (kmp 306 to kmp 264) was 9.0±0.2 km d$^{-1}$ (0.104±0.002 m s$^{-1}$), corresponding to a transit time of seven days from Fort Edward to Troy 65 km if the same rate is applied to the remaining 23 km. Multiplying the observed advective rate by the cross-sectional area of the river at the injection point just south of Ft. Edward (585 m$^2$) yields a flow rate of 61 m$^3$ s$^{-1}$. This rate is in reasonable agreement with the rate of 53 m$^3$ s$^{-1}$ reported by the USGS (2002) at Ft. Edward for the whole of October 2001.

Dispersion

On day 2 (at kmp 289) and day 3 (at kmp 280), it was possible to obtain pairs of bidirectional profiles of the main body of the SF$_6$ patch suitable for extracting the variance ($\sigma^2$) by fitting a Gaussian curve to each profile (Fig. 4). The variances for each pair were...
averaged and the longitudinal dispersion coefficient \( K_x \) was calculated using Eq. (4), resulting in values of 15.7±2.0 \( \text{m}^2\text{s}^{-1} \) for day 2.3 (55 h from injection) and 19.0±2.4 \( \text{m}^2\text{s}^{-1} \) for day 3.2 (77 h from injection). The average \( K_x \) was 17.3±4.0 \( \text{m}^2\text{s}^{-1} \) (1.50±0.35 km$^2$ d$^{-1}$). The data supporting this value were collected along the top 31 km of the study reach. Downstream of kmp 280, no further determinations of \( \sigma^2 \) were possible. In this lower reach (kmp 280 to kmp 248), significant changes in the character of the channel or the volume of the flow are not evident, suggesting that the estimate of \( K_x \) obtained from the upper reach should be a reasonably good approximation for this region as well. A check of the ratio of the injection time (1 min) to the quantity \( \sigma/\nu \) for the first survey (6.7 h) confirms that the injection can be treated as instantaneous for the purposes of Eq. (2), thereby justifying the solution given in Eq. (3).

### Gas Transfer

Results from applying Eq. (6) to three successive pairs of peak tracer concentration measurements are shown in Table 1. According to these limited results, the dams are the major cause of SF$_6$ tracer loss in the study area, and SF$_6$ losses between dams are a very small term in the overall loss rate. No detectable loss was measured over a 9 km reach (measurements were 23 h apart) between kmp 289 and kmp 280 that contained no dams, but a 72% loss was recorded over a 14 km reach (measurements were 39 h apart) between kmp 280 and kmp 266 that contained two dams.

### SF$_6$ Trapped in the Canal above Lock No. 6

The mean cross-sectional area of the canal above Lock No. 6 was estimated (field observations) to be 90±25 \( \text{m}^2 \). The total SF$_6$ inventory in this canal was calculated from a profile taken on day 3 (3.1 days after injection) as 72±13 \( \mu\text{mol} \) (using a cross-sectional area of 90 \( m^2 \)). This inventory is approximately 5% of the inventory measured in the main channel on day 3. Tracer mass in the canal was also calculated from profiles on days 2 and 4 (Fig. 5), but the data do not show a significant trend that would enable an estimate of gas transfer rate. Gaussian curves were fitted to each profile (Fig. 5). Advection was determined from the peaks of the curve fits to be 0.7±0.06 \( \text{km} \text{d}^{-1} \). Longitudinal dispersion \( K_x \) in the canal was estimated as 1.2±0.3 \( \text{m}^2\text{s}^{-1} \) (0.11±0.025 km$^2$ d$^{-1}$) by a least square fit of \( \sigma^2 \) versus time, corresponding to the relationship predicted by Eq. (4).

### Discussion

The advection rate along the measured reach was constant within measurement errors (Fig. 3). Intuitively, this result seems surprising in view of the irregularities along this reach. To examine the uniformity of flow per unit area, cross-sectional area, and average contributions from tributaries were obtained from USEPA (2000) and plotted versus river kilometer (Fig. 6). The plot illustrates that as tributaries and runoff enter the main stem of the river, the cross-sectional area increases to accommodate the additional flow, producing a near-constant mean velocity.

### Table 1. SF$_6$ Losses with and without Dams, Calculated from Peak Concentrations

<table>
<thead>
<tr>
<th>Day</th>
<th>kmp</th>
<th>( \sigma_{peak} ) (km)$^b$</th>
<th>Dispersion only</th>
<th>Dispersion and Dilution</th>
<th>Actual peak (fmol L$^{-1}$)</th>
<th>( f_{12} )</th>
<th>( t_2-t_1 ), days</th>
<th>Number of dams 1→2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.31</td>
<td>288.8</td>
<td>2.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.21</td>
<td>279.9</td>
<td>3.10</td>
<td>239</td>
<td>223</td>
<td></td>
<td>222</td>
<td>0.00</td>
<td>0.9</td>
</tr>
<tr>
<td>4.90</td>
<td>266.1</td>
<td>3.83</td>
<td>193</td>
<td>155</td>
<td></td>
<td>43$^b$</td>
<td>0.72</td>
<td>1.7</td>
</tr>
</tbody>
</table>

$^a$The values given for \( \sigma \) are not the measured values; these are expected values based on the measured mean \( K_x \). This approach is necessary to apply the peak inventory estimate evenly to locations where \( \sigma \) could not be measured directly, as well as those where it could be measured.

$^b$The peak was tentatively identified twice more shortly after this time. Both measurements were similar to this one. Following these measurements, the patch crossed another dam and signal levels fell below the detection limit.

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**Fig. 5.** Gaussian curves fit to 3 days of profiles in the canal above lock No. 6. All profiles shown are northbound. Average inventories from bidirectional profiles are shown above each curve.

**Fig. 6.** Variation of cross-sectional area and flow in the upper Hudson River (bathymetry from EPA 2000)
Deng et al. (2001) compile a set of 73 empirical data points containing longitudinal dispersion coefficient, width, depth, mean velocity, and shear velocity, drawn from studies of 29 different rivers in the United States. The data set, derived primarily from dye tracer studies, contains 58 data points from Seo and Cheong (1998) and 15 data points from Rutherford (1994). These measurements were made on reaches without dams. As no similar data set exists for rivers with dams, the Deng et al. (2001) data is used here as a means for comparing the upper Hudson River to channels without dams.

The median and mean \( K_x \) for all 73 data points are 40.5 and 132 m\(^3\) s\(^{-1}\), respectively. 77% of the \( K_x \) measurements were higher than that found for the upper Hudson River. These data were used to construct plots of \( K_x \) versus nominal discharge (width \( \times \) depth \( \times \) mean velocity), mean velocity, and nominal cross-sectional area (width \( \times \) depth), respectively (Fig. 7). Two upper Hudson River data points were added to the plots: The solid circles represent the results of the October 2001 SF\(_6\) study. The solid triangles (“normal-flow prediction”) represent the upper Hudson River after applying a correction for the anomalously low flow conditions prevalent during the experiment. An increase by a factor of 2.4 in the discharge of the upper Hudson River during the study would correspond to 25-year mean conditions. Hibbs et al. (1998), on the basis of earlier work by McQuivey and Keefer (1974), suggest that a linear relationship between \( K_x \) and discharge is successful within \( \pm 80\% \) in natural streams if at least one measured value of \( K_x \) has already been found. Applying a linear factor of 2.4 to \( K_x \) in the upper Hudson River results in an increase from 17 to 42 m\(^2\) s\(^{-1}\). It is further assumed, for the purposes of this prediction, that the cross-sectional area would not change significantly with discharge (the dams control the cross-sectional area), but that the mean velocity would increase by an amount proportional to the increase in discharge.

In comparison to the undammed channels from the Deng et al. data, \( K_x \) in the upper Hudson River (for both the observed and normal-flow cases) falls within or slightly above the range expected from the measured velocity [Fig. 7(b)], but well below the range expected from the discharge [Fig. 7(a)] or nominal cross-sectional area [Fig. 7(c)]. It can be generally observed from Fig. 7 that \( K_x \) is a stronger function of velocity than of cross-sectional area. Taken together, these two points imply that the addition of dams to the upper Hudson River reduced \( K_x \) below the values expected from a natural channel of the same discharge. The effect of dams is to increase cross-sectional area and reduce velocity proportionally. The greater dependence of \( K_x \) on velocity results in a net reduction of \( K_x \), causing a sharp reduction of \( K_x \) below the value expected based on cross-sectional area [Fig. 7(c)]. This hypothesis should be verified by further empirical confirmation, e.g., additional tracer studies on dammed channels.

The prediction of \( K_x \) from theoretical relationships based upon physical stream parameters was reviewed (e.g., Deng et al. 2001, 2002), as were regressions of peak unit concentration (a measure of longitudinal dispersion normalized by flow rate) against time (Jobson et al. 1997). Although these models offer some promise for undammed reaches, none have demonstrated reliable predic-
tion of \( K \), within a factor of 2 on a river not used for the development of the model. For example, the relationship of Deng et al. (2001) predicts \( K = 6.9 \text{ m}^2 \text{s}^{-1} \) for the upper Hudson River under the study conditions (using measured \( B, h, \) and \( u \) and assuming \( u^* / u = 0.1 \)). Despite advances in analytical prediction, tracer studies continue to hold value in the determination of longitudinal dispersion, particularly in dammed channels.

The relatively slow mixing and advection in the canals, together with expected seasonal variance in these properties (as a function of boat traffic and lock cycling rate), is of significance for spill forecasting and modeling. The canals appear to have the capacity to trap small amounts of solutes from the main channel for long periods of time (days to weeks). It follows that contaminants introduced directly into the canals would persist for a much longer time, and at higher concentrations, than they would if spilled in the main channel. Five days into the experiment, parcels of water containing significant amounts of SF\(_6\) were still working their way through the canal above Lock No. 6 (only several km south of the injection point). At this time the main patch had passed 30 km downriver and peak concentrations there were an order of magnitude smaller than in the Lock No. 6 canal.

## Predicted Gas Transfer at Dams

Erie (2002) tabulated the change in river elevation at each lock on the upper Hudson; these drops are good estimates for the heights of the dams. Based upon a review of literature, a model was developed (Appendix I) to approximate the loss of SF\(_6\) over the dams of the upper Hudson River as a function of dam height, \( h \), and specific flow, \( q \) (flow per unit of dam width). The net flow rate at these dams was calculated as 76 \( \text{m}^3 \text{s}^{-1} \) by adding 24% tributary dilution (USEPA 2000) to the flow determined at the injection site from the measured advection. Three values for \( q \) were examined, corresponding to 50, 100, and 200% of the net flow rate divided by an approximate mean dam width of 200 m (observed from satellite photos) (see Fig. 8). Predicted values for \( f_{12} \) from the model (75–86%) are compared to the measured (72%) cumulative losses over the two dams spanned by the tracer patch peak measurements on days 3 and 5 (Table 2). The model developed here is a rough approximation which appears to capture the major effects. A dedicated experiment would be necessary to understand all of the factors affecting gas exchange over dams of this type.

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## Table 2. Net Tracer Loss over Two Dams Compared with Analytical Predictions

<table>
<thead>
<tr>
<th>kmp</th>
<th>( h ) (m)</th>
<th>( q=0.19 \text{ m}^2 \text{s}^{-1} )</th>
<th>( q=0.38 \text{ m}^2 \text{s}^{-1} )</th>
<th>( q=0.76 \text{ m}^2 \text{s}^{-1} )</th>
<th>Predicted ( f_{12} ) per dam</th>
<th>Cumulative ( f_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>270</td>
<td>4.8</td>
<td>0.59</td>
<td>0.53</td>
<td>0.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>267</td>
<td>5.9</td>
<td>0.66</td>
<td>0.60</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where \( E = \text{gas transfer efficiency (achieved gas transfer divided by total potential gas transfer)} \); \( r = \text{the excess ratio (or deficit ratio, depending upon the gas)} \); \( C_e = \text{equilibrium gas concentration} \); and \( C_u \) and \( C_d \) gas concentrations upstream and downstream of the structure, respectively.

According to Cirpka et al. (1993) and Gulliver et al. (1998), gas transfer at hydraulic structures of moderate size (1 to 10 m high) is dominated by diffusion across the surface of air bubbles that are entrained as the falling water enters the catch basin below the dam. The formation and division of bubbles greatly magnify the available surface area for gas transfer, while the physical turbulence in the zone where these bubbles rise to the surface accelerates dispersion away from the bubble–water interface. This process can be parameterized in a number of ways. Gulliver et al. (1998) investigated 12 formulations and settled on the following classical relationship, originally formulated by Avery and Novack (1978), as the best predictor for oxygen invasion at weirs:

\[
E_{O_2,\text{weir}} = 1 - \frac{1}{1 + 2.4 \times 10^{-3} F^{1.78} R^{0.33}}
\]

\[
F = \left( \frac{8gh^2}{\nu^2} \right)^{1/4}
\]

\[
R = \frac{q}{\nu}
\]

where \( E_{O_2,\text{weir}} = \text{gas transfer efficiency for oxygen} \); \( F = \text{Froude and Reynolds numbers of the jet, respectively} \); \( g = \text{gravitational constant} \); \( h = \text{drop over the weir} \); \( q = \text{specific flow per horizontal length of weir (m}^2 \text{s}^{-1} \) ); and \( \nu = \text{kinematic viscosity of the water} \) (Systeme International units are used for all variables). This relationship was developed for water at 15°C. Over 95% of upper Hudson River water temperature observations from the October 2001 study were within ±2°C of 15°C.

In order to apply Eq. (8) to SF\(_6\), the difference in molecular properties between \( O_2 \) and SF\(_6\) must be considered. Cirpka et al. (1993) show that both \( O_2 \) and SF\(_6\) have sufficiently high Henry’s Law constants to ensure that gas exchange is limited by the liquid.

## Appendix I. Model for SF\(_6\) Loss over Dams on the Upper Hudson River

Both Gulliver et al. (1998) and Cirpka et al. (1993) present predictive relationships found in the literature for gas transfer efficiency as a function of physical parameters of flow over hydraulic structures (dams, weirs, spillways, cascades, and gates). It is standard in these relationships to employ the dependent variables \( E \) and \( r \):

\[
E = 1 - \frac{1}{r}
\]

\[
r = \frac{C_u - C_e}{C_d - C_e}
\]

Both Gulliver et al. (1998) and Cirpka et al. (1993) present predictive relationships found in the literature for gas transfer efficiency as a function of physical parameters of flow over hydraulic structures (dams, weirs, spillways, cascades, and gates). It is standard in these relationships to employ the dependent variables \( E \) and \( r \):
side of the air–water interface, even inside of a rising bubble of entrained air. Their formulation for the excess ratio, \( r \), for gases sharing this characteristic takes the form:

\[
r = 1 + Y \sqrt{D}
\]

where \( Y \) = constant specific to the flow and to the weir; and \( D \) = aqueous molecular diffusion coefficient. Designating \( R_D = D_{SF_6}/D_O_2 \) (the ratio of the molecular diffusion coefficients of \( SF_6 \) and \( O_2 \)) and using Eqs. (7) and (9) and, the \( SF_6 \) transfer efficiency can be expressed as a function of \( O_2 \) and \( R_D \):

\[
E_{SF_6} = \left( 1 + \frac{(O_2)^{-1} - 1}{\sqrt{R_D}} \right)^{-1}
\]

In all calculations, \( R_D \) is taken as 0.53.

As written, Eq. (8) is intended for oxygen invasion, and therefore Eq (10) is an estimate for the invasion of \( SF_6 \), rather than its evasion (loss). Asher et al. (1996) demonstrate that gas transfer in bubble plumes should not be assumed to be symmetrical with regard to invasion or evasion of a particular gas. Invasive transfer includes a contribution from a certain fraction of bubbles that will collapse completely before surfacing, adding gas to the water in a nonequilibrium process. Evasive transfer lacks this term and therefore will be overestimated by parameterizations developed from observations of invasion. Asher et al. (1996) point out that this asymmetry will increase with decreasing solubility (because the contribution of a collapsed bubble becomes more significant relevant to the contribution of a surfacing bubble), and also suggest that evasion for highly insoluble gases such as \( SF_6 \) will be further reduced (relative to invasion) by depletion of dissolved gas in the immediate vicinity of the rising bubble plume.

Asher et al. (1996) conducted experiments in a seawater tank, revealing that under sufficiently bubbly conditions, transfer efficiency for \( SF_6 \) invasion could be up to 50% greater than for \( SF_6 \) evasion. However, their results also suggest that the equations derived here from \( O_2 \) data can be expected to underestimate \( SF_6 \) invasion by roughly the same amount that they overestimate \( SF_6 \) evasion; thus the total overestimate of \( SF_6 \) loss from a river dam is expected to be substantially less than 50%. The current model is intended to illustrate the effect of various parameters (such as dam height) on gas exchange while providing no more than a crude estimate of the magnitude. Lacking methodical studies of \( SF_6 \) loss over dams of the type encountered in the upper Hudson River, any additional precision would be conjectural at this point. Accordingly, the model neglects asymmetry with regard to direction.

Eq. (8) was originally derived for sharp crested weirs with a separated free jet of water. On the upper Hudson River, the majority of dams were observed to be of the rounded crest type (ogee crest), with an inclined spillway and attached flow. Based on 54 small dams, Butts and Evans (1983) developed a predictive equation for oxygen deficit ratio \( r \) that included a shape parameter. As presented in Tang et al. (1995), their equation has the form:

\[
r = 1 + b \cdot f(a,h,T)
\]

where \( a \) = factor to account for water quality; \( h \) = dam height; \( T \) = temperature; and the shape parameter \( b \) ranges from 0.6 for a broad-crested dam to 1.05 for a sharp-crested weir. In order to convert transfer efficiencies derived from the sharp-crested weir, Eq. (8) for use with the broad crested dams of the upper Hudson River, consider two dams of equal height under equal conditions, for which \( f(a,h,T) \) is a constant, \( Z \):

\[
r = 1 + Zb
\]

Comparing this relationship with the top part of Eq. (7) reveals:

\[
E = \left( \frac{1}{Zb + 1} \right)^{-1}
\]

This equation holds for a dam of either shape. To convert from a sharp weir to a broad crest:

\[
\frac{E_{\text{broad}}}{E_{\text{weir}}} = \left( \frac{1}{Zb_{\text{broad}}} + 1 \right) \left( \frac{1}{Zb_{\text{weir}}} + 1 \right)^{-1}
\]

which after manipulation yields:

\[
E_{\text{broad}} = b_{\text{weir}} \left( \frac{1}{E_{\text{weir}} - 1} + 1 \right)^{-1}
\]

Finally, considering that \( C_e < C_d < C_u \) for all of the \( SF_6 \) concentrations in Table 1 allows Eq. (7) to be restated for \( SF_6 \):

\[
E_{SF_6} = 1 - \frac{C_d}{C_u} = f_{12}
\]

where \( f_{12} \) = loss fraction introduced in the main body of this work, applied across the dam. Combining Eqs. (16) and (17) and restating Eqs. (10) and (8) for clarity gives a complete expression for \( SF_6 \) extraction efficiency as a function of flow rate \( q \) and dam height \( h \):

\[
f_{12} = \left( b_{\text{weir}} \left( \frac{1}{E_{SF_6,\text{weir}} - 1} + 1 \right)^{-1}
\]

\[
E_{SF_6,\text{weir}} = \left( 1 + \frac{(O_2,\text{weir})^{-1} - 1}{\sqrt{R_D}} \right)^{-1}
\]

\[
\beta = b_{\text{weir}} / b_{\text{broad}}
\]

\[
E_{O_2,\text{weir}} = \frac{1}{1 + 2.4 \times 10^{-5} F 1.78 R^{0.53}}
\]

\[
R = \frac{q}{v}
\]

where \( \beta = 1.75 \) and \( R_D = 0.53 \) as discussed, and values taken for \( g \) and \( v = 9.81 \) m s\(^{-2}\) and 1.15 \( \times 10^{-6} \) m\(^2\) s\(^{-1}\), respectively.

It is interesting to note that USEPA (2000) make use of Cirpka et al. (1993) as their primary source for modeling volatilization of certain contaminants [polychlorinated biphenyls (PCBs)] at these same dams in the upper Hudson River. The EPA concluded that PCB losses at each dam were less than 3%, and thus chose to neglect volatilization at dams in their composite PCB model of the upper Hudson River.

**Notation**

The following symbols are used in this paper:

- \( A \) = cross-sectional river area;
- \( a \) = water quality factor;
- \( B \) = mean river width;
- \( b \) = dam shape factor;
- \( C_d \) = downstream concentration;
- \( C_e \) = equilibrium concentration;
- \( C_i \) = initial concentration;
- \( C_u \) = upstream concentration;
- \( c \) = tracer concentration;
\[ c_p = \text{peak tracer concentration}; \]
\[ D = \text{aqueous molecular diffusion coefficient}; \]
\[ E = \text{gas transfer efficiency}; \]
\[ F = \text{Froude number}; \]
\[ f_{12} = \text{fraction of tracer lost between observations}; \]
\[ g = \text{gravitational constant}; \]
\[ H = \text{mean river depth}; \]
\[ h = \text{dam height}; \]
\[ K_X = \text{longitudinal dispersion coefficient}; \]
\[ K_Y = \text{transverse dispersion coefficient}; \]
\[ M = \text{mass of trace gas injected}; \]
\[ q = \text{flow per unit width}; \]
\[ R = \text{Reynolds number}; \]
\[ R_D = \text{ratio of molecular diffusion coefficients}; \]
\[ r = \text{excess ratio}; \]
\[ T = \text{temperature}; \]
\[ T_t = \text{transverse mixing time}; \]
\[ t = \text{time}; \]
\[ t_0 = \text{injection time}; \]
\[ u = \text{mean advective velocity}; \]
\[ u^* = \text{bed shear velocity}; \]
\[ v = \text{specific volume}; \]
\[ x = \text{distance traveled}; \]
\[ Y, Z = \text{arbitrary constants}; \]
\[ \alpha = \text{coefficient defining complete transverse mixing}; \]
\[ \beta = \text{ratio of dam shape factors}; \]
\[ \Delta c = \text{air-water concentration difference}; \]
\[ \lambda = \text{gas exchange loss term}; \]
\[ \nu = \text{kinematic viscosity}; \]
\[ \sigma^2 = \text{variance (second moment) of a distribution}. \]

References


Erie Lock 2 Employees (2002). http://www.yolles.net/canals/pagef.htm, viewed 01/20/02.


