Multilevel Upscaling for Two-Phase Porous Flow

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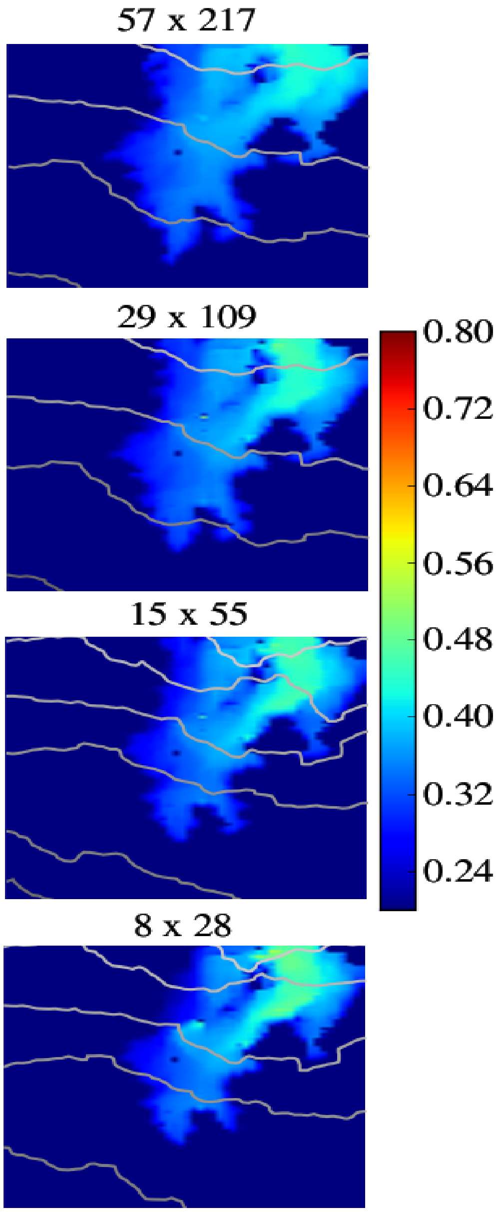
The study of two-phase flows in porous media is fundamental to our understanding of many important problems, including the maintenance of groundwater and the optimization of oil reservoir production. In these problems, fine-scale heterogeneity of the subsurface plays a significant role in even large-scale features of the flow. However, when simulating kilometer-scale reservoirs, modern computing power is insufficient to directly incorporate subsurface material properties that naturally vary on scales as small as millimeters. In particular, absolute permeability, a local measure of the resistance to flow through the medium, may vary by several orders of magnitude over very short length scales. For even simple heterogeneous structures, simple spatial averages of model parameters are known to produce highly inaccurate flows.

In order to capture accurately the influence of fine-scale heterogeneous structure while computing on much coarser scales, multilevel upscaling techniques are being developed. Previous work in single-phase flows [1] has resulted in a multilevel upscaling method (hereafter MLUPS) in which variational coarsening provides a hierarchy of coarse-scale models. Here, we begin to investigate the use of this multilevel upscaling approach in addressing the problem of two-phase porous flow in oil reservoir flooding.

Reservoir Flooding

The two-phase modeling of secondary oil recovery, or reservoir flooding, is a popular and challenging benchmark problem for upscaling methods. In this process, water is pumped into oil reservoirs, effectively pushing out the oil and, thereby, increasing the production rate of the reservoir. Flow in the reservoir is determined by Darcy's Law coupled with conservation of mass



Saturation and pressure contours of a two-phase flow model for the flooding of oil reservoirs to increase production. In this simulation, a well in the upper right corner injects water, while a well at the lower left removes the bulk fluid. All boundaries are impervious. The top plot is a fine-scale simulation on a 57×217 *grid, while the lower plots use a MLUPS based velocity field at the indicated resolution. The essential features of the flow are captured at all resolutions.*

of each phase. This results in a diffusion equation where flow is driven by gradients in pressure and scales with the relative permeability. The relative permeability is a function of the saturation, a measure of the water/oil content of the fluid, and the absolute permeability of the subsurface medium. In this way, the saturation, which is advected by the flow, also couples with the determination of the flow.

To solve this problem we use a standard operator splitting technique called IMPES (implicit in pressure, explicit in saturation). First, the hyperbolic saturation equation is solved, calculating the saturation at the next time step. Then, the pressure equation is updated implicitly using the MLUPS upscaling method, obtaining both pressures and velocities.

In general, finite element pressure solvers that do not explicitly enforce local mass conservation, such as MLUPS, are not locally conservative. However, a locally conservative velocity field is important for avoiding spurious behavior the advection of saturation. To address this issue, we extended the post-processing algorithm of Cordes and Kinzelbach [2]. Specifically, we introduced a method for handling source terms, such as wells, tensorial permeability fields, and non-trivial boundary conditions. However, MLUPS introduces additional approximations to the fine-scale pressure space. Hence, additional smoothing of the reconstructed fine-scale pressure is required. In this study, we used two sweeps of line relaxation before post-processing the velocity field.

This approach is applied to a simplified version of the Tenth SPE Comparative Solution Project's three-dimensional benchmark problem [3]. The two-phase flow equations are solved on a 2160 ft by 1120 ft subdomain of a two-dimensional slice of the benchmark problem. This allows an exact coarsening of up to three levels, by factors of two. The domain initially consists of oil with a residual water saturation of 20%. In this flooding problem, a well injecting water at a rate of 5000 bbl/day is positioned in the upper right corner, and a production well that removes the bulk fluid is positioned at the lower left (see figure). At the top is the saturation from the fine-scale simulation, using 57×217 grid points. Below this plot, we show the saturation obtained with the MLUPS velocity field. In these cases the pressure is solved at the coarser resolutions, coarser by a factor of 8 in the last case. Here, the dominant features of the saturation front are well preserved in all cases. However, it is interesting to observe that some fine-scale features farthest from the injection are diminished or distorted at the coarsest resolution.

Conclusions and Future Work

In this preliminary work, we have demonstrated the potential for MLUPS to be used in two-phase flow modeling. MLUPS, like other multiscale methods, has the advantage that it upscales the fine-scale model and not just its parameters. However, many other multiscale methods use only two levels; the hierarchy of models given by MLUPS allows much more versatility in resolution and accuracy. The variational multigrid approach provides not just corrections, but also solutions on each level. This suggests a method for determining error estimates at each level, and employing adaptivity in the hierarchy to meet the desired accuracy.

Acknowledgements

Los Alamos Report LA-UR-07-xxxx. Funded by the Department of Energy at Los Alamos National Laboratory under contracts DE-AC52-06NA25396 and the DOE Office of Science Advanced Computing Research (ASCR) program in Applied Mathematical Sciences.

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