Two-dimensionalisation of rapidly rotating turbulence

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Over the last decade renewed interest in rapidly rotating turbulence has led to a number of studies employing numerical simulations, laboratory experiments and asymptotic analysis. The point of departure for much of this work has been the Taylor-Proudmann theorem. This classical result merely states that if a rapidly rotating flow evolves slowly with respect to the inverse Coriolis parameter (in other words if the Rossby number, Ro, is small), then the flow must be at least close to two-dimensional. It says nothing about how such a state is achieved starting from, say, isotropic initial conditions.

In problems involving fast waves and slow vortical motions, the important nonlinear interactions are the resonant ones. Only these interactions drive a particular mode's amplitude on the slow timescale. Using this fact and an "instability hypothesis", Waleffe (1993) argued that the nonlinear transfer in rotating turbulence is preferentially towards smaller, but non-zero, $|k_z|$. However, since resonant interactions between a 2D mode ($k_z = 0$) and two inertial wave modes (with $k_z \neq 0$) have a vanishing coupling coefficient, a decoupling of the slow 2D modes from other degrees of freedom is predicted in this limit. Early simulations of decaying rotating turbulence starting from isotropic initial conditions (Bartello, Metais and Lesieur JFM 1994) showed a more rapid decrease of the energy contribution from wave modes than from slow modes. The resulting quasi-2D flow emerged via a more efficient downscale cascade of 3D (or wave) energy. Later, Mahalov et al (1997 Theor. Comput. Fluid Dyn.) predicted not only a decoupling of slow 2D vortical modes from wave modes, but in addition a freezing of the nonlinear transfer at all vertical wavenumbers. This was due to the emergence of an infinite set of adiabatic invariants in the limiting equations at infinite rotations. Recently Cambon, Rubinstein and Godeferd (NJP 2004) have argued that previous work has ignored a wave-vortex interaction that remains active even at large rotation rates, provided the geometry of the flow permits it. This analysis is also not valid if $k_z = 0$.

In this study we use direct numerical simulation to study the mechanism of two-dimensionalisation in inviscid and decaying rotating turbulence. In addition, we perform simulations starting from a quasi 2D state, observing the generation of three-dimensional inertial wave motion, as well as simulations in which there is no initial energy in two dimensional modes. The inviscid truncated model is compared with equilibrium statistical mechanics and decaying simulations were also performed. The resulting behaviour supports the decoupling of the two in the asymptotic limit. However, the Rossby number scaling is quantified and the inhibition of vertical transfer of wave energy will be reported on.

Recent forced simulations of rotating turbulence by Smith and Waleffe (Phys. Fluids 1999) and Chen et al (unpublished 2004) have shown an upscale cascade of quasi 2D motion. These results are interpreted in light of the present study.