Identification of Mantle plumes using Second Generation Wavelets

Gordon Erlebacher, David A. Yuen, and Oleg V. Vasilyev

Mantle convection is a strongly time-dependent processe and has many different scales. Identifying plumes at high Rayleigh numbers and computing their quantitative properties, such as their width and the local heat-flow within them, is an outstanding problem. Automating this process in a reliable manner will help process the increasingly large time-dependent datasets that result from many numerical simulations, needed because of the vast parameter space. Applications of plume identification include computing and constraining the heat budget of the core-mantle boundary, the birthplace of plumes, and computing various heat contributions from various sources, such as viscous heating. Unless the plumes are identified reliably, auxiliary computations will include other contributions from the adjacent mantle outside the plumes, which will skew the results.

One difficulty in reliable plume identification emerges from the continuous nature of the variable (usually temperature, but also composition or both) used to characterize the plumes. Thermal and Thermal-chemical fields have no distinct boundary (i.e., constant temperature, constant temperature gradient, constant composition or a mixture of both). When considering mantle convection in a heated-from below configuration, plumes are usually considered to be regions where the horizontal gradient, or gradients in the case of thermal-chemical convection, is locally stronger than outside the plume. This criterion if of course too simplistic since at high Rayleigh numbers, false plume identification is likely from the strong temperature fluctuations, or from the thinner thinner still chemical fluctuations. Thus far there have been very few studies exist with the intent of identifying plumes; one such uses pattern-recognition techniques (Labrosse 2002).

We use second generation wavelets as a means to identify these plumes (Erlebacher *et al.* 2002). Second generation wavelets are a multiscale technique, which are a generalization of the original wavelets formulations (Sweldens 1997), which compute a wavelet basis by scaling and translating a single mother wavelet. Rather, second generation wavelets transform physical data into a wavelet basis solely through in-place transformations in physical space. This is particularly useful for large datasets. Furthermore, it is possible to establish a one to one correspondance between wavelet coefficients and positions in physical space. Wavelets allow the simultaneous analysis of frequency and spatial structure. Thus, it should be possible to better characterize the structure of plumes: large and small scale components can be distinguished, and selectively retained or removed. The frequency content at both scales can be controlled independently.

To identify the scales of relevance in a plume, we considered the temperature field, which nicely characterizes upwelling and downwelling plumes. After transformation to wavelet space, we performed a hard thresholding operation, which sets to zero all wavelet coefficients whose magnitude falls below a a user-set value. This effectively removes all low and high frequency components that do not contribute significant "energy" to the temperature. The structure location is identified by plotting the positions of the wavelet coefficients in physical space, shown in the bottom three panels of Figure 1. From left to right, we show the strongest 0.8%, 1.2% and 6% of the wavelets. It is clear that the plumes are well captured. The top three panels shown a volumetric reconstruction of the temperature field after performing an inverse wavelet transform (having performed the thresholding operation).

Figure 2 shows the temperature field one plane away from the cold plate (the dataset is 97^3 with $Ra = 10^6$. The wavelet coefficients that exceed a fixed threshold are plotted as black squares. The figure illustrates clearly that the thin, cold filaments are well captured by the wavelet coefficients.

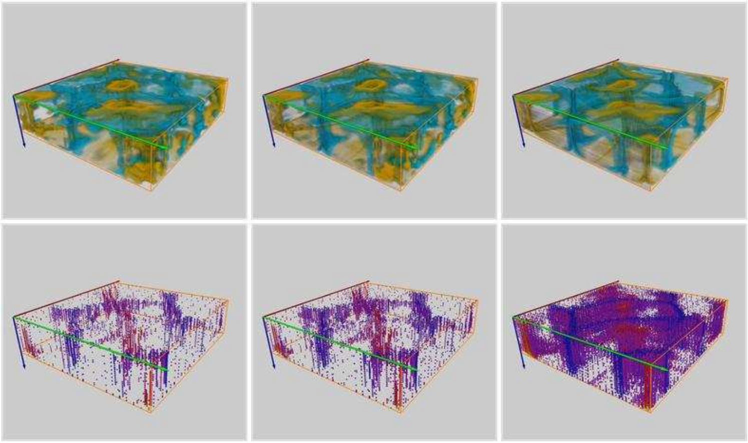

Figure 1: Wavelet coefficients (top) and volumetric reconstruction (bottom) of the temperature field. Left: 0.8%, middle: (1.2%), right: (6%) of the wavelet coefficients are retained during thresholding.

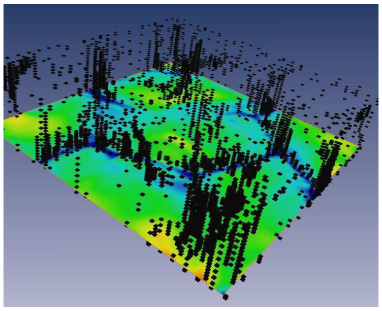

Figure 2: Temperature field in a plane with the positions of the wavelet coefficients of strongest magnitude.

We will discuss our approaches to plume identification, data compression, and heat flux computation within the plumes. Pitfalls, and issues will be detailed. In addition, we will describe some preliminary results in which we use discontinuous second generation wavelets to detect phase diagrams in systems with two degrees of freedom.

Additional work is necessary to determine appropriate values for the threshold, and perhaps identify better variables with stronger correlations to whatever geoscientists consider to be a plume, whether it be thermal or thermal-chemical. To some degree, plume identification is akin to coherent vortex identification in fluid turbulence. Despite several decades of research, scientists have not yet converged on a single working definition.

References

Labrosse, S., Hotspots, mantle plumes and core heat loss, Earth Planet. Sci. Lett., 199, 147-156, 2002.

W. Sweldens. The lifting scheme: A construction of second generation wavelets. SIAM J. Math. Anal., 29:511526, 1997.

G. Erlebacher, D.A. Yuen, F. Dubuffet, Case Study: Visualization and Analysis of High Rayleigh Number – 3D Convection in the Earth's Mantle, *IEEE Visualization '2002/*, pp. 529–532, (October 2002).