An Analytic Solution of Steady Stokes Flow on a Rotating Spherical Cap

Hideaki Kitauchi*, Harper Simmons† and Motoyoshi Ikeda‡

*Frontier Research System for Global Change, 3173-25 Showamachi, Kanazawa-ku, Yokohama 236-0001

†International Arctic Research Center-Frontier, 930 Koyukuk Drive, Fairbanks, Alaska 99775-7335

‡Graduate School of Environmental Science, Hokkaido University, N10, W5, Sapporo 060-0810

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Abstract

An analytic solution of steady linear viscous flow on a spherical cap (Figure 1) rotating about its center is obtained. An inflow and an outflow on the boundary of spherical section drive the fluid motion. The solution of the stream function $\psi(\vartheta, \varphi)$ is expressed as the Fourier series in longitude $\varphi \in [0, 2\pi)$ and the first-kind associated Legendre functions of complex degrees in colatitude $\vartheta \in [0, \vartheta_b]$.

Figure 2 shows contours of the stream function ψ in a frame rotating with the spherical cap ($\vartheta_b = \pi/9$) viewed from the north pole. We assume uniform flow within an inlet and an outlet. The centers of the inlet and outlet are placed at $\varphi = 0, \pi$, respectively, making the straight angle, and their inscribed angles are the same $\pi/9$. No-slip boundary condition is imposed. The inflow turns westward immediately after entering the spherical cap and separates into cyclonic and anticyclonic flows along the boundary. These two branches merge and turn eastward before exiting the outlet. The streamline passing through the center of the inlet passes the north pole and the center of the outlet; this divides the inflow into the above two branches. The results in part support the approximation analysis and laboratory experiment done by Imawaki and Takano (1974).

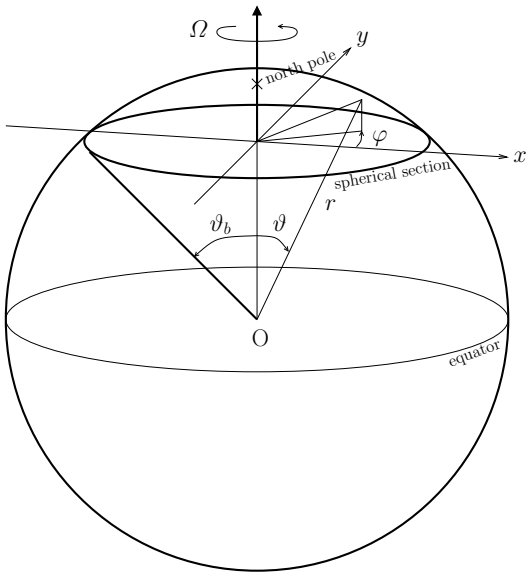

Figure 1: Configuration of a spherical cap.

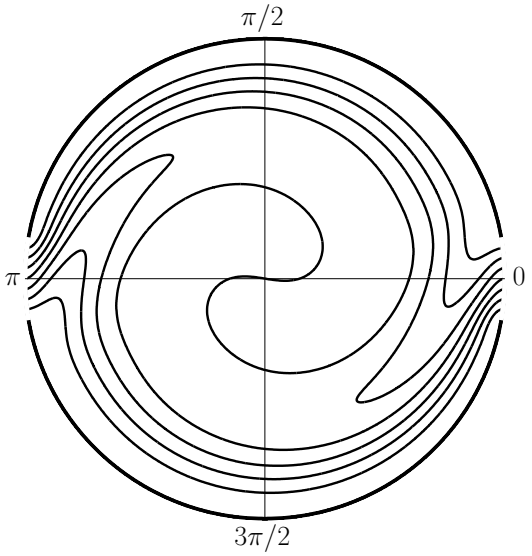

Figure 2: Contours of stream function $\psi(\vartheta, \varphi)$.

*E-mail: kitauchi@jamstec.go.jp

†E-mail: hsimmons@iarc.uaf.edu

‡E-mail: mikeda@ees.hokudai.ac.jp