Reduced atmospheric models: proper basis functions, dimensionality, replacing fast degrees of freedom by stochastic noise

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The construction of reduced atmospheric models, i. e., models that explicitly deal only with a limited number of essential degrees of freedom while keeping as much realism as possible has attracted some attention in recent years. In the present paper, nonlinear reduced models of large-scale atmospheric dynamics are derived using a quasigeostrophic three-level spectral model, truncated to T30, with realistic variability as dynamical framework. The study focuses on three issues: (i) finding appropriate basis functions for efficiently spanning the dynamics and a comparison between different choices of basis functions; (ii) the minimal dimensionality of the reduced model necessary to faithfully reproduce certain aspects of the long-term behavior of the full spectral model; (iii) the question of whether the influence of unresolved fast-evolving degrees of freedom onto the resolved slowly-evolving large-scale degrees of freedom can be represented by stochastic terms.

In the first part of the study, nonlinear deterministic reduced models are obtained by projection of the equations of motion onto a truncated basis spanned by empirically determined modes. The total energy metric is used in the projection; the nonlinear terms of the low-order model then conserve total energy. Apart from retuning the coefficient of horizontal diffusion, no empirical terms are fitted in the dynamical equations of the low-order model in order to properly preserve the physics of the system. Using the methodology of principal interaction patterns (PIPs), basis functions are derived that are optimized to capture the strongly non-normal linear interactions between the mean state and the anomalies as well as the nonlinear terms. The optimized modes are calculated easily and robustly as eigenvectors of a linear eigenvalue problem. This eigenvalue problem is determined out of a class of eigenvalue problems using an only low-dimensional nonlinear search based on a dynamically motivated cost function. The mean state and the standard deviation of the sreamfunction as well as the momentum fluxes in the T30 model are well reproduced in a long-term integration by a PIP model with only about 50 modes; however, a correct modeling of probability distributions and especially spectral densities of the system requires a quite large number of modes. At any truncation level, reduced models based on PIPs are substantially superior to reduced models based on empirical orthogonal functions (EOFs) that have recently been proposed.

In the second part of the work, the reduced models are augmented by additive stochastic terms in order to represent the influence of neglected fast-evolving modes onto the resolved slowly-evolving modes. Both white and red noise processes are tested. The variances, correlation structure and autocorrelation functions of the noise terms are determined empirically from time series of the tendency errors of the deterministic low-order model. Again, the coefficient of horizontal diffusion is retuned for the stochastic model to exhibit in a long-term integration the same amount of variance in the resolved modes as the full spectral model. The noise terms lead to a great improvement of the performance of the reduced model over the deterministic reduced model.