Rheology of magmas and transition in lava flow dynamics inferred from percolation theory

Martin Saar
Department of Geological Science, University of Michigan (msaar@umich.edu

Michael Manga
Department of Earth and Planetary Science, University of California, Berkeley (manga@seismo.berkeley.edu)

Transport properties of multiphase materials may either reflect the deformation of a material as a whole under applied stress (rheology) or the transfer of some medium, such as fluids, within the material (conductivity). Both types of transport are fundamentally different and depend on the relevant material properties in different ways. However, rheology and conductivity of composite materials are both determined in part by the interconnectivity of their individual elements (objects) that constitute their phases.

Percolation theory describes interconnectivity of such objects in a random multiphase system as a function of the geometry, distribution, volume fraction, and orientation of the objects. The structure of the composite material may evolve with time due to chemical reactions or temperature changes. A critical threshold may be passed during the structural evolution and as a result some material properties such as yield strength or conductivity can change abruptly and may exhibit a power-law behavior above, and close to, the so-called percolation threshold.

Multiple processes in the Earth sciences may exhibit threshold properties and may thus be investigated employing percolation theory concepts. Examples include permeability changes in the crust or in the mantle due to partial melting, formation or closure of pores and fractures in solids, or growth and coalescence or degassing of bubbles in liquids. Similarly, the rheology of magmas can depend on the amount, geometry, and interconnectivity of the suspended crystals (Saar et al., EPSL, 2001) as described next in more detail.

We present continuum percolation results (Saar and Manga, Phys. Rev. E, 2002) for three-dimensional randomly oriented soft-core polyhedra (prisms) representing for example crystals in suspensions or fracture networks for rheology and permeability studies, respectively. The prisms are biaxial or triaxial and range in aspect ratio over 6 orders of magnitude. Percolation threshold results for prisms are compared with studies for ellipsoids, rods, ellipses, and polygons and differences are explained using the concept of the average excluded volume (Balberg et al., Phys. Rev. B, 1984). For large shape anisotropies we find close agreement between prisms and most of the above mentioned shapes for the critical total average excluded volume. Cubes exhibit the lowest shape anisotropy of prisms minimizing the importance of randomness in orientation with respect to percolation threshold results. We also derive an equation that allows scaling of percolation thresholds for suspensions of different particle shapes and with given respective average excluded particle volumes.

We provide an example from magma rheology (Saar et al., EPSL, 2001) as an application of the above continuum percolation results. The formation of a continuous crystal network in magmas and lavas can provide finite yield strength, τ_y, and can thus cause a change from

Newtonian to Bingham rheology. The rheology of crystal-melt suspensions affects geological processes, such as ascent of magma through volcanic conduits, flow of lava across the Earth's surface, melt extraction from crystal mushes under compression, convection in magmatic bodies, or shear wave propagation through partial melting zones. Here, three-dimensional numerical models are used to investigate the onset of 'static' yield strength in a zero-shear environment. Crystals are positioned randomly in space and can be approximated as convex polyhedra of any shape, size, and orientation. We determine the critical crystal volume fraction, ϕ_c, at which a crystal network first forms. The value of ϕ_c is a function of object shape and orientation distribution, and decreases with increasing randomness in object orientation and increasing shape anisotropy. For example, while parallel-aligned convex objects yield $\phi_c = 0.29$, randomly oriented cubes exhibit a maximum ϕ_c of 0.22. Approximations of plagioclase crystals as randomly oriented elongated and flattened prisms (tablets) with aspect ratios between 1:4:16 and 1:1:2 yield $0.08 < \phi_c < 0.20$, respectively. The dependence of ϕ_c on particle orientation implies that the flow regime and resulting particle ordering may affect the onset of yield strength and that ϕ_c in zero-shear environments is a lower bound for ϕ_c.