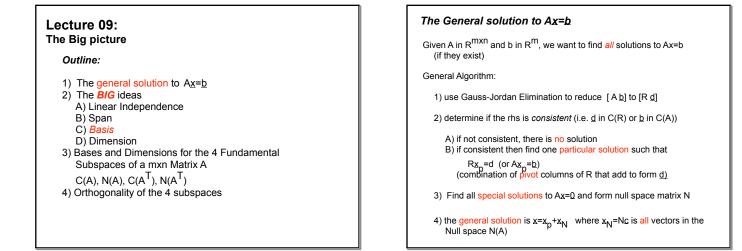
# Lecture 09



The General solution to Ax=b

Example #1: A=[ 1 2 1 0 1 ; 2 4 1 0 0; 1 2 0 1 -4], <u>b</u>=[ 1 1 1]'

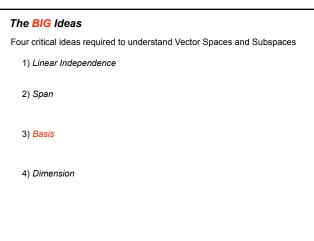
# The General solution to A<u>x</u>=<u>b</u>

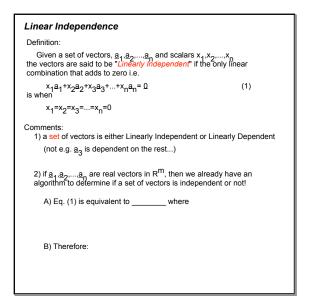
Example #2: A=[ 1 1 ; 1 -2 ; -2 1 ],  $\underline{b}$ =[  $b_1 b_2 b_3$  ]'



# Lecture 09

| The General solution to A <u>x</u> = <u>b</u>                  | The Blo            |
|--|--------------------|
| Types of solutions   | Four criti         |
| Three important numbers for any matrix, m,n,r                  | 1) <i>Lin</i>      |
| Several types of problems                                      | .,                 |
| 1) Full Column Rank (r=n):<br>all columns linearly independent | 2) Spa             |
| N(A)=Z   |                    |
| Either one or none solution                                    | 3) <mark>Ba</mark> |
| 2) Full Row Rank (r=m):  |                    |
| no zero rows   |                    |
| C(A) = R <sup>m</sup><br>N(A) always non-trivial if n>m        | 4) Din             |
| At least one solution, usually infinite solutions              |                    |
| 3) Full Row and Column Rank (r=n=m): Invertible square         |                    |
| matrices: exactly 1 unique solution.                           |                    |





# Linear Independence

Example:  $\underline{a}_1 = [0 \ 1 \ 0]', \ \underline{a}_2 = [1 \ 0 \ 1]', \ \underline{a}_3 = [1 \ -2 \ 1]'$ 

## Span

Definition:

Given a set of vectors, <u>a</u><sub>1</sub>,a<sub>2</sub>,...,a<sub>n</sub> span(a1,a2,...,an) is the vector subspace formed by all linear combinations of the vectors

Comments:

we also say a set of vectors  $\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n$  some subspace S

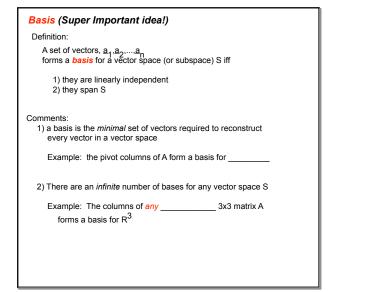
Examples: the columns of a matrix A span \_\_\_\_\_

the special solutions span

# Span

Examples:

# Lecture 09



### **Basis:** Important properties

Theorem:

given a particular basis for a vector space S any other vector  $\underline{v}$  in S is described by a unique linear combination of the basis vectors.

Proof:

1) let  $\underline{x}_1, \underline{x}_2, ... \underline{x}_n$  be a basis for a vector space S 2) let  $\underline{v}$  be any other vector in S

### Dimension of a vector space

Definition:

The dimension of a vector space S is the number of vectors in any basis for S

Comment:

While there are an infinite number of bases for any vector space S, the number of basis vectors (i.e. the dimension of S) is constant.

Example: Give 3 different bases for R<sup>2</sup>

The dimension of R<sup>2</sup> is \_\_\_\_\_

### **Dimension** of a vector space

Theorem:

Given a vector space S of dimension n, there are exactly n vectors in every basis for S.

Proof:

 $\begin{array}{l} \text{let } \underline{v}_1, \underline{v}_2, \ldots \underline{v}_m \text{ be one basis for a vector space S} \\ \text{let } \underline{w}_1, \underline{w}_2, \ldots \underline{w}_n \text{ be another basis S} \\ \text{show that } \underline{\qquad} \end{array}$ 

1) First assume n>m

| Basis and Dimension of the 4 subspaces of a matrix A   |        |          |            |
|--|--------|----------|------------|
| Given A in R <sup>mxn</sup> there are actually four fundamental subspaces associated with the matrix A |        |          |            |
| Name   | Symbol | Subspace | Spanned by |
| 1) Column Space  |        |          |            |
| 2) Null Space  |        |          |            |
| 3) Row Space   |        |          |            |
| 4) Left Null Space   |        |          |            |
|  |        |          |            |

# Basis and Dimension of the 4 subspaces of a matrix A Find the dimension and a basis for each of the 4 subspaces: a recipe 1) Reduce A to R=rref(A) 2) Get C(A) from R and A basis = pivot columns of \_\_\_\_\_\_ dim(C(A)) = \_\_\_\_\_\_ 3) Get N(A) from R basis = dim(N(A)) = 4) Get C(A<sup>T</sup>) from R basis = dim(C(A<sup>T</sup>)) = \_\_\_\_\_\_\_ 5) Get N(A<sup>T</sup>) the hard-way (find Null space of pivot columns of A) dim(N(A<sup>T</sup>))=

| Basis and Dim<br>Example: | ension of the 4 subspaces of a matrix A |
|---------------------------|---|
| A=                        | R=                                      |
|                           |   |
|                           |   |
|                           |   |
|                           |   |
|                           |   |
|                           |   |
|                           |   |
|                           |   |

