Lecture 17:
EigenProblems: Diagonalization and Matrix powers

Outline:

1) One more example (a rank one matrix)
2) $A\underline{x}=\lambda\underline{x}$ as $AS=S\Lambda$
3) Diagonalization and Factorization $A=S\Lambda S^{-1}$ (or $\Lambda=S^{-1}AS$)
4) Application of Diagonalization #1: Matrix Powers A^n
5) Requirements for Diagonalization: distinct eigenvalues
6) Application of Matrix Powers: Iterative Maps $\underline{u}_{k+1}=A\underline{u}_k$
   A) Fibonacci numbers
   B) Iterative methods for solving $A\underline{x}=\underline{b}$

Mechanics of finding Eigenvalues and Eigenvectors

One More Example: A=[1 1 1; 1 1 1; 1 1 1]

Eigenvalues and Eigenvectors

Diagonalization and Factorization

1) All Eigenproblems can be written in matrix form as $AS=S\Lambda$

Definition: A matrix can be diagonalized if it has n linearly independent eigenvectors i.e.

A=__________ or Λ=_________________

Eigenvalues and Eigenvectors

Diagonalizable matrices have nice properties

1) Easy to prove the general checks for |A| and Tr(A)

2) Gives a simple formula for Matrix Powers A^n

Eigenvalues and Eigenvectors

Diagonalization:

1) Unfortunately, not all matrices can be diagonalized:

Example: A=[1 1 ; 0 1]

Eigenvalues and Eigenvectors

Diagonalization

Theorem: all matrices with distinct eigenvalues can be diagonalized

Proof: (sketch for 3x3 case)

Consider a system with 3 eigenvectors $\underline{x}_1, \underline{x}_2, \underline{x}_3$ with corresponding distinct eigenvalues

show that $\underline{x}_1, \underline{x}_2, \underline{x}_3$ are linearly independent

Eigenvalues and Eigenvectors

Diagonalization: The Rules

1) If a matrix A has n distinct eigenvalues (no repeats), it will have n linearly independent eigenvectors and can always be diagonalized.

2) If a matrix has repeated eigenvalues it might be diagonalizable.

Need to check the eigenvectors:
A) if n linearly independent: diagonalizable
B) if repeated eigenvectors (not linearly independent): not diagonalizable

3) If a matrix is symmetric (A^T=A): always diagonalizable (and more)

Eigenvalues and Eigenvectors

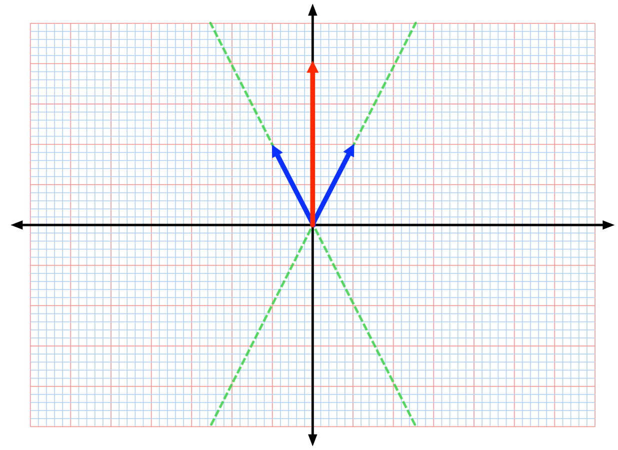
Applications of Matrix Powers: Iterative maps

Consider the iterative map $\underline{u}_{k+1}=A\underline{u}_k$

Eigenvalues and Eigenvectors

Applications of Diagonalization: Iterative maps

An alternative interpretation of the general solution: change of basis



Eigenvalues and Eigenvectors

Applications of Iterative Maps:

Example 1: Fibonacci Numbers

fibonacci sequence 0,1,1,__________________

Recursion relationship $f_{n+1}=$

Question: what is the ratio of f_{n+1}/f_n *in the limit n-> infinity?*

Answer: A cute linear algebra trick

Eigenvalues and Eigenvectors

Applications of Iterative map: Fibonacci Numbers

The trick: turn the recursion relationship into an iterative map...

Eigenvalues and Eigenvectors

Applications of Iterative maps

Example #2: Iterative methods for solving A<u>x</u>=<u>b</u>