

Homework #2: RootFinding Fest

Just 4 Questions: Useful support and example codes can be found in ProblemSet02.tar.gz

1. Let's calculate square roots. Consider the function $f(x) = x^2 - M$ which has a positive root at $x = \sqrt{M}$. Here we'll try to find \sqrt{M} without calling `sqrt`. For each of the following fixed point iterations $x_{k+1} = g(x_k)$ do the following

- Confirm that $x = \sqrt{M}$ is a fixed-point of $g(x)$
- Analyze the convergence behavior close to the fixed-point.
- Calculate the first 3 iterates x_1, x_2, x_3 and their relative errors for $M = 3, x_0 = 2$

(a) $g_1(x) = 1 + x - \frac{x^2}{M}$

(b) $g_2(x) = M/x$

(c) $g_3(x) = x - \frac{f}{f'}$ (Newton's Method: write this out)

2. Choose any rootfinding algorithm of your choice (other than MATLAB's `fzero`) and write a short matlab program with the interface

```
function sqrtM = estimateSqrt(M)
```

that solves $f(x) = x^2 - M = 0$ to robustly estimate \sqrt{M} to within 1 ulp (you can use any of my example codes, e.g. `bisection`, `newton`, `secant`, `newtSafe` or write your own).

Similar to the first problem set, I have supplied a test function `testSqrtEstimate.m` to check your results. Turn in the plot produced by `testSqrtEstimate(@estimateSqrt)` and a print-out of your code.

3. Use any rootfinder of your choice (including MATLAB's `fzero`) and solve the following problems (don't turn in code, just the answers and how you found them).

(a) find the first 10 zeros of Bessel's function $J_0(x)$ for $x \geq 0$ (Hint #1: define the matlab anonymous function `J0=@(x) besselj(0,x)`; Hint #2 compare J_0 to $\cos(x)$); Hint#3: a plot showing $J_0(x)$ with its zeros marked with symbols is a nice test that you've done this correctly).

(b) Estimate the number of terms n in the Taylor expansion of $\exp(x)$ such that the relative error

$$\frac{x^n}{n!} = \epsilon_{\text{mach}}$$

Give solutions for $x = 10, 100, 1000$ (hint: Stirling's approximation $\log n! \approx n \log n - n$ could be useful).

(c) The annoying but fun ladder problem. Consider two ladders of length 20 ft and 30ft, that cross in an alleyway of width w (see Fig. ??). If the height at which they cross is 8ft, what is the width of the alley? (hint: check your answer against the analytic case where the ladders are the same length).

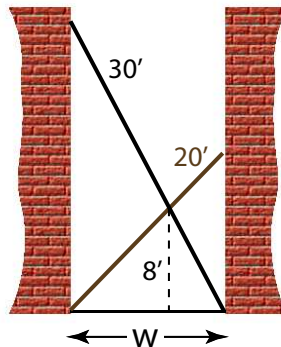


Figure 1: The Ladder problem

4. Fun with optimization: Use any minimization routine you want (including `fminbnd`) and your solutions from Question 3a, to find the values of the first 10 extrema (maxima and minima) of $J_0(x)$. *Extra Credit:* write your own optimization scheme (e.g. Golden Section Search or successive parabolic interpolation) and use that.