

data used in earthquake engineering are occasionally collected in the near field. But when one takes up in more detail the question of where the near field ends and where the far field begins, it becomes apparent that far-field terms also can be big enough to cause earthquake damage to engineering structures. (See Problem 4.1.)

4.2.1 PROPERTIES OF THE FAR-FIELD P-WAVE

We introduce here the *far-field P-wave*, which for (4.23) has the displacement $\mathbf{u}^P$ given by

$$u_i^P(\mathbf{x}, t) = \frac{1}{4\pi\rho\alpha^2} \gamma_i \gamma_j \frac{1}{r} X_0 \left(t - \frac{r}{\alpha} \right). \quad (4.24)$$

As in (4.23), this is for a point force $X_0(t)$ in the x_j -direction at the origin. Along a given direction $\boldsymbol{\gamma}$ from the source, it follows from (4.24) that this wave

- (i) attenuates as r^{-1} ;
- (ii) has a waveform that depends on the time-space combination $t - r/\alpha$, and therefore propagates with speed α (recall that $\alpha^2 = (\lambda + 2\mu)/\rho$);
- (iii) has a displacement waveform that is proportional to the applied force at retarded time; and
- (iv) has a direction of displacement at $\mathbf{x}$ that is parallel to the direction $\boldsymbol{\gamma}$ from the source. This follows from the property $u_i^P \propto \gamma_i$ (see (4.24)). The far-field *P-wave* is therefore *longitudinal* (sometimes called *radial*) in that its direction of particle motion is the same as the direction of propagation. If $t = 0$ is chosen as the time at which $X_0(t)$ first becomes nonzero, then r/α is the *arrival time* of the *P-wave* at r .

4.2.2 PROPERTIES OF THE FAR-FIELD S-WAVE

The *far-field S-wave* in (4.23) has displacement $\mathbf{u}^S$ given by

$$u_i^S(\mathbf{x}, t) = \frac{1}{4\pi\rho\beta^2} (\delta_{ij} - \gamma_i \gamma_j) \frac{1}{r} X_0 \left(t - \frac{r}{\beta} \right). \quad (4.25)$$

As in (4.23), this is for a point force $X_0(t)$ in the x_j -direction at the origin. Recall that $\boldsymbol{\gamma}$ is the unit vector directed from the source to the receiver. Along a given direction $\boldsymbol{\gamma}$, this wave

- (i) attenuates as r^{-1} ;
- (ii) propagates with speed β and has arrival time r/β at $\mathbf{x}$;
- (iii) has a displacement waveform that is proportional to the applied force at retarded time; and
- (iv) has a direction of displacement $\mathbf{u}^S$ at $\mathbf{x}$ that is perpendicular to the direction $\boldsymbol{\gamma}$ from the source. (From (4.25) it is easy to show that $\mathbf{u}^S \cdot \boldsymbol{\gamma} = 0$.) The far-field *S-wave* is therefore a *transverse* wave, because its direction of particle motion is normal to the direction of propagation.

Radiation patterns for $\mathbf{u}^P$ and $\mathbf{u}^S$ are given in Figure 4.2.

Associated far-field displacements are then

$$\begin{aligned}\mathbf{u}^P(\mathbf{x}, t) &= \frac{\mathcal{F}^P \mu A}{4\pi \rho \alpha^3 r} \dot{\bar{u}} \left(t - \frac{r}{\alpha} \right) \mathbf{I}, \\ \mathbf{u}^{SV}(\mathbf{x}, t) &= \frac{\mathcal{F}^{SV} \mu A}{4\pi \rho \beta^3 r} \dot{\bar{u}} \left(t - \frac{r}{\beta} \right) \hat{\mathbf{p}}, \\ \mathbf{u}^{SH}(\mathbf{x}, t) &= \frac{\mathcal{F}^{SH} \mu A}{4\pi \rho \beta^3 r} \dot{\bar{u}} \left(t - \frac{r}{\beta} \right) \hat{\boldsymbol{\phi}}.\end{aligned}\tag{4.92}$$

4.5.3 ADAPTING THE RADIATION PATTERN TO THE CASE OF A SPHERICALLY SYMMETRIC MEDIUM

We have taken care to obtain P , SV , and SH displacements (4.92) in a form comparable with the geometric ray solutions derived in Section 4.4 (see (4.62), (4.65), (4.66)). To complete the comparison for P -waves, it remains only to identify and generalize r/α as the ray travel time T^P , $1/r$ as the geometrical spreading factor $1/\mathcal{R}^P(\mathbf{x}, \boldsymbol{\xi})$, and $\mu/(\rho\alpha^3)$ as the factor $\mu(\boldsymbol{\xi})/[\sqrt{\rho(\boldsymbol{\xi})\rho(\mathbf{x})\alpha(\boldsymbol{\xi})\alpha(\mathbf{x})\alpha^2(\boldsymbol{\xi})}]$. This last result follows from generalizing $\mu/(\rho\alpha^3)$ to a term proportional to $1/\sqrt{\rho(\mathbf{x})\alpha(\mathbf{x})}$ (as required by (4.62)), in which the constant of proportionality can depend only on properties at the source. Then

$$\mathbf{u}^P(\mathbf{x}, t) = \frac{\mathcal{F}^P \mu(\boldsymbol{\xi}) A \dot{\bar{u}}(t - T^P) \mathbf{I}}{4\pi \sqrt{\rho(\boldsymbol{\xi})\rho(\mathbf{x})\alpha(\boldsymbol{\xi})\alpha(\mathbf{x})\alpha^2(\boldsymbol{\xi})} \mathcal{R}^P(\mathbf{x}, \boldsymbol{\xi})},\tag{4.93}$$

and similarly

$$\mathbf{u}^{SV}(\mathbf{x}, t) = \frac{\mathcal{F}^{SV} \mu(\boldsymbol{\xi}) A \dot{\bar{u}}(t - T^S) \hat{\mathbf{p}}}{4\pi \sqrt{\rho(\boldsymbol{\xi})\rho(\mathbf{x})\beta(\boldsymbol{\xi})\beta(\mathbf{x})\beta^2(\boldsymbol{\xi})} \mathcal{R}^S(\mathbf{x}, \boldsymbol{\xi})},\tag{4.94}$$

$$\mathbf{u}^{SH}(\mathbf{x}, t) = \frac{\mathcal{F}^{SH} \mu(\boldsymbol{\xi}) A \dot{\bar{u}}(t - T^S) \hat{\boldsymbol{\phi}}}{4\pi \sqrt{\rho(\boldsymbol{\xi})\rho(\mathbf{x})\beta(\boldsymbol{\xi})\beta(\mathbf{x})\beta^2(\boldsymbol{\xi})} \mathcal{R}^S(\mathbf{x}, \boldsymbol{\xi})}.\tag{4.95}$$

The radiation patterns here are exactly the same as for a homogeneous medium, and are given in (4.89)–(4.91). The only noteworthy symmetry is a reversal in sign of $\mathcal{F}^P$, $\mathcal{F}^{SV}$, $\mathcal{F}^{SH}$ if the rake is changed by 180° . Particularly, one should note that there is no symmetry to changes of 180° in strike ϕ_s , or takeoff azimuth ϕ , so that care must be taken to follow the definitions given in Figures 4.13 and 4.20, in which these angles increase clockwise round from North.

The principal use of our final formulas (4.93)–(4.95) lies in estimating the seismic moment. From a far-field observation of the displacement, one can obtain $\mu A \dot{\bar{u}}(t - T)$ after correction for the radiation pattern, geometrical spreading, and scaling factors at source and receiver. (In practice, correction is typically required also for the effects of transmission across material boundaries, attenuation, and instrument response.) It often occurs that the

We begin in Figure 9.12 with a look at the S -wave rays that are present for a surface source in a model of the upper mantle. Clearly, several rays might arrive at a given receiver, and the travel-time function (Figure 9.12b) is multivalued for a certain range of distances. However, each point along the travel-time curve has a unique slope, the value decreasing from A to B , B to C , etc. This suggests a useful independent variable. It follows from Figure 5.2 that this slope, $dT/d\Delta$, which is the horizontal slowness for an

obliquely propagating wave, is nothing but the ray parameter p . (Horizontal distance is Δ in dimensionless units and is $r\Delta$ in units of length.)

Thus consider how the distance function depends on p . From Figure 9.12b it is clear that distance and slope decrease together for the branch BC , but along AB and CD distance is increasing while slope decreases. This is shown explicitly in Figure 9.13, and some special significance is attached to the points B and C . Note that $d\Delta/dp$ changes sign at these points, and it can happen that $d\Delta/dp$ remains continuous, so that $d\Delta/dp = 0$ at C or at B and C . Since the geometrical spreading function $\mathcal{R}^{-1}$ is proportional to $1/\sqrt{d\Delta/dp}$

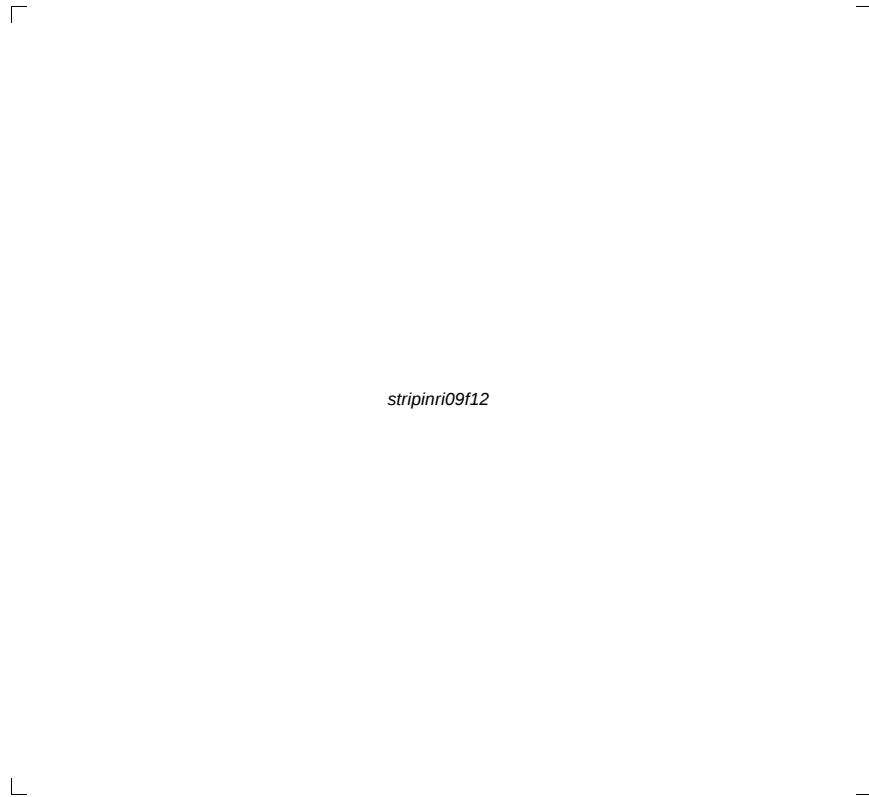


FIGURE 9.12

(a) The S -wave velocity for the upper mantle, taken from model CIT 11 GB. (b) Corresponding reduced travel-time curve. Point C is clearly identified with strong focusing of rays in (c) at Δ near 14° , and amplitudes there will be large. Lines AB , BC , and CD together constitute a *triplication*, as do the lines CD , DE , and EF , and each of these two triplications is associated with a major velocity increase (with depth) in the Earth model. (c) Corresponding S -wave rays for a point source at the surface, calculated for take-off angles increasing from 28° to 50° in $\frac{1}{2}^\circ$ increments. Note that distance between source and receiver in the Earth is measured by the angle Δ subtended at the Earth's center. [After Julian and Anderson, 1968.]

(see Problem 4.4), ray theory predicts a singular amplitude for the displacement. This is the phenomenon of a *caustic*, and an example is shown in Figure 9.12 at the distance 14° . A caustic is the envelope of a system of rays, and for the source and Earth model of Figure 9.12 the envelope in three dimensional space is a surface inside the Earth, which intersects the Earth's surface at a circle centered on the seismic source. Of course, the prediction of ray theory here is incorrect: there is no singularity at finite frequencies, although amplitudes may be large in the vicinity of a caustic (as shown by the focusing of rays in Figure 9.12c).

In practice, the sensitivity of amplitudes (calculated by ray theory) to the quantity $d\Delta/dp$ leads to some difficulties in computation. The problem is that Earth models are ordinarily specified by giving the values of density (ρ) and P - and S -wave speeds (α and β) at several different radii. But different methods of interpolation between such discrete values can



FIGURE 10.7
Slip function for a circular
fault with uniform slip.

For subsonic rupture propagation, (10.24) can be written as

$$\Omega(\mathbf{x}, t) = v^2 \left(t - \frac{r_0}{c} \right) H \left(t - \frac{r_0}{c} \right) \int \frac{\Delta U \left(\frac{t - r_0/c}{q_c/v}, \phi' \right)}{q_c^2} d\phi', \quad (10.25)$$

where the integral is taken over the range of ϕ' for which $[(t - r_0/c)/(q_c/v)] < \rho_b$.

Suppose that the final slip ΔU is uniform except near the fault perimeter, and suppose that we look at the beginning of the far-field displacement waveform, when $t - r_0/c$ is small and the range of integration for ϕ' covers 0 to 2π . In that case, we see from (10.25) that the displacement waveform is a linear function of time (a ramp function). The linearity will hold until the rupture front reaches the perimeter of the prescribed fault surface.

Thus a subsonically spreading rupture with a uniform step-function slip generates a far-field displacement waveform $(t - r_0/c)H(t - r_0/c)$ until the stopping signal arrives from the perimeter of the fault. The corresponding particle-velocity waveform due to this type of nucleation is a step function with a discontinuity at $t = r_0/c$. The acceleration is a δ -function, reaching infinity at $t = r_0/c$. The spectral density of acceleration, velocity, and displacement are therefore constant, and proportional to ω^{-1} and ω^{-2} , respectively.

In order to see what happens when the rupture stops propagating, let us consider the case of a circular fault with uniform slip, in which $\rho_b = \rho_0$ (constant), $\Delta U(\rho, \phi') = \Delta U_0$ (constant). Then $\Delta u(\xi, t)$ is a function of ρ and t , as shown in Figure 10.7. The simplest result is obtained in the direction normal to the fault plane. For $\theta = 0$, $q_c = 1$ and equation (10.25) shows that the integral with respect to ϕ' is constant for $v(t - r_0/c) < \rho_0$ and vanishes for $v(t - r_0/c) > \rho_0$. In other words, the far-field displacement waveform $\Omega(\mathbf{x}, t)$ for $\theta = 0$, which grows like a ramp function beginning at $t = r_0/c$, subsequently has a jump discontinuity at $t = r_0/c + \rho_0/v$, when $\Omega(\mathbf{x}, t)$ suddenly becomes zero. This jump discontinuity gives infinite particle velocity and acceleration. The spectral density of displacement, in the high-frequency limit, will decay in proportion to ω^{-1} . A seismic signal associated with the stopping of rupture was named a “stopping phase” by Savage.

For $\theta \neq 0$, $q_c = 1 - (v/c) \sin \theta \cos(\phi - \phi')$ is a function of ϕ' , taking its minimum value in the azimuth $\phi' = \phi$ to the station and its maximum in the opposite azimuth $\phi' = \phi + \pi$. Since $\Delta U(\rho, \phi')$ is constant and q_c is a smooth function, the integral in equation (10.25) is proportional to the range of ϕ' for which $[v(t - r_0/c)]/q_c < \rho_0$. As long as the locus of $\rho = [v(t - r_0/c)]/q_c$ is contained inside the circle $\rho = \rho_0$, the integration range of ϕ' is 2π . Since the minimum of q_c is at $\phi' = \phi$, the locus of $\rho = [v(t - r_0/c)]/q_c$ will touch the

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FIGURE 12.3

Amplitude response $|X(\omega)|$ and phase delay $\phi(\omega)$ of an inertial seismometer according to (12.4). h is the dimensionless damping constant, ε/ω_s . **Correction: the horizontal axes here correspond to ω_s/ω (not to ω_0/ω).**

we find

$$|X(\omega)| = \frac{\omega^2}{\sqrt{(\omega^2 - \omega_s^2)^2 + 4\varepsilon^2\omega^2}} \quad \text{and} \quad \phi(\omega) = -\tan^{-1} \frac{2\varepsilon\omega}{\omega^2 - \omega_s^2} + \pi. \quad (12.4)$$

For $\omega \gg \omega_s$, $|X(\omega)| \rightarrow 1$ and $\phi(\omega) \rightarrow \pi$. In other words, the sensor displacement ξ records the ground displacement faithfully at **high** frequencies, but with reversed sign. The sign difference is usually eliminated by indicating the direction of ground motion properly on the record. Figure 12.3 shows $|X(\omega)|$ and $\phi(\omega)$ without the π term in (12.4). The curves are shown with $h = \varepsilon/\omega_s$ as a parameter; h is the *damping constant*, equal to half the reciprocal of the Q -value (the quality factor of a damped oscillator).

The performance of an inertial seismometer can also be completely described by its response $f(t)$ to a unit impulsive acceleration $\ddot{u}(t) = \delta(t)$, the Dirac δ -function. From equation (12.2), $f(t)$ satisfies

$$\ddot{f} + 2\varepsilon \dot{f} + \omega_s^2 f = -\delta(t). \quad (12.5)$$

Taking the Fourier transform of both sides of (12.5) and putting

$$\int_{-\infty}^{\infty} f(t) e^{i\omega t} dt = f(\omega), \quad (12.6)$$