

# Slip-length scaling in large earthquakes: Observations and theory and implications for earthquake physics

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**Abstract.** For twenty years there has been a dilemma in earthquake physics, because the observed scaling law for large earthquakes did not appear to be consistent with the stress-drop invariance of small earthquake scaling. Surprisingly, slip was seen to continue to increase with rupture length  $L$  even for events with lengths much longer than the event widths  $W$  (the brittle crust down-dip depth), whereas it might have been expected to saturate for lengths much beyond the width. If this implies that the physics of great earthquakes is somehow different from that of their smaller counterparts, this casts serious doubts on predicting the effects of the rare and damaging great events from observations of the more common smaller events. Here we bring together recently compiled observations of very large aspect ratio earthquakes with results of a 3 dimensional dynamic earthquake model to show that slip-length scaling observations are, in fact, consistent with a scale-invariant physics. Further, we discuss the origin of the large earthquake scaling in the model.

## Introduction

Earthquakes have long been expected to scale like cracks with scale-invariant stress drops. Thus for small earthquakes, with dimensions less than the seismogenic thickness, displacement should scale with radius, and for long earthquakes, with lengths  $L$  greater than the seismogenic thickness  $W$ , displacement  $D$  should scale with  $W$  [Kanamori and Anderson, 1975]. Although this is the observed scaling for small earthquakes [Hanks, 1977], it was pointed out in 1982 [Scholz, 1982] that the observations indicate that displacement scales with  $L$  rather than  $W$  for large earthquakes. There have been continued discussions about the data and their interpretation [Romanowicz, 1992; Scholz, 1994a; Peger and Das, 1996; Mai and Beroza, 2000], with little resolution, and various interpretations have been suggested for the meaning of this finding, such as the suggestion that higher stress drop earthquakes simply propagate farther [Heaton, 1990]. There was always the unsettling possibility that new physics, such as frictional melting [McKenzie and Brune, 1972], may accompany the large earthquakes but not the small ones. If that were so, the study of small events may not be helpful in predicting the effects of the much rarer but destructive great events, which would undercut a key assumption of earthquake hazard analysis.

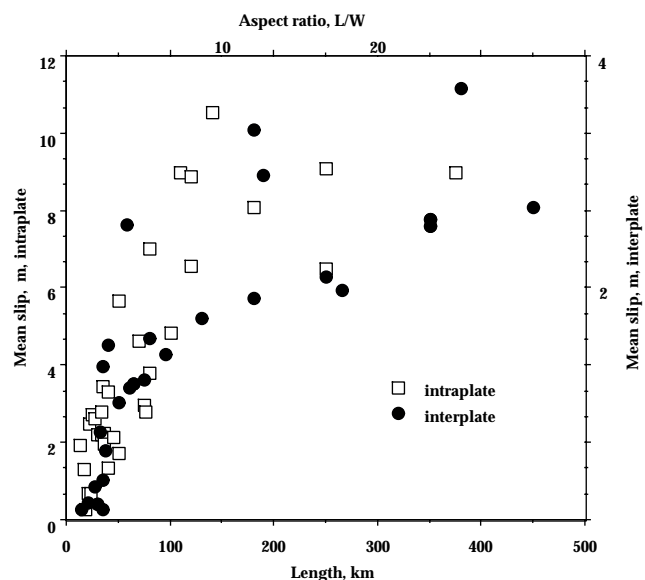
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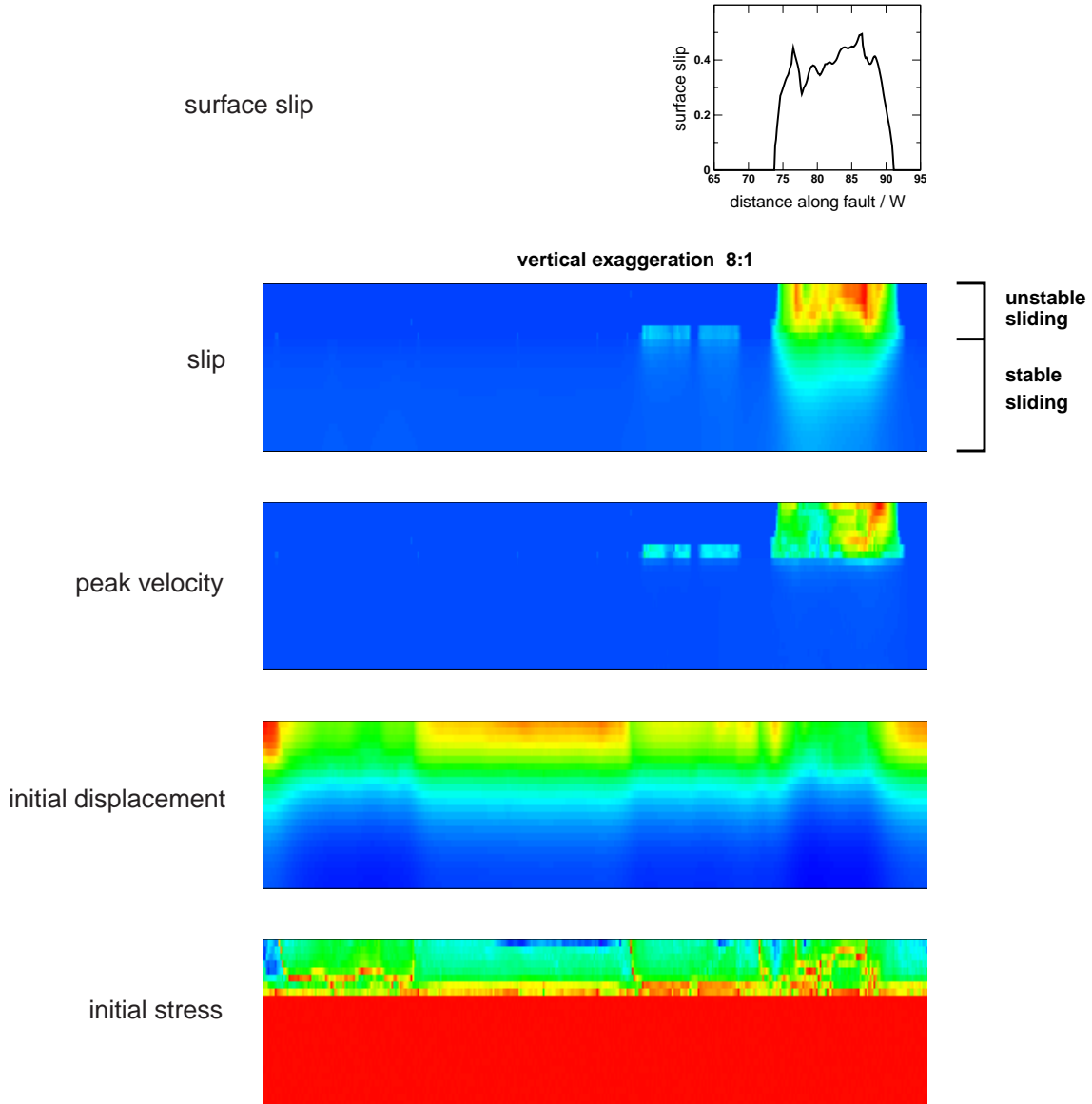
## Observations

The scaling question hinges on observations of slip in very large aspect ratio earthquakes, which are very rare and were few in the early data sets. Gradually, however, such data has accumulated, and the most recent compilation does suggest a rollover to constant slip for very long earthquakes [Scholz, 1994b]. These data, shown in Figure 1, are for large earthquakes, in which  $L > W$ , where  $W$  has a constant value of 15–20 km. The different scales for interplate and intraplate earthquakes reflect the systematically greater stress drops of the latter events, here shown to be about a factor of three. The difference in the two types of earthquakes is that the intraplate earthquakes occur on faults with geologic slip rates one or two orders of magnitude slower than the interplate ones, and their larger stress drops are thought to result from greater fault healing due to their longer recurrence times [Marone, 1998].

The linear trend in  $D$  vs.  $L$  is clearly evident for the shorter events in Figure 1, but now one can also see, for earthquakes with  $W/L > 10$  a tendency for slip to attain a constant value, as expected from a 2-D crack model. What is surprising is that this crossover is so gradual and occurs at such a high  $L/W$  ratio [Bodin and Brune, 1996]. The large scatter in the data also makes one wonder if the observed trend is real [Bodin and Brune, 1996]. We explore both of these issues now with a 3-D dynamic earthquake model.



**Figure 1.** Compilation of mean slip vs length for large crustal earthquakes (modified after [Scholz, 1994b]). The aspect ratio is based on an assumed value of  $W = 15$  km.



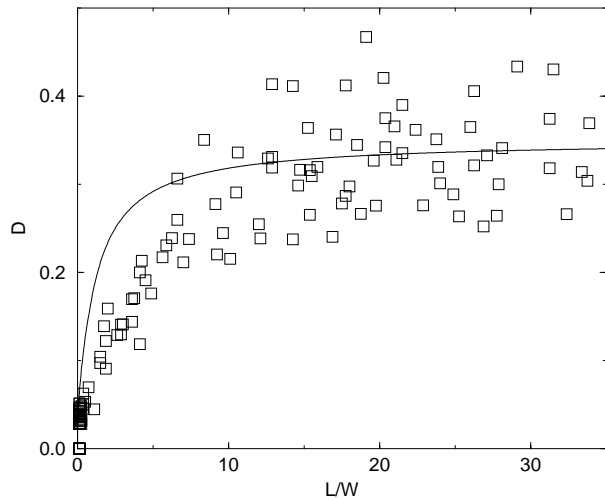
**Figure 2.** Five different views of an example event. From top to bottom, we see the slip at the surface, and then four different views of the fault plane: the slip in the event, the maximum velocity, the initial net displacement, and the initial stress.

## Theory

Our model both retains a number of complications of the Earth, while simplifying others. The model is three dimensional, retaining the large scale geometry of faults. We use a planar fault, neglecting the complication of geometric irregularity. Our model uses fully inertial Newtonian dynamics, so that waves mediate interactions. We consider just one scalar elastic mode, and thus the bulk satisfies the wave equation. The fault is loaded from a steady, slowly moving, distant boundary. The upper surface is a free surface, as in the Earth.

Friction on the fault plays a central role in the dynamics. When friction increases with slip or slip rate, the fault slides stably, creeping along at the plate loading rate. In contrast, when friction decreases with slip or slip rate, the fault slides unstably, rupturing in sudden stick-slip events [Brace and Byerlee, 1966; Carlson and Langer, 1989]. In

the Earth, a seismogenic unstably sliding brittle crust of roughly 15 km depth overlays a stably sliding ductile lower crust [Blanpied et al., 1987; Tse and Rice, 1986]. We model this using friction with two layers, with frictional weakening in the top layer and frictional strengthening in the bottom layer. As with the geometry, we simplify the friction in our model, taking a friction which is uniform along the fault, and with constant properties with depth within each of the two layers. The friction we use has been described in detail and studied in lower dimensional models [Shaw, 1995; Shaw and Rice, 2000]. It combines a mixture of slip and velocity weakening effects. While it differs from the more elaborate rate and state laboratory derived friction [Dieterich, 1979; Ruina, 1983], the scaling results we present appear to be insensitive to many of the details of the frictional instability. It has a crucial advantage of allowing for a faster numerical scheme, which is useful since we are at the limits of our computational resources.



**Figure 3.** Average surface slip as a function of surface rupture in the model. Each point corresponds to an individual event. Note the remarkable similarities with Figure 1, both in terms of the mean behavior, and the variation about the mean. The solid line is the scaling expected from a simple constant stress drop estimate.

Beginning from any nonuniform initial condition, the system self-organizes into a sequence of events, on an attractor which appears to be independent of the initial conditions. Two types of attractors are seen. When there is insufficient weakening or when there is only slip weakening without any velocity weakening in the upper layer, throughgoing events which break the entire fault length develop. Alternatively, for sufficient weakening and when there are some velocity weakening effects, a nonperiodic, chaotic sequence of events ensues. This chaotic regime, which most resembles nature, is our focus here.

Figure 2 shows a few different views of the fault surface for one example event. This figure illustrates what large surface breaking events typically look like in the model, and some aspects of the complex attractor that develops. It also illuminates an important aspect of the behavior: when the fault is long enough, there are attractors on which events die out without propagating the whole length of the fault. They die out because neighboring regions have slipped in a large event in the not too distant past.

Figure 3 shows, as in Figure 1, a plot of the average surface slip as a function of the length of the surface rupture for the model events. The events plotted are the events which break the surface, taken from one long sequence of events. Notice the striking similarity to Figure 1, not only in the mean properties of the scaling, but in the scatter about the mean as well.

For the mean, we see a rapid initial increase, followed by a rollover to roughly constant, or at least slowly increasing slip with rupture length, with the rollover occurring at roughly an aspect ratio 10. An assumption of constant stress drop, taking into account the finite brittle depth, captures some, but not all, of the scaling. Roughly, we can estimate it as follows. For an average slip  $D$ , the strain is given by  $D/L_x + D/L_z$  where  $L_x$  is the rupture length along the fault and  $L_z$  is the rupture length in the depth direction.

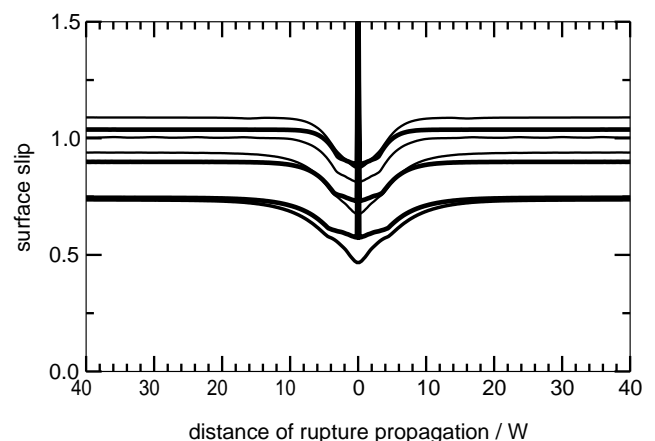
Equating this to a constant stress change and taking into account the finite depth  $W$  gives

$$D \sim \begin{cases} \frac{1}{L_x + L_z} & L \leq 2W; \\ \frac{1}{L_x + 2W} & L > 2W. \end{cases} \quad (1)$$

where the factor of 2 comes from the effect of the free surface. With this estimate there is only one free parameter, the overall amplitude, which is fit to the slip of the largest events. The solid line in Figure 3 shows this scaling, for comparison to the model. It captures some of the mean behavior, including the rapid increase and rollover at several times the crust depth. This result indicates a constant stress drop of 4 MPa for the interplate events and 12 MPa for the intraplate events.

While the scaling argument captures the basic first order effect, there seem to be some systematic second order effects in the results which appear to go beyond this simple scaling. We see, in particular, a slower increase in the slip with length in the measurements. Interestingly, the model data fit the observed data, in this respect, even better than either fit the simple scaling argument!

What appears to be underlying the surprisingly large lengthscales in the problem are dynamic effects. Slip pulses carrying along potential and kinetic energy concentrations are seen to take very long distances, both to get going, and to die away. Though the complex attractor contains a very heterogeneous stress field, as Figure 2 illustrates, these effects are easiest to see by looking at pulses propagating over faults with constant initial stress conditions. In Figure 4, we plot the slip that results from a kick to a fault having an initially uniform stress. Here we see long transients in the initiation of ruptures through constant stress fields. The lengthscales of these transients are only weakly dependent on friction parameters, and weakly dependent on initial stress. What we see is that, in general, only over very long lengthscales, of order 5 to 10  $W$  (10 to 20  $W$  bilaterally), do the slip pulses saturate in slip. These long transients give us basic insight into our scaling problem. It suggests that it takes some quite long distance to reach the maximum amount of slip, and that for very long ruptures we can



**Figure 4.** Surface slip for kicks into initially uniform stress. Different lines are for different initial stresses, and different friction parameters (thicker lines for friction parameter corresponding to less weakening). Note the large distances it takes for pulses to saturate in slip: lengths of order 10  $W$ .

expect to see peak slip not necessarily at the epicenter, but away from it. It is these long transient lengths which stretch the saturation of the scaling out to such large aspect ratios. Dynamic energies are central to the long saturation lengths in the model, and, we expect, in the Earth.

## Implications and Conclusions

Thus far, we have focussed our attention on the mean behavior of the scaling. The scatter, however, also contains important information. Here again, there is a remarkable similarity of the model to the observations, with both showing roughly a factor of 2 variation between events in the average slip for a given rupture length. The scatter in the model data comes from the intrinsic roughness of the slip which has developed along the fault. That is, it arises from naturally developed variations of initial conditions, and not from intrinsic strength variations, which are absent in the model. This has significant implications for the source of heterogeneities in earthquake behavior, where a debate has raged about the relative importance of geometrical and material heterogeneities versus stress heterogeneities. The fact that the model scatter is already of order the scatter in the observations suggests that stress heterogeneities cannot be neglected compared with geometrical and material heterogeneities.

Earthquake scaling laws provide constraints on the physics of the earthquake source. Here we have shown that a longstanding mystery, why slip in earthquakes continues to increase with rupture length even for lengths many times the crust depth, can be reproduced by dynamic models having scale invariant physics. This provides evidence for the notion that there is nothing essentially different about the physics of great earthquakes relative to large earthquakes, and provides support for the use of measurements of the more numerous magnitude 7's in preparing for the rarer and more devastating magnitude 8's.

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