Response of convection to relative SST: Cloud-resolving simulations in 2D and 3D

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Abstract

The properties of equilibrated tropical convection are studied using a cloud-resolving model with large-scale dynamics parameterized by the weak temperature gradient (WTG) approximation. Model integrations are carried out in both two-dimensional (2D) and three-dimensional (3D) geometries. The target profile towards which horizontal mean free tropospheric temperature is relaxed is held fixed while SST is varied. Consistent with previous studies, large-scale ascent and precipitation increase under WTG as the SST is increased, but more rapidly in 2D than 3D. This is related to greater extremes of near-surface buoyancy in 2D as well as a lower gross moist stability, and perhaps also to weaker entrainment. In both 2D and 3D, the vertical profiles of large-scale vertical velocity are top-heavy and remarkably self-similar in shape as SST is increased. When all integrations are analyzed together, precipitation increases with column-integrated relative humidity once the latter reaches a threshold, as in observations and other models. However, within each integration, the two quantities are correlated negatively, albeit over a very narrow range.
1. Introduction

Cloud-resolving models (CRMs; also known as “cloud system resolving models”) are an increasingly important resource as we seek to understand the role of moist convection and its interaction with larger-scale dynamics. Though still limited by resolution and the need to parameterize some physical processes, CRMs at least partially resolve deep convection, avoiding the need for cumulus parameterization and thus presumably giving results more faithful to the behavior of the real atmosphere. It has recently even become feasible to run CRMs on global domains, though this remains sufficiently expensive that it is not a methodology in wide use. More common is to run a CRM on a limited domain representing a small region of the atmosphere with periodic boundary conditions in the horizontal. In this context one can aim to examine the interaction between the convection (which is at least partly resolved) and larger-scale dynamics, which is represented in this context by domain-averaged quantities.

The weak temperature gradient (WTG) approximation is a methodology which can be used to parameterize the large-scale vertical motion in a way broadly consistent with our understanding of large-scale tropical atmosphere dynamics. Otherwise, the large-scale vertical motion must be specified, which strongly constrains the domain-averaged convective activity. Under WTG, a CRM can be used to examine the bulk response of convection to external parameters such as sea surface temperature and surface fluxes (for example). Under specified large-scale vertical motion these parameters will not exert a control on deep convection similar to that which they do in the real system under a fixed mean climate. Rather, under fixed large-scale vertical motion, a change in SST (for example) will cause a change in domain-averaged tropospheric temperature but little change in precipitation. Such a response may represent the effect of a change in SST due to a mean climate change, but not the effect of an SST variation (in
space or time) for fixed mean climate. In the real tropical atmosphere where tropospheric
temperature gradients must remain small, the free-tropospheric temperature profile can be
thought of as related to the tropical mean SST [e.g., Sobel et al., 2002]. Varying SST while
holding free tropospheric temperature fixed (or relaxing it strongly to a target profile) can be
thought of as varying the relative SST, or difference between the local and tropical mean SSTs
[e.g., Vecchi and Soden, 2007; Ramsay and Sobel, 2010] and it is this relative SST which our
simulations use as the primary control parameter.

A number of studies have been performed with single-column models (SCMs) under WTG
[Sobel and Bretherton, 2000; Chiang and Sobel, 2002; Shaevitz and Sobel, 2004; Sobel et al.,
2007; Sobel and Bellon 2009; Ramsay and Sobel 2010]. A few have been performed with CRMs.
These have either used an implementation similar to ours, but in two dimensions [Raymond and
Zeng, 2005; Raymond, 2007; Sessions et al., 2010] or three dimensional geometry, but with a
different implementation of WTG, in which a one-dimensional wave equation is solved for the
large-scale vertical velocity [Kuang 2008; Blossey et al., 2009; Kuang, 2010]; still other studies
have used related, but more different parameterizations of large-scale dynamics [Mapes, 2004;
Bergman and Sardeshmukh, 2004]. In the present study, we use the Weather Research and
Forecasting model (WRF) in both 2D and 3D configurations under WTG. Our goals are to
determine whether key aspects of the results obtained in previous studies will be reproduced
when a different model is used, to analyze some of those results in more detail, and in particular
to examine the differences and similarities between 2D and 3D simulations.

Past CRM studies [e.g., Grabowski et al., 1998; Tompkins, 2000; Phillips and Donner, 2006;
Petch et al., 2008; Stephens et al., 2008; Wu and Guimond, 2006] have found that, although
some aspects of deep convection can be captured in a 2D domain, 2D convection may be
distorted in comparison to that in a 3D domain. These studies indicate that convection in two dimensions may exhibit excessive higher frequency variability; time- and domain-averaged quantities such as temperature, relative humidity, and bulk cloud microphysical properties, may also be different in 2D than 3D. Bretherton and Smolarkiewicz [1989] demonstrated that 2D geometry artificially spreads the subsidence associated with a heat source, while Tompkins [2000] suggested that cold pools may be artificially enhanced in 2D. Zeng et al. [2007] showed that buoyancy is insufficiently damped in 2D such that excessive buoyancy oscillations are spuriously produced and further cause high frequency variabilities, while Petch et al. [2008] showed that entrainment of dry air into convective cells in 2D is much weaker than in 3D; both effects favor stronger convection. Perhaps most dramatically, Held et al. [1993] showed that strong interactions between convection and the mean momentum field in 2D can lead, if not actively suppressed (e.g. by an imposed external restoring forcing on the horizontal wind field), to self-sustained long-period oscillations by a mechanism broadly similar to that of the stratospheric quasi-biennial oscillation. Whether these various distortions introduced by a 2D configuration are acceptable depends on the situation and the question being asked.

Understanding the capabilities and limitations of 2D simulations is perhaps more relevant than in the past as 2D CRMs are used in place of a convective parameterization in the multiscale modeling framework [Khairoutdinov et al., 2005].

The rest of this article is outlined as follows. Section 2 describes the details of the WTG implementation in WRF and the configurations of our numerical experiments. Section 3 presents results from a broad range of diagnostics applied to the experiments. Section 4 summarizes the conclusions.
2. Methodology

2.1. Weak Temperature Gradient

We use the Advanced Research Weather Research and Forecasting (WRF) Model version 3.0 [Skamarock et al., 2008]. We have configured WRF to run under either the WTG or radiative-convective equilibrium (RCE) mode in either 2D or 3D geometry. Our implementation of WTG has some minor differences in detail from that presented in previous studies, due to the particular vertical coordinate and configuration of WRF. Apart from these, it largely follows Raymond and Zeng [2005] and Sobel and Bretherton [2000].

The first step to implement WTG is to add a term representing large-scale vertical advection of potential temperature to the thermodynamic equation. This term is taken to relax the horizontal mean potential temperature in the troposphere to a prescribed profile:

\[
\frac{\partial \theta}{\partial t} + \cdots = -\frac{\bar{\theta} - \theta_{RCE}}{\tau}
\]  

(1),

where \(\theta\) is potential temperature (or temperature), \(\bar{\theta}\) is the mean potential temperature of the CRM domain (overbar indicates the CRM horizontal domain average), \(\theta_{RCE}\) is the target potential temperature for the relaxation, and \(\tau\) is the Newtonian relaxation time scale. When \(\tau\) is taken close to 0 (a limit one may not be able to reach due to numerical issues), this becomes a strict implementation of WTG and the horizontal mean free troposphere temperature must equal \(\theta_{RCE}\). In general, \(\tau\) is interpreted as the time scale over which gravity waves propagate out of the domain, thus reducing the horizontal pressure and temperature gradients. Finite \(\tau\) allows the temperature to vary in response to convective and radiative heating.

The large-scale vertical circulation implied by this relaxation constraint is \(W_{wtg}\), the WTG vertical velocity,
where $\rho$ is the density, $\eta$ is the dry mass based vertical coordinate of WRF, and $-\overline{\rho g / \mu}$ is part of the coordinate transformation from $z$ to $\eta$. $\eta$ is defined as $\eta = \frac{p_d - p^T_d}{\mu}$, where $p_d$ is the dry pressure, $p^T_d$ is a constant dry pressure at the model top, and $\mu$ is the dry column mass.

The above equation (2) implies the dominant balance in the tropics, i.e., adiabatic cooling by large-scale vertical motion tends to balance large-scale diabatic heating. In the CRM, the horizontal average over the domain of all other terms besides the tendency and relaxation terms in equation (1) would be considered diabatic heating from the point of view of a large-scale model with parameterized convection. Thus in steady state, equation (1) implies that the dominant balance will hold with large-scale vertical motion parameterized by equation (2).

Using the dry pressure differs from the conventional pressure coordinate, but it is numerically consistent and can be justified from the fact that the mass of the moisture in the troposphere is negligible (on the order $O(10^{-2})$) compared to the dry air mass. Within the boundary layer, following Sobel and Bretherton [2000] we do not apply equation (1), but instead obtain $W_{wtg}$ by linear interpolation from surface to the PBL top. Unlike in previous studies, with our implementation of WTG, the PBL top is not fixed, but is diagnosed in the boundary layer parameterization scheme (discussed below). The scheme determines a PBL top at each grid point, and we use the maximum value in the computational domain at each time step as the PBL top for the computation of $W_{wtg}$. 

\[
W_{wtg} \frac{\partial \overline{\theta}}{\partial \eta} \left( -\overline{\rho g / \mu} \right) = \frac{\overline{\theta} - \theta_{ref}}{\tau} \tag{2},
\]
Transport of moisture by the large-scale motion introduces additional source and sinks for moisture. The moisture equation is updated at each time step by adding the following terms associated with the WTG vertical velocity:

$$\frac{\partial Q}{\partial t} + \cdots = -W_{\text{wtg}} \frac{\partial Q}{\partial \eta} (-\rho g / \mu)$$  (3),

where $Q$ is the moisture mixing ratio. The right hand side of equation (3) is the advection by the large-scale vertical velocity $W_{\text{wtg}}$. Here we assume that the moisture field is horizontally uniform on large scales, such that the modeled convection is protected from the injection of reference profile humidities. Raymond and Zeng [2005] and Sobel and Bellon [2009] also incorporated parameterizations of the horizontal advection of moisture by large-scale flow. Raymond and Zeng's model assumes that the humidity of the air immediately surrounding the explicitly simulated convection is that of the reference profile, allowing parameterized horizontal advection by the large-scale divergent flow when the modeled domain’s mean humidity differs from that of the reference. Sobel and Bellon [2009] modeled horizontal advection by relaxation towards a reference profile of zero humidity within a specified layer on a specified advective time scale (by a flow presumed entirely rotational, with horizontal advection by the divergent component neglected). While it can be represented different ways, qualitatively horizontal advection is expected to reduce the precipitation when it is large, assuming regions with large precipitation are among the most humid on the planet so that nearby ones are drier. Here we exclude this effect to simplify the parameter space, but plan to explore it further in future work.

### 2.2. The WRF model

Here we briefly discuss the physical parameterization schemes used in this study. Boundary layer turbulence and the vertical sub-grid scale eddy diffusion are treated with the YSU scheme
This is a first order closure scheme, but also includes the non-local counter gradient transport [Troen and Mahrt; 1986]. In this scheme, boundary layer height, a parameter also used in WTG, is determined by the local Richardson number, temperature and wind speed. The horizontal sub-grid eddy mixing is parameterized using the 2D Smagorinsky first order closure scheme performed in physical space. The surface moisture and heat fluxes are parameterized following the Monin-Obukhov similarity theory. The bulk microphysics scheme is the Purdue-Lin scheme in WRF 3.1.1 [Lin et al., 1983; Rutledge and Hobbs, 1984; Chen and Sun, 2002]. This scheme has six species: water vapor, cloud water, cloud ice, rain, snow and graupel.

Radiation is a simple Newtonian relaxation scheme as in Pauluis and Garner [2006],

\[
Q_R = \begin{cases} 
-1.5 \text{ K} \cdot \text{day}^{-1} & \text{for } T > 207.5K \\
\frac{200 \cdot K - T}{5 \text{ day}} & \text{elsewhere}
\end{cases}
\]  (4),

The troposphere is cooled at constant rate 1.5 K/day, which is close to the observed climatology. The stratospheric temperature is near constant 200 K. This radiative cooling has a very weak dependence on temperature. In the RCE and WTG experiments discussed below, the vertically integrated \( Q_R \) yields an energy loss of \~145 W/m\(^2\) to the column, varying by no more than 1-2 W/m\(^2\) (at very high SST). Hence we can consider radiative cooling to be approximately fixed in our experiments. Such simple radiation cooling eliminates the complication of cloud-radiation feedback, so that only the parameterized large-scale circulation impacts the simulated convection and its organization. This simplifies the problem, at the cost of some loss of realism. In reality, long wave radiative cooling can be suppressed by the abundant upper tropospheric ice cloud in the deep tropics, an effect which is absent in our results.
WRF conserves dry mass to computer precision. However, conservation of moisture and energy are not guaranteed. Under RCE, in a sufficiently long time average, the surface moisture flux, $E$, should be exactly balanced by the precipitation, $P$, and the vertical integral of the radiative cooling $<Q_R>$ should be exactly balanced by the sum of the surface latent heat flux ($LE$) and sensible heat fluxes ($H$): $<Q_R> = LE + H$. These balance constraints can be evaluated using model output from RCE integrations. When we do this, our choice of the above schemes leads to a relative error in the moisture budget $(P - E)/P \sim 0.5\%$, while the residual of the radiative cooling and the sensible and latent heat fluxes has relative error $\sim 1\%$. We consider these to be good results, but they are not independent of the choice of physics packages. Our tests showed that with different choices of physical parameterizations (such as eddy viscosity schemes, microphysics schemes, and boundary layer schemes) non-conservation of water vapor and energy can occur, with relative errors up to several tens of percent. Our choices of physical parameterizations were influenced in part by our desire to maintain good conservation properties.

2.3. Experiment design

Numerical experiments are carried out in 2D and 3D under RCE and WTG. Statistical steady states are first achieved in RCE mode (no large-scale vertical advection applied in either temperature or moisture) over an SST of 28°C in both 2D and 3D. The time-averaged vertical temperature profile from these two experiments will be referred as the “reference” or RCE profiles. In our WTG integrations, SST is then varied from 27°C to 31.5°C while the horizontal mean potential temperature is relaxed towards the same RCE profiles in both 2D and 3D. Newtonian relaxation time scale $\tau$ in equation (1) is 3 hours for all 2D and 3D experiments.
Sensitivity experiments to this time scale is explored in Section 3.7. The moisture field is integrated including advection by the implied large-scale vertical velocity as discussed above. The experiments are performed at the equator (the Coriolis parameter $f = 0$). We use 50 vertical levels with 10 levels in the lowest 1 km while grid spacing is gradually stretched to 1.5 km near the model top (~22 km). The horizontal and vertical advections are 5th order and 3rd order accurate, respectively. Moisture and other scalars are advected using a positive definite scheme [Skamarock et al., 2008]. Vertically propagating gravity waves are absorbed in the top 5 km to prevent unphysical wave reflection off the top boundary by using the implicit damping vertical velocity scheme [Klemp et al., 2009]. The horizontal grid spacing is 2 km. This horizontal resolution is close to that of the 2D CRMs used in the multiscale modeling framework or “super-parameterization” [e.g., Khairoutdinov et al., 2005].

The model domain has 96x96 horizontal grid points in the 3D experiments. All the 2D experiments are chosen to have the same horizontal grids as 3D for a fair comparison between 2D and 3D under WTG. Our choice of the resolution and domain size is a result of the balance between high computational expense for long term integrations and the desire to resolve both fine details and convective organization. We also perform experiments with smaller domain size: 64x64 grid points and 32x32 horizontal grid points in 3D, and 64 grid points and 32 grid points in 2D. The primary results are not sensitive to domain size within this range. It is possible that in sufficiently larger domains convection would self-aggregate [Bretherton et al., 2005], behavior which does not occur in our integrations.

To avoid the pathological behavior of convective organization in 2D geometry [Held et al., 1993], the domain mean wind is relaxed to a vertically uniform value of 6 m/s with a relaxation time scale of 2 hours for the 2D experiments. Similarly, mean wind in one horizontal direction is
relaxed for the 3D experiments for the sake of fair comparison. The relaxation of mean wind also has some impact on the convection: it reduces the difference between 2D and 3D, because it strengthens winds in 3D (surface drag would otherwise eventually reduce the mean wind to near zero), while limiting the artificially strong perturbations in 2D which would otherwise occur [Tompkins, 2000].

The 3D experiments under RCE are integrated for 6 months, and the 2D experiments for 8 months. The last 150 days are averaged to obtain the RCE state for 3D, and last 300 days for 2D. The WTG experiments are integrated for 30 days for 3D and 120 days for 2D. The last 10 days of 3D experiments and last 60 days of 2D experiments are averaged to obtain the statistically steady state under relative SST forcing. Unless otherwise mentioned, diagnostics of vertical profiles are produced by averaging results for the last 10 days from the 3D experiments, and 150 days for 2D. The longer sampling period for 2D is meant to partially compensate for the missing third dimension and accordingly reduced sample size in 2D.

All the WTG experiments are initialized from one random snapshot of the RCE runs. Interestingly, previous studies have demonstrated that multiple equilibria are possible under WTG in both SCM [Sobel and Bellon, 2007] and CRM [Sessions et al., 2010] integrations, given sufficiently different initial conditions. It is possible that for very dry initial conditions, some of our integrations which result in states with significant deep convection would instead reach dry states with no deep convection. This possibility is beyond the scope of the present study; our initial condition has sufficient moisture that we expect deep convection to occur unless the boundary conditions render it impossible, i.e., sufficiently low SST for a given target RCE temperature profile.
3. Results

3.1 Precipitation and convective organization under WTG

Precipitation in 3D is shown in Figure 1 for SST=28°C and 30°C under WTG. The snapshots of hourly rain rates in Figures 1a and 1b are randomly sampled at two different times and indicate the spatial structure of convective organization. The impact of the relative SST is readily seen. There is virtually no organized convection at 28°C (also the SST for RCE runs) in these two snapshots. In contrast, at SST=30°C, convection appears to aggregate and linear mesoscale convective systems (e.g., squall lines) are common. In fact, examination of the rain rate at other times indicates that there are always 1-3 well organized convective clusters. To further quantify the impact of the relative SST on convection and precipitation, Figure 1c plots time series of the hourly and daily rain rate. At SST=28°C, the daily rain rate remains approximately unchanged at the RCE value after WTG is turned on at day 0. At SST=30°C, it increases smoothly during the first 5 days, after which it reaches a statistical equilibrium value of ~22 mm/day.

Figures 2a and 2b show Hovmöller diagrams of the hourly rain rate at RCE and SST = 30°C in 2D. At SST=28°C, there are a few bands of organized convection, as well as dry regions. At SST=30°C, convective events are much more frequent while the dry region shrinks considerably. Cloud clusters appear to be able to propagate in both directions, although left moving clusters are preferred, presumably because the mean wind is always close to 6 m/s. On the other hand, there is also no sign of organized cloud clusters in the Hovmöller diagrams from the 3D experiments. Quantitatively, rain rate at SST=28°C (Figure 2c) is nearly steady, while it is much higher at SST=30°C with increased variability. The daily rain rate reaches a statistically steady state ~32 mm/day after first 20 days at SST=30°C. The 2 K increase of relative SST forces a much higher rain rate (~32 mm/day) in 2D than that (22 mm/day) in 3D. On the other hand, the rain rate at
both hourly and daily time scales exhibits higher frequency variability in 2D than in 3D, as first noted by Grabowski et al. [1998]. This greater temporal variability in 2D may be related to weaker damping of buoyancy in 2D, as suggested in Zeng et al. [2007], but may also be to some extent simply a result of sampling given the much smaller number of grid points used to compute daily rainfall in 2D.

Figure 3 shows the equilibrated precipitation versus SST ranging from 27°C to 31.5°C for all the 2D and 3D experiments. In general, precipitation increases above 28°C and vanishes below 28°C. We will refer to these two regimes as the “wet” and “dry” regimes below, with the RCE state at 28°C falling between the two. We first briefly discuss the dry regime. In both the dry and wet regimes, the surface heat fluxes (latent plus sensible) are greater than those found at SST=28°C, the RCE value. This may be induced by enhanced thermodynamic disequilibrium between the ocean and lowest model levels. It also has implications for the energy budget, as discussed in more detail in Section 3.6. At SST=28°C, the rain rates, as well as surface fluxes, for 2D and 3D are almost the same, around 4.8 mm/day. This illustrates that introducing WTG at the RCE SST does not cause the model to diverge from the RCE state; the RCE is robust in this sense, as has been found in some single column models [Sobel and Bretherton, 2000, Sobel and Bellon, 2009] though not in the CRM calculations of Raymond and Zeng [2005] and Raymond [2007]. Nevertheless, the steady state in 2D under WTG is slightly drier than that in 3D. The rain rate is about 4.3 mm/d under WTG, nearly 10% lower than that in RCE.

In the wet regime, rain rate and surface fluxes both increase with SST. The increases in surface heat fluxes in 2D and 3D are quantitatively very similar, while the increases in precipitation in 2D and 3D are qualitatively similar but quantitatively different. As noted above, at 2 K warming (30°C), $P$ is ~22 mm/day for 3D and ~32 mm/day for 2D (Figure 3) nearly 60%
more in 2D than 3D. The difference in precipitation can not be attributed to surface fluxes, since those are similar; we show below (Section 3.6) that instead the difference in precipitation is consistent with a smaller gross moist stability in 2D than 3D.

3.2 Parameterized large-scale circulation

Figure 4 shows the large-scale vertical velocity $W_{wtg}$ at statistically steady state under WTG for both 2D and 3D. $W_{wtg}$ is top-heavy for all SSTs greater than 28°C; at that value $W_{wtg}$ is close to zero, as expected. $W_{wtg}$ peaks near 12 km. As SST increases further above 28°C, this peak value $W_{wtg}$ also increases (Figure 4b): it reaches ~12 cm/s at SST=30°C, and ~25 cm/s at SST=31.5°C in 3D. On the other hand, the shape of $W_{wtg}$, which is obtained by normalizing $W_{wtg}$ by the peak value (Figure 4d), remains remarkably self-similar in all experiments despite substantial variation in the SST forcing.

As SST is decreased below 28°C, precipitation ceases to occur. The dominant heat balance is between adiabatic warming by large-scale descent and radiative cooling. This explains why $W_{wtg}$ is the same at dry SST regime 27°C and 27.5°C, as the radiative cooling is nearly identical in those two calculations.

As documented by, e.g., Back and Bretherton [2006, 2009], top-heavy large-scale vertical velocity profiles are typical of the western Pacific warm pool region, and bear self-similar vertical structure in the rainy regions [See Figure 4 in Back and Bretherton, 2006]. In contrast to the self-similar shape of $W_{wtg}$ in our model, Kuang [2010] obtains $W_{wtg}$ profiles with varying degree of top-heaviness: as the scales of the coupled large-scale wave become smaller, $W_{wtg}$ becomes more top-heavy. We show below that variations in the $W_{wtg}$ profile greater than those
shown in Figure 4, and in some respects similar to those of Kuang [2010], can be obtained in our
model by varying the relaxation time scale $\tau$.

On the other hand, our model in the present configuration does not produce any bottom-heavy profiles of $W_{\text{wgt}}$, such as are found in the east Pacific in reanalysis datasets [Back and
Bretherton, 2006]. Raymond and Sessions [2007] obtained bottom-heavy $W_{\text{wgt}}$ profiles by
simultaneously warming the upper troposphere and cooling the lower troposphere in their target
RCE temperature profile; when they used the unperturbed RCE profile from their model as the
target, they obtained top-heavy $W_{\text{wgt}}$ profiles qualitatively similar to ours.

Although $W_{\text{wgt}}$ behaves qualitatively similarly in response to the SST changes in 2D and 3D,
there are significant quantitative differences. Figure 4b shows that $W_{\text{wgt}}$ is significantly larger in
2D than in 3D at the same SST; its maximum value for 30°C (a 2K warming in SST relative to
the RCE value) is ~16 cm/s for 2D and ~12cm/s for 3D. This is consistent with the rain rate
difference between 2D and 3D at equilibrium states. How precipitation is related to $W_{\text{wgt}}$ will be
further discussed in detail through budget analysis of moist static energy in Section 3.6.

3.3 Thermodynamic structure

In this section, we examine mean profiles of thermodynamic variables in the convecting
atmosphere under WTG, including temperature, relative humidity, moist static energy, and
buoyancy.

Figure 5a shows the profiles of mean troposphere temperature (SST=28°C) in both 2D and
3D. Variation of mean temperature under WTG with respect to the RCE profile is plotted in
Figures 5a and 5b for 3D and 2D. In all the cases, unlike in strict WTG as has been enforced in
some SCM studies [Sobel and Bretherton, 2000; Ramsay and Sobel, 2010], the free tropospheric
temperature increases with SST, due to the finite relaxation time $\tau$. $T$ increases by 1.3 K in 3D and 1.8 K in 2D at 2 K SST warming near 12 km, where latent heat release is maximum. Ramsay and Sobel [2010] show in an SCM that upper troposphere can warm by $\sim$ 5 K with 2 K increase in SST in RCE, because the temperature profile approximately follows a moist adiabat tied to the SST. So while warming in the upper troposphere under WTG in our model is quite different from zero, it is also considerably less than it would be in the RCE state given the same 2K SST perturbations as expected since WTG strongly inhibits upper troposphere warming. In the dry SST regime, free tropospheric temperature decreases as SST does, but the maximum decrease occurs near 1 km, the top of the boundary layer. This local minimum in temperature is associated with a deeper boundary layer, as seen from potential temperature profiles (not shown). Turbulence in the boundary layer maintains an approximately dry adiabatic temperature profile starting at the surface air temperature, which itself is close to the SST. When the SST is relatively low, the temperature just below the top of the PBL is cold compared to that above in the reference profile. In a deeper PBL, layers which were free-tropospheric become part of the PBL and thus colder, resulting in the minimum around 1 km.

Near surface temperature increases weakly with SST - less than 0.5°C even with a 3.5°C SST (31.5°C) forcing, in contrast to $\sim$ 2°C warming at upper levels. There is apparently a very strong negative feedback on near-surface air temperature, presumably due to convective downdrafts. A similarly strong negative feedback was found in the SCM calculations of Ramsay and Sobel [2010], who noted that the air-sea disequilibrium varies less with relative SST in simulations with global climate models than in the WTG SCM. We speculate that the strong negative feedback within the PBL in WTG calculations in both SCMs and CRMs may result from the quasi-steadiness of convection. In nature (and presumably in climate models) there is greater
intermittency in the occurrence of deep convection, so that there are presumably periods without
downdrafts when the boundary layer is able to recover more completely than in our WTG
calculations.

Here we show the association between enhanced convection at high SST and equivalent
potential temperature $\theta_e$ in the sub-cloud layer, a rough indicator of parcel buoyancy at higher
levels. Figure 6 plots the normalized histogram of $\theta_e$ at 100 m for SST $\geq 28^\circ$C. In both 2D and
3D experiments at RCE, $\theta_e$ values at 100 m are concentrated around the peak but are slightly
skewed toward smaller values. In contrast to the left tail, the right tail of the histogram plummets
to zero quickly a couple of degrees above the peak. At higher SST, the histograms are flatter.
The distributions remain skewed toward smaller values, particularly in 3D. In 2D, the skewness
becomes small at high SST. The increase of $\theta_e$ at high SST is associated with more frequent
mesoscale convective systems as seen in both the 3D and 2D experiments (Figures 1 and 2).

Figure 6c compares the $\theta_e$ distributions in the sub-cloud layer at 28$^\circ$C and 30$^\circ$C. At 30$^\circ$C, the
distribution has a slightly fatter right tail in 2D than in 3D, consistent with stronger convection as
indicated by the difference in 2D and 3D rain rates at high SST shown above. Tompkins [2001]
showed that cold pools were well formed and new convection can be initiated at the cold pool
front even without mean wind shear. On the other hand, convection is also affected by the
entrainment of less buoyant surrounding air into convective cells. Both factors - entrainment and
cold pools - can be influenced by dimensionality. Petch et al. [2008] suggested that the
entrainment rate was significantly larger in 3D than 2D. It is not easy to quantify the relative
roles of these two factors, and we do not attempt to do so here.

Figure 7 displays relative humidity (RH). RH at the RCE temperature is intermediate
between the extremes of the dry and wet regimes. In the dry regime, free-tropospheric RH is near
zero. This is as expected due to the lack of convective moistening to counteract the drying effect of large-scale descent, and has been found in earlier WTG calculations with both SCMs and CRMs [e.g., Sobel and Bretherton, 2000; Raymond and Zeng, 2005; Sobel et al., 2007; Sessions et al., 2010]. In the wet regime, upper tropospheric RH increases rapidly with the first 1°C increase of relative SST, but then saturates so that further SST increases only lead to slight increases in RH. The minimum just below 10 km which is present at 28°C is absent in the wet regime, in which the slope of RH with respect to height is nearly constant. This disappearance of the dry minimum is probably related to the enhanced vertical transport associated with enhanced convective fluxes. Compared to the SCM study in Ramsay and Sobel [2010, their Figure 3b], the increase of RH at upper levels is greater here; in their SCM integrations the RH minimum remained present at higher SST.

Figure 7c compares RH at RCE and 30°C. In RCE, the 2D integration has a significantly drier free troposphere than does the 3D one. In the boundary layer both temperature and moisture are very similar in 2D and 3D. At 30°C, the difference in RH between 2D and 3D is much smaller. It is slightly moister below 8 km in 2D. The relation between RH and precipitation will be discussed further below.

Figure 8 shows vertical profiles of moist static energy

\[ h = C_p T + gz + L_v Q \]  

(5).

Definitions of symbols are standard. \( h \) is approximately conserved during phase changes (without considering ice). At RCE, \( h \) is lower in 2D at its minimum (~5 km), probably because of the drier mean atmosphere and more cloud at lower atmosphere (see Figure 9 below). In response to SST forcing, \( h \) increases with SST throughout the free troposphere in the wet regime. The minimum value (like the whole profile) increases with SST, and becomes a rather shallow
minimum; however it occurs at a lower altitude (~3 km) at high SST than at RCE. Dry static energy behaves similarly to temperature under WTG with a maximum increase near 12 km (not shown).

3.4 Cloud properties: cloud fraction and convective mass fluxes

Here we turn our attention to bulk cloud properties under WTG. Previous studies using CRMs with traditional forcing methods have noted that bulk cloud properties are different in 2D and 3D experiments [e.g., Tompkins, 2000; Phillips and Donner, 2006; Petch et al., 2008]. We find that the difference increases with relative SST under WTG.

Cloud fraction is shown in Figure 9. The cloud is defined as the set of grid points where the mixing ratio of cloud hydrometeors (ice and water) is greater than 0.005 g/kg. At SST=28°C, cloud fraction peaks at two levels, near 1 km and 12 km, in both 2D and 3D. These two peaks indicate shallow and deep cumulus. There is also an indication of mid-level cloud near the melting level (~ 4 km, see Figure 5a). Compared to 3D, the 2D RCE experiments have more shallow and mid-level cloud but less deep cloud.

Under WTG, cloud fraction in the upper troposphere increases quickly with SST, more rapidly in 2D than in 3D. It increases from 0.07 at 28°C to 0.6 at 30°C in 2D, and from 0.12 at 28°C to 0.55 at 30°C in 3D. Below 2 km, there is a small increase in cloud fraction with SST in 3D but much more in 2D.

The profile of cloud fraction is normalized by the maximum value (Figures 9b and 9d), as was the WTG velocity in Figure 4. Unlike the self-similar structure of $W_{wtg}$, cloud fraction shows more variations in low and mid troposphere, particularly for 2D. Low cloud (below 2 km) is much less at large SST.
Convective mass fluxes are key quantities for a statistical description of convection. Here we examine both updraft ($M_u$) and downdraft mass fluxes ($M_d$). $M_u$ is estimated, similarly to Robe and Emanuel [1996], as the sum over the points with positive liquid water content greater than 0.005 g/kg and vertical velocity greater than +1 m/s, normalized by total number of grid points,

$$M_u = \sum_{q_i,w>1m/s,>0.005g/Kg} \frac{\rho_{i,j} w_{i,j}}{N}$$

where $i$ and $j$ indicate grid points, $\rho$ is density, and $N$ is the total number of horizontal grid points. Downdraft mass flux $M_d$ is estimated over points where $w < -1$ m/s; no threshold on liquid water content is applied as we are interested in unsaturated as well as saturated downdrafts.

Figure 10 shows $M_u$ and $M_d$ for all experiments with SST greater than or equal to 28°C. At the RCE SST, $M_u$ is larger at lower levels than at upper levels. At lower levels, $M_u$ is slightly larger in 3D than in 2D. $M_u$ increases rapidly with SST; the increase is more rapid in 2D than in 3D, consistent with the results for precipitation. The absolute value is much higher than that found in the SCM of Ramsay and Sobel [2010], but this may be partly due to the threshold used here, which is necessarily somewhat arbitrary.

Updraft mass flux $M_u$ does not always peak in the upper troposphere, as does cloud fraction and WTG velocity. In RCE (28°C), it peaks at ~ 2 km, suggesting a greater role for shallow convection. Figures 10b and 10d show convective mass flux normalized by its peak value, which clearly reveals the difference between the shallow and deep convection. Like cloud fraction, convective mass flux is not self–similar. As SST increases above RCE, convective mass flux becomes more top-heavy with relatively less mass flux in the low troposphere.

Downdraft mass flux $M_d$ generally peaks at heights near those where updraft mass flux $M_u$ does. Comparing panels a and c in Figure 10 suggests that $M_d$ is stronger in 2D than in 3D.
Here we quantify the relative strength of $M_d$ to $M_u$ using the ratio $M_d / M_u$. $M_d / M_u$ at the top of boundary layer is an important parameter in the theory of boundary quasi-equilibrium [Emanuel, 1995; Raymond, 1995; Raymond et al. 2009]. In our WTG experiments, $M_d / M_u$ at 1 km (close to top of boundary layer) is 0.75 for 2D and 0.35 for 3D at SST=28°C. At SSTs greater than 28°C, it is 1.0-1.5 for 2D and 0.4-0.6 for 3D. This ratio varies, however, with the $w$ threshold chosen in equation (6). Figure 10 (f) shows $M_d / M_u$ decreases from 2-5 to zero as threshold value of $w$ increases from 0 to 3 m/s, as $M_d$ decreases more rapidly with larger $w$ threshold than does $M_u$. [Note that the ratio does not reach unity for a $w$ threshold of zero because of the condensate threshold applied to $M_u$ but not $M_d$. If no condensate threshold is applied to either one, the ratio does reach unity for a $w$ threshold of zero (not shown), reflecting the fact that the WTG vertical velocity appears only in the thermodynamic budgets and not in the mass budget, so that the domain-averaged mass flux resolved by the model must vanish at each level.] The ratio in 2D is distinctly larger than in 3D for all $w$ thresholds and all SST.

3.5 Precipitation and column relative humidity

Observational studies [Bretherton et al., 2004; Peters and Neelin, 2006; Neelin et al., 2009; Holloway and Neelin, 2009] have shown relationships between precipitation and the column relative humidity (CRH, the column-integrated water vapor divided by its saturation value). Theoretical and modeling studies have also examined the implications of these relationships [e.g., Raymond, 2000; Raymond and Zeng, 2005; Sobel and Bellon, 2009; Muller et al., 2010]. Here we examine such relationships in our experiments.

Figure 11 illustrates the daily rain rate versus column relative humidity at two different periods: the latter portions of the calculations, during which a statistical equilibrium is reached
for any particular SST, and the earlier portions, which are transition periods from RCE to the
ew equilibrium. The equilibrium periods are sampled during the last 10 days for 3D and last 30
days for 2D. The transition periods are sampled during the first 10 days for 3D and the first 30
days for 2D after WTG is switched on. Analysis is performed based on daily accumulated
quantities as well as hourly quantities. The latter leads to similar mean relationships but greater
spread, as expected.

Compact relationships qualitatively similar to those seen in observations and other models
are found between the daily rain rate and CRH for both periods. There is virtually no
precipitation for CRH < 0.6, followed by a very sharp increase for CRH greater than 0.6. In the
transition period (Figures 11a and 11b), two differences between 2D and 3D emerge. First,
considerable spread of CRH and precipitation is seen in 2D. Also, the rain rate reaches greater
values in 2D, consistent with the other statistics discussed above. The transition from near-zero
precipitation to rapidly increasing precipitation with CRH also appears sharper in 3D than 2D,
though it is possible that this is a result of greater scatter in 2D.

In the equilibrium period (Figures 11c and 11d), we still have compact relationships between
CRH and precipitation similar to those in the transition period. However, a fine scale feature
emerges somewhat more clearly. For any given integration at a particular SST, the rain rate
decreases slightly with respect to CRH, contrary to its general increase with respect to CRH
when multiple integrations with different SST are considered together. The spread of
precipitation appears to be larger at higher CRH. The decline of precipitation with CRH at
constant SST presumably reflects negative feedbacks between the two variables, such as the
drying effect of precipitation. We have not investigated this further, but there could be useful
clues here about the mechanisms responsible for the overall CRH-precipitation relationship.
We have also examined the rain rate accumulation at shorter time scales (e.g., 6 hours). This substantially increases the sample size of rain rate in 3D (by a factor of 4 compared to daily sampling). All the features discussed above and the difference between 2D and 3D still hold, suggesting that it is unlikely the small sample size (10 days from each experiment) in 3D is an issue for the features discussed above.

3.6. Budget analysis

In our experiments, precipitation increases nonlinearly with respect to the SST forcing (Figure 3), confirming earlier results from single column models [Sobel and Bretherton, 2000; Ramsay and Sobel, 2010], both qualitatively and quantitatively: 2 K warming leads to 4-5 fold increase of daily rain rate. Here we analyze the moist static energy budget in order to understand the controls on precipitation in more detail.

The budgets of vertically integrated moist static energy ($h$) and dry static energy ($s$, similar to $h$ but without the moisture term in equation 6) in the statically steady state can be written as

$$<w \frac{\partial s}{\partial z}> = H + P + <Q_R> \quad \text{and} \quad <w \frac{\partial h}{\partial z}> = E + H + <Q_R>$$

where $P$, $E$, $H$ and $Q_R$ denote precipitation, surface sensible flux, latent flux and radiative heating, and $<>=\int_{p_0}^{p_f} dp / g$ denotes the mass-weighted vertical integral from the bottom to the top of the domain.

In the wet regime, it is useful to consider the following diagnostic relation [Sobel, 2004] to understand $P$,

$$P = \frac{1}{M} (E + H + <Q_R>) - <Q_R> - H$$
Where \( M \) is the normalized gross moist stability, a dimensionless number estimated as

\[
M = \frac{\langle \frac{\partial h}{\partial z} \rangle}{\langle \frac{\partial S}{\partial z} \rangle},
\]

where the overbar indicates time averaging (over whatever time scale we wish to compute \( P \)) encapsulates the influence of the divergent atmospheric circulation on the moist and dry static energy budgets in the way relevant to precipitation [Neelin and Held, 1987].

Given estimate of \( M \), knowing energy fluxes will allow us to diagnose the precipitation. The parameter \( M \) is similar to the “normalized gross moist stability” in Raymond et al. [2009], with the minor difference that the denominator here is the dry static energy divergence rather than the moisture convergence (Note however that Neelin and Held [1987] and many subsequent authors have used the notation \( M \) for the gross moist stability defined with a different normalization and different dimensions; our \( M \) is dimensionless.)

We utilize the relation (8) to estimate precipitation using the modeled energy fluxes and \( M \) evaluated from the model output, all sampled every 12 hours. The radiation flux from our simple radiative scheme leads to nearly constant radiative cooling of \( \sim 145 \) W/m\(^2\). The surface fluxes increase roughly linearly, as discussed before. Comparing this to the precipitation output directly by the model gives a good idea of the degree to which energy is conserved in the model, as well as that to which fluctuations on time scales shorter than 12 hours can be neglected in the budget [as they are in equation (8); i.e., if all quantities individually are time-averaged then the precisely correct equations should have also the time-averaged products of shorter timescale fluctuations in all nonlinear terms]. The relative magnitudes of the different terms then provide information about the relative roles of circulation and surface fluxes (radiative cooling again varying little here) in producing the precipitation variations with SST.

Figure 12a shows \( M \), computed from the domain and time-averaged data. Alternatively, we have also diagnose \( M \) by replacing the time averaged quantities with median values, or by
performing time average of instantaneous $M$, or by taking the median value of instantaneous $M$. These different estimates yield nearly identical values for all 3D experiments, while estimates using the time average of instantaneous $M$ show greater variations for the 2D experiments. This indicates that transients play a greater role in 2D, as might have been expected from the other results above.

Generally, the gross moist stability $M$ decreases with SST in the wet regime. This is true in 3D over the entire range of SST and in 2D for SST>29.5°C. $M$ decreases gradually from 0.36 at SST=28.5°C to ~0.25 at SST=31.5°C for 3D. Computations of $M$ holding either WTG vertical velocity or mean moist static energy fixed (not shown) suggest that changes in moist static energy explain almost entirely the variations in $M$ in 2D, as might be expected from the self-similarity of $W_{wtg}$ profiles shown in fig. 4. In 3D, these calculations suggest that while changes in moist static energy explain much of the $M$ variation with SST, changes in $W_{wtg}$ also play a role despite their smallness.

The values of $M$ in 2D are all smaller than those in 3D. This indicates higher $P$ in 2D than in 3D given similar surface forcing, as seen from equation (8). Figure 12b shows the precipitation computed from equation (8). This diagnosis of $P$ based on the energy budgets is quantitatively close to the actual daily mean precipitation for the 3D experiments. $P$ is underestimated slightly more at high SST in 2D, consistent with the greater role of neglected high-frequency transients.

The smaller values of $M$ in 2D than in 3D are the main reason for the greater $P$ in 2D. This can be seen from the budget of precipitation (equation 8), which states that three factors can contribute to the increase of precipitation with SST in steady state: surface fluxes, radiative cooling, and $M$. In our experiments, the surface fluxes are slightly smaller in 3D than 2D (Figure 3) and radiation cooling is nearly the same in 2D and 3D, but $M$ is significantly smaller in 2D.
3.7 Sensitivity to WTG relaxation time $\tau$

In all experiments discussed above, the Newtonian relaxation time scale $\tau$ in equation (1) is 3 hours. Here we explore the solution sensitivity to $\tau$. Given the same $2^\circ$C SST forcing (SST=30$^\circ$C) in both 2D and 3D experiments, $\tau$ is varied from 3 minutes to 2 days: 0.05, 0.1, 0.5, 1, 2, 3, 4, 6, 9, 12, 15, 18, 24 and 48 hours. Presumably, interaction between convection and WTG vertical velocity $W_{wtg}$ is the strongest in the zero limit of $\tau$ and the weakest in the infinity limit. As we show below, at finite $\tau$ there is a local minimum and a maximum in the precipitation as a function of $\tau$.

Figure 13a shows that $P$ achieves a maximum at $\tau = 5$-6 hours, and the minimum in at $\tau = 0.5$ hours. The local minimum is associated with large value of gross moist stability (Figure 13c). The local maximum is also seen in previous WTG studies [Figure 9 of Sessions et al. 2010, Kuang 2010]. Kuang [2010] interpreted the dependence of gross moist stability on horizontal wavelength – a parameter which (squared) plays the same role as $\tau$ in our calculations – as potentially responsible for the scale selection in the Madden-Julian Oscillation, assuming the dynamics of that mode to be described by “column MSE instability”, or (almost equivalently) as a “moisture mode” [Neelin and Yu, 1994; Sobel et al., 2001; Fuchs and Raymond, 2002; Sobel and Bretherton, 2003; Bretherton et al. 2005; Fuchs and Raymond, 2005; Fuchs and Raymond, 2007; Raymond and Fuchs, 2007; Raymond and Fuchs, 2009; Sugiyama, 2009a,b; Maloney et al. 2010]. The decrease in $P$ in Figure 13a at large $\tau$ is easily understood; the system must return to RCE as $\tau$ goes to infinity, and in that limit the precipitation must return to the value determined by radiative cooling. Consistent with the dependence on horizontal wavelength found by Kuang [2010], as $\tau$ increases, $W_{wtg}$ becomes less top-heavy and gross moist stability
decreases (Figure 13 c). This is explains the increase of $P$ with $\tau$ at small $\tau$ shown in Figure 13a. As $\tau$ increases to larger values, the decline of $P$ can presumably be understood as the beginning of the approach to the limit as $\tau$ goes to infinity. In this limit the system must approach RCE, implying that surface fluxes balance radiative cooling, surface evaporation balances precipitation, and $M$ becomes undefined as $W_{wtg}$ vanishes, This limit is not yet close at $\tau = 24$ hours, but the decrease of surface latent heat flux (not shown) is already sufficient to overcome the decrease in $M$ and cause reduction in $P$.

Figures 13b and 13d shows normalized $W_{wtg}$ in 2D and 3D. As $\tau$ is varied, vertical structure in $W_{wtg}$ changes systematically. Below 4 km, $W_{wtg}$ is smaller for small $\tau$ than large $\tau$; above 4 km $W_{wtg}$ is almost independent of $\tau$. In other words, the profile is less top-heavy as $\tau$ increases, unlike the self-similar behavior of $W_{wtg}$ as SST is varied. This explains the decrease in gross moist stability with $\tau$ mentioned above. The change of $W_{wtg}$ with respect to $\tau$ is not uniform over the range of $\tau$ considered: it is relatively large as $\tau$ increases up to 6 hours and very small as $\tau$ varies from 6 hours to 24 hours. The latter is consistent with the self-similar behavior of $W_{wtg}$ in SST forcing experiments.

The difference between 2D and 3D is evident for all $\tau$ values, but is largest in the limit $\tau = 0$ and the smallest in the limit $\tau = \infty$.

### 3.8 Sensitivity to domain size

In the results discussed in previous sections, the domain size is 192 km. Here we repeat our 2D and 3D experiments in smaller domains: 128 km and 64 km, respectively. In both 2D and 3D experiments, varying domain size seems have little impact on the equilibrated precipitation (not
shown). On the other hand, precipitation in 2D remains clearly different from that in 3D, indicating that the comparisons between 2D and 3D are robust.

4. Conclusions

In this study, equilibrated tropical convection is explored using a cloud-resolving model. The weak temperature gradient (WTG) approximation is used to parameterize the coupling between explicitly resolved convection and large-scale motion. Sea surface temperature is varied while other parameters, including the target temperature profile towards which the horizontal mean free tropospheric temperature is relaxed, are held fixed. Model integrations are carried out with both 2D and 3D geometry. The primary findings are as follows.

1. As in previous studies with parameterized convection, and as expected from basic considerations, precipitation increases with relative sea surface temperature. The increase is not independent of geometry, however. While the RCE states in 2D and 3D are similar, precipitation increases more rapidly in 2D than 3D as SST is increased above the RCE value. As an example, given an SST increase of 2 K above the RCE value, $P$ is 60% more in 2D than 3D.

2. When the time scale at which the horizontal mean free tropospheric temperature is relaxed to the target profile is held fixed at a small value (e.g., 3 hours), the shape of the WTG vertical velocity profile is remarkably invariant with SST (as long as SST is above the RCE value), although the magnitude varies strongly with SST. Cloud fraction and convective mass flux show a lesser degree of self-similarity with SST.

3. As might be expected given the precipitation results, both the WTG vertical velocity, and other measures of the intensity of the resolved convection - such as convective mass fluxes and cloud fraction - increase more rapidly in 2D than 3D. We expect that the difference between 2D
and 3D may be attributed to differences in the buoyancy of PBL parcels and entrainment. Entrainment is not quantified in this study, but is expected to be weaker (thus diluting updraft buoyancy less) in 2D than 3D. We do analyze the distributions of near-surface equivalent potential temperature, and show that larger values occur more frequently in 2D than 3D.

4. Precipitation as a function of column-integrated relative humidity (CRH) – when results from all model integrations are plotted together – shows the expected qualitative behavior, with no precipitation when CRH is below a threshold value and a rapid increase above that. However, within each individual model integration, CRH and precipitation show a clear negative correlation as both vary within narrow ranges of their respective quasi-equilibrium states. This is presumably indicative of the negative feedbacks which maintain the tight coupling observed over the larger ranges filled by the ensemble of model integrations. This behavior is found in both 2D and 3D.

5. An analysis of the moist static energy budget shows that the normalized gross moist stability is smaller in 2D than 3D. This is consistent with the greater precipitation in 2D, as it leads to greater moisture convergence for the same surface evaporation. Surface evaporation as a function of SST is very similar in 2D and 3D. The normalized gross moist stability decreases smoothly with SST, but remains positive.

6. When the time scale at which the horizontal mean free tropospheric temperature is varied, the shape of the WTG vertical velocity profile changes. At longer relaxation time, corresponding to larger warming of the upper troposphere in response to convection, the WTG vertical velocity in the lower troposphere increases while the upper tropospheric peak narrows. To the extent that these results can be compared with those of Kuang [2010], the lower tropospheric change is consistent while the upper one is inconsistent. These differences likely result from the different
formulations of WTG in the two models, and are worthy of further investigation. However, the overall decrease of top-heaviness, and gross moist stability, with relaxation time is qualitatively consistent with the results of Kuang [2010], who proposed this as a mechanism for the scale selection of the Madden-Julian Oscillation.

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Figure 1. Snapshots of hourly precipitation in the 3D WTG experiments for (a) SST=28°C and (b) SST=30°C. Panel (c) show the time series of daily (dark solid, mm/day) and hourly rain rate (thin blue, mm/day) for SST=28°C and 30°C.
Figure 2. Hovmöller diagram of hourly precipitation in the 2D WTG experiments for (a) SST=28°C and (b) SST=30°C. Panel (c) shows the time series of daily and hourly rain rate (mm/day) for SST=28°C and 30°C.
Figure 3. Daily rain rate $P$ (mm/day, solid) and surface fluxes (latent and sensible heat flux, $E+H$, in the unit of mm/day, dashed) versus SST for 2D and 3D. Surface fluxes in 3D are almost the same as in 2D.
Figure 4. WTG vertical velocity for (a) 3D and (c) 2D. The maximum value of WTG vertical velocity is shown in (b). Normalized WTG vertical velocity is shown in (d).
Figure 5. (a) Mean temperature from the 3D and 2D WTG experiments at 28°C. (b) and (c) show temperature difference between all WTG experiments and the WTG experiment at 28°C for (b) 3D and (c) 2D.
Figure 6. Normalized histogram of equilibrium potential temperature $\theta_e$ at 100 m for (a) 3D and (b) 2D. (c) shows histogram of $\theta_e$ at SST= $28^\circ$C and SST= $30^\circ$C for 3D and 2D. Bin size of $\theta_e$ is 0.2 K. $\theta_e$ for SST less than $28^\circ$C is not shown.
Figure 7. Relative humidity in (a) 3D and (b) 2D WTG experiments. Relative humidity at SST=28°C and 30°C for 3D and 2D is compared in (c).
Figure 8. Moist static energy in all the (a) 3D and (b) 2D WTG experiments.
Figure 9. Cloud fraction in (a) 3D and (b) 2D WTG experiments. (b) and (d) are normalized cloud fraction of (a) and (c). (e) cloud fraction at SST=28°C. Cloudy points are chosen if cloud water \( q_c \) and cloud ice \( q_i \) is greater than 0.005 g/kg.
Figure 10. Updraft and downdraft mass flux in (a) 3D and (c) 2D WTG experiments for SST no less than 28°C. (b) and (d) are normalized convective mass flux of (a) and (c). (e) Convective mass flux at RCE (28°C). Updraft mass flux is estimated on cloudy grid points (where \( q_c + q_i > 0.005 \text{ g/kg} \)) with \( w > 1 \text{ m/s} \), and downdraft flux is estimated on points where \( w < -1 \text{ m/s} \) (f) shows \( M_d / M_u \) (the ratio between downdraft and updraft mass flux) at 1 km for SST no less than 28°C as the \( w \) threshold varies from 0 to 3 m/s.
Figure 11. Daily rain rate versus column relative humidity for 3D in (a) and (c) and 2D in (b) and (d). Top panels show data at the transition periods from RCE to WTG (sampled for the first 10 days for 3D and first 30 days for 2D). Bottom panels show data at the equilibrium periods of WTG (sampled for the last 10 days for 3D and last 30 days for 2D).
Figure 12. (a) Normalized gross moist stability $M$ estimated using the model output. (b) Daily rain rate (mm/day) diagnosed using equation (8) with estimated $M$ (solid), and daily rain rate from model output (dashed).
Figure 13. (a) Precipitation versus the relaxation time scale $\tau$ at SST=30°C for 2D and 3D experiments. (c) Normalized gross moist stability versus $\tau$. (b) and (d) show normalized $W_{WTG}$ in 2D and 3D, respectively. For clarity, only the experiments with $\tau=0.5$, 1, 3, 8, 12 and 24 hours are plotted in (b) and (d).