Kinetic energy budget for the Madden-Julian Oscillation in a multi-scale framework

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Abstract

A kinetic energy budget for the Madden-Julian Oscillation (MJO) is established in a three-scale framework. The three scales are the zonal mean, the MJO scale with wavenumbers 1-4, and the small scale with wavenumbers larger than 4. For the composite MJO event, the dominant balance at the MJO scale is between conversion from potential energy and work by the pressure gradient force (PGF). This balance is consistent with the view that the MJO wind perturbations can be viewed as a quasi-linear response to a slowly varying heat source. A large residual in the upper troposphere suggests that much kinetic energy dissipates there by cumulus friction. Kinetic energy exchange between different scales is not a large component of the budget for the composite MJO event. There is a transfer of kinetic energy from the MJO scale to the small scale; that is, this multi-scale interaction appears to damp rather than strengthen the MJO. There is some variation in the relative importance of different terms from one event to the next. In particular, conversion from mean kinetic energy can be important in some events. In a few other events, the influence from the extra-tropics is pronounced.
1. Introduction

The Madden-Julian Oscillation (MJO) is the dominant mode of intraseasonal variability in the tropics (Madden and Julian 1971, 1972, 1994). There have been many theories and model studies of the MJO thus far, as summarized in Wang (2005) and Slingo et al. (2005). Different theories focus on different mechanisms, such as wave-CISK (e.g., Lindzen 1974; Lau and Peng 1987), upscale cascade from small scale convection (Moncrieff 2004; Majda 2007a, b), recharge-discharge (e.g., Blade and Hartmann 1993; Hu and Randall 1995; Kemball-Cook and Weare 2001; Sobel and Gildor 2003), feedbacks with surface heat fluxes and radiation (e.g., Emanuel 1987; Neelin et al. 1987; Raymond 2001; Sobel et al. 2008, 2010), and interactions with synoptic scales (Biello and Majda 2005; Biello et al. 2007). However, none of the existing theories can explain all the key features of the MJO. The diversity of conclusions, both from theoretical and modeling studies, suggests that the observed MJO may be controlled by a multitude of factor. Previously proposed mechanisms (each of which focuses on a specific physical process) may not be inconsistent. For example, Benedict and Randall (2007) argued that the discharge-recharge and frictional moisture convergence mechanisms together explain many features of the MJO.

In this study, we examine the MJO from the point of view of the kinetic energy budget, constructed from a reanalysis data set. The results can be used to provide some constraints on theory, though they do not clearly validate or rule out any specific theory that has been proposed. Analysis of the budget for individual events can shed light on the differences between events as well as the features common to most of them.

The Lorenz energy cycle (Lorenz 1955; Holton 2004) is a well-known quantitative framework within which to study energy and its transformation in the atmosphere. However, it was established for the mid-latitudes. Thus, some assumptions are not suitable for the tropics,
e.g., the neglect of vertical fluxes (advection terms by the vertical velocity in the momentum
equations). In addition, the MJO has been hypothesized to involve processes at multiple scales.
For example, Biello and Majda (2005) established a multi-scale model in which both the upscale
and downscale energy transfer associated with the MJO are important. We would like to
construct a set of energy equations from the primitive equations by adopting assumptions
suitable for the tropics. We will focus our discussion on the energy conversion related to the
MJO. It is impossible to construct a completely closed budget for kinetic energy from reanalysis
data sets due to the lack of some key variables (such as friction, convective heating, and other
unresolved sub-grid processes). Our discussion focuses on the dominant components in the
energy budget for which most parts are resolved; regions of significant residual are discussed
specifically.

Derivation of the energy equation is described in Section 2. Definition of composite MJO
event in this study is described in Section 3. Detailed results for every term in the energy
equation for the composite MJO event are presented in Section 4. Case studies on some special
MJO events, in which the energy balance is different from the one in the composite MJO event,
are presented in Section 5. Finally, a summary is offered in Section 6.

\section{Kinetic Energy Equation in Three Scales}

The primitive equations on the $\beta$-plane in isobaric coordinates are

\begin{align}
\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + \omega \frac{\partial}{\partial p}\right) u - \beta yv &= - \frac{\partial \Phi}{\partial x} + X \ldots \quad (1a), \\
\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + \omega \frac{\partial}{\partial p}\right) v + \beta yu &= - \frac{\partial \Phi}{\partial y} + Y \ldots \quad (1b), \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} &= 0 \ldots \quad (1c),
\end{align}
\[
\frac{\partial \Phi}{\partial p} = -\frac{RT}{p} \quad \text{(1d)},
\]

where \(u\), \(v\), \(\omega\) are zonal, meridional, and vertical velocities; \(\beta\) is the meridional gradient of the Coriolis parameter; \(\Phi\) is the geopotential; \(T\) is the temperature; while \(X\) and \(Y\) are residuals denoting all unresolved processes. We develop a spectral multi-scale framework consisting of three components: (1) the zonal mean component; (2) the MJO scale (zonal wavenumbers 1-4); and (3) “small-scale” processes which have zonal wavenumbers higher than 4. The latter may include, but are not restricted to convectively coupled equatorial waves (CCEW; Kiladis et al. 2009). This decomposition into three scales is supported by the wavenumber spectra of key dynamic and thermodynamic variables, as shown in Fig. 1a. Taking zonal winds at 850 hPa (the green line in Fig. 1a) for example, a wavenumber spectrum is made with the meridionally averaged (between 10°N and 10°S) zonal winds for every day. The result is normalized by the corresponding 95% significance level (the number of degrees of freedom is taken to be 68, estimated as \(2 \times \frac{N_{\text{record}}}{nfft}\) where \(N_{\text{record}} = 8644\) is the record length and \(nfft = 256\) is the FFT length). Finally, the normalized wavenumber spectra for the days during MJO events, which are defined based on a multi-variate MJO index (Wheeler and Hendon 2004; see Section 3 for details), are averaged; the mean spectrum is presented in Fig. 1a. Where the normalized spectrum in Fig. 1a is larger than 1, the peak is significant at the 95% significance level and is persistent with time during the MJO. For all variables, there are significant peaks around wavenumber 3, but none between wavenumber 5 and wavenumber 10. There are also some significant peaks with wavenumbers higher than 10, but they are not consistent in all variables. The purpose of showing these spectra in Fig. 1a is not to quantitatively determine all spectral peaks, but to verify that the MJO scale (wavenumbers 1-4) is distinct from variability with higher wavenumbers. The rapid drop in the zonal wind spectrum (the green line in Fig. 1a) is due to the inability of the
reanalysis products to resolve zonal winds with wavelengths smaller than 800-1000 km (Milliff et al. 2004).

Given that the spatial scales are distinct, all variables are decomposed into three components. Taking the zonal velocity \( u \) for example, the zonal mean is denoted with \( \bar{u} \). After removing the zonal mean, the zonal wind at the MJO scale (denoted \( u' \)) and the zonal wind at the small scale (denoted \( u'' \)) are reconstructed from the corresponding Fourier components. Thus, the zonal velocity can be written as \( u = \bar{u} + u' + u'' \). In Eq. (1), all terms are linear except for the advection terms, which enable interactions between different scales. By analogy to the standard decomposition into two scales (mean and perturbation), the advection terms are categorized into three scales in the following way (see Appendix A for technical details), taking the term \( v \partial u / \partial y \) for example,

(1) The equation for the zonal mean velocity includes \( \bar{v} \partial \bar{u} / \partial y \), \( \bar{v}' \partial u' / \partial y \), and \( \bar{v}'' \partial u'' / \partial y \);

(2) Advection at the MJO scale includes \( \bar{v} \partial u' / \partial y \), \( v' \partial \bar{u} / \partial y \), and \( v'' \partial u'' / \partial y \);

(3) Advection at small scales includes \( \bar{v} \partial u'' / \partial y \), \( v'' \partial \bar{u} / \partial y \), \( v' \partial u'' / \partial y \), and \( v'' \partial u' / \partial y \).

As argued in Appendix A, \( v'' \partial u'' / \partial y \) is much smaller than \( v'' \partial u'' / \partial y \), thus it is neglected in the following calculation. \( v' \partial u' / \partial y \) and \( v'' \partial u'' / \partial y \) are grouped into the scale larger than the scale of the velocities themselves, because these two terms represent the energy transfer from smaller scale to larger scale or up-scale energy cascade. Considering these approximations and some additional manipulations as shown in Appendix B, the kinetic energy equations at the MJO scale can be written as

\[
\frac{\partial KE'}{\partial t} = -\bar{u} \cdot \nabla KE' + [KE' \cdot KE''] + [KE' \cdot \bar{KE}] - \nabla (\bar{U}' \Phi') + [KE' \cdot PE'] + R \ldots (2),
\]
where $KE' = \frac{(u'^2 + v'^2)}{2}$ represents the MJO scale kinetic energy; $\nabla = \partial / \partial x \hat{i} + \partial / \partial y \hat{j} + \partial / \partial p \hat{k}$; 

$\tilde{U} = u\hat{i} + v\hat{j} + \omega\hat{k}$. The kinetic energy equations for the other two scales (the zonal mean and the small scale) can be obtained in the same way. The latter are not the focus of this study, and are not presented here. Physical interpretations of the terms in (2), which are analogous to the physical meanings of the corresponding terms in the Lorenz energy cycle (Holton 2004), are listed below.

(1) $\frac{\partial KE'}{\partial t}$ is the local tendency of $KE'$. 

(2) $-\vec{u} \cdot \nabla KE'$ is the advection of $KE'$ by zonal mean winds. 

(3) $[KE' \cdot KE'']$ denotes the kinetic energy conversion between the MJO scale ($KE'$) and the small scale ($KE''$). The explicit form is $-u' \cdot \nabla (u'' \cdot \tilde{U}'') - v' \cdot \nabla (v'' \cdot \tilde{U}'').$

(4) $[KE' \cdot \overline{KE}]$ denotes the kinetic energy conversion between the MJO scale ($KE'$) and the zonal mean scale ($\overline{KE}$). The explicit form is $-u' \cdot (\overline{U}' \cdot \nabla \overline{u}) - v' \cdot (\overline{U}' \cdot \nabla \overline{v}).$

(5) $\nabla (\overline{U}' \Phi')$ denotes the work done by the pressure gradient force (PGF). 

(6) $[KE' \cdot PE']$ denotes the energy conversion between the kinetic energy ($KE'$) and the potential energy ($PE'$) at the MJO scale. The explicit form is $-\frac{R\omega' T'}{p}$. 

(7) $R$ is the residual term with the explicit form of $u'X' + v'Y'$, which cannot be directly calculated with the resolved variables from the reanalysis products.

The total resolved energy source (denoted with $TS$ hereafter) for the local kinetic energy tendency is defined as the sum from Term (2) through Term (6), i.e.,

$$TS = -\vec{u} \cdot \nabla KE' + [KE' \cdot KE''] + [KE' \cdot \overline{KE}] - \nabla (\overline{U}' \Phi') + [KE' \cdot PE'] \cdots (3).$$

The residual terms $X$ and $Y$ are obtained from the momentum equation,
\[ X = \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + \omega \frac{\partial}{\partial p} \right) u - \beta y v + \frac{\partial \Phi}{\partial x} \cdots (4a), \]

\[ Y = \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + \omega \frac{\partial}{\partial p} \right) v + \beta y u + \frac{\partial \Phi}{\partial y} \cdots (4b). \]

Then, \( X' \) and \( Y' \) are the residual terms at the MJO scale, which have wavenumbers 1-4. All terms in the energy budget are calculated with ERA-40 (Uppala et al. 2005) and NCEP (Kalnay et al. 1996) reanalysis products. The two reanalysis yield qualitatively similar results. Most results presented in the following (Sections 4 and 5) are obtained from ERA-40, unless otherwise specified.

Similarly, one can also construct the budget for MJO-scale potential energy (see Appendix B). The major source for \( PE' \) is \( \frac{R' f' T'}{s_p} \), where \( f' \) is the external heating and \( S_p \) is the static stability parameter. However, the external heating, which is mainly due to convection, is not a standard output in reanalysis products. (In the operational analysis produced in the project of Year of Tropical Convection (YOTC), the convective heating is stored, but this product only starts from May 2008 and is too short to include enough MJO events for an analysis of the type we present here.) Therefore, the potential energy budget is not discussed in detail here.

### 3. Composite Madden-Julian Oscillation

We employ the MJO index defined by Wheeler and Hendon (2004; referred as WH04 hereafter; http://www.bom.gov.au/climate/mjo/). The amplitudes of MJO events are determined with the sum of the squares of the first two leading principal components of the combined OLR and wind fields (i.e. MJO index = \( RMM1^2 + RMM2^2 \)). A daily MJO index is used for this study. Significant MJO events are identified according to the criterion that the MJO index is larger than
two for a period longer than 30 days; the MJO index is permitted to be less than two for up to three days during the period without disqualifying the event. With this criterion, there are 35 MJO events in all from 1979 to 2009, which are listed in Table 1. Our criterion allows only strong MJO events, which thus are fewer in number than might be obtained with more standard criteria. The lifetimes of the events in Table 1 range from 30 days (only 2 events) to 85 days, with a mean of 47 days. Because we use a daily MJO index, there are occasional breaks for a few days during the strong MJO events. Therefore, the allowance of the occasional breaks for a few days prevents an MJO event from being segmented into smaller pieces. Atmospheric intraseasonal oscillations of all types were carefully categorized by (Wang and Rui 1990; referred as WR90 hereafter). The above criterion along with the WH04 MJO index only allows eastward-propagating events. For example, event No. 9 (Table 1) is the same as the one shown in WR90 (their Fig. 1). However, the independent northward-propagating events (e.g. Sep 18, 1984 – Oct 17, 1984; Fig. 6 in WR90) and the westward-propagating events (e.g. Aug 19, 1982 – Sep 12, 1982; Fig. 9 in WR90) are excluded.

Two kinds of composites are used in the following analysis. The first is the Hovmöller diagram of the meridionally and vertically averaged (between 10°N and 10°S; through the troposphere from 1000 hPa to 100 hPa) terms in the energy budget during the composite MJO event. Usually, there are two convection centers, identified by negative OLR anomalies, during an MJO event in the Indo-Pacific region: one in the central Indian Ocean and the other in the western Pacific Ocean. Two central points are chosen along the equator, one at 80°E and the other at 130°E. For each convection center, the day when the OLR anomaly reaches its minimum is chosen as Day 0. Then, 30 days before and after Day 0 are taken from all MJO events (Table 1)
to make the composite Hovmöller diagrams, separately for the two points. For the composite vertical profile, only that constructed for 80°E is shown.

4. Results for composite MJO event

4.1 Kinetic Energy at the MJO scale

The Hovmöller diagram of the composite $KE'$ is shown in Fig. 2. A broad maximum in kinetic energy propagates from the Indian Ocean to the western Pacific Ocean during the composite MJO event, as does the OLR minimum. The $KE'$ maximum lags the OLR minimum in longitude and time, and is stronger in the Pacific than Indian Ocean region in both composites. Positive $TS$ (the total resolved energy source defined in Eq. 3) occurs a few days before the enhanced $KE'$ (Fig. 3). The advection of $KE'$ by the zonal mean winds is small (not shown). The dominant energy source for $\frac{\partial KE'}{\partial t}$ in the kinetic energy budget is the energy gain from $PE'$ via the term $[KE' \cdot PE']$, which is shown in Fig. 4. Positive $[KE' \cdot PE']$ occurs to the east rather than to the west of the convection center. Since the location of the greatest energy source and that of maximum kinetic energy do not match, some process must shift the gained kinetic energy from one to the other. This is the PGF; the term $-\nabla \left( \overline{u'} \Phi' \right)$, as shown in Fig. 5 (Yanai et al. 2000). The negative work done by PGF to the east of the convection center compensates the energy source from $PE'$. Meanwhile, it transfers the kinetic energy to the west. As a result, $KE'$ is greatest to the west of the convection center (Fig. 3). Since both $[KE' \cdot PE']$ and $-\nabla \left( \overline{u'} \Phi' \right)$ are obtained from the linear terms in the momentum equations (Eq. 1), the balance between these two terms indicates that the kinetic energy associated with MJOs is mainly controlled by linear processes, at least in the coarse view offered by the vertically- and meridionally-integrated
kinetic energy budget. The compensation between $[KE' \cdot PE']$ and the work done by PGF also holds, for example, in the Gill model (Gill 1980). From numerical solutions using the code of Bretherton and Sobel (2003; the major parameters are listed in Table 2), the energy budget is shown in Fig. 6. The perturbed kinetic energy is shown in Fig. 6a with black contours. In this idealized model, since the only energy source is the heating (denoted with $M$, see Bretherton and Sobel (2003) for more details) centered on the origin of the domain, the energy source is proportional to $M \cdot \phi$ which is also pronounced around the center of the domain (black contours in Fig. 6b). The energy source is compensated by the work done by PGF (white contours and shades in Fig. 6b) and the energy is shifted mainly to the west of the energy source (a small amount of energy is also shifted to the east). The sum of $M \cdot \phi$ and $-\frac{\partial(u \cdot \phi)}{\partial x} - \frac{\partial(v \cdot \phi)}{\partial y}$ (white contours and shades in Fig. 6a) is consistent with the enhanced energy (black contours in Fig. 6a).

The similarity between the linear Gill-type model and major energy balance for the composite MJO event suggests that linear dynamics of a forced, steady response to heating is qualitatively relevant to the MJO. This same conclusion has been reached by other investigators (e.g., Hendon and Salby 1994; Sugiyama 2009) and is built into the idealized model of Sobel and Maloney (2011) as a simplifying assumption.

Both $[KE' \cdot PE']$ and the work done by PGF are energy conversions at the same scale, i.e., the MJO scale. Energy can also be exchanged between different scales. The energy conversion between $KE'$ and $\overline{KE}$ is shown in Fig. 7. Positive energy conversion indicates energy gain of $KE'$ from the zonal mean field. However, this energy exchange is one order of magnitude smaller than the two dominant terms discussed above. While this term is not important in the composite MJO event, it may be important in a few specific MJO events, which will be discussed in Section 5.2. The energy conversion between $KE'$ and $KE''$ is shown in Fig. 8.
pattern is noisy, since the scale of $KE''$ is small. But almost all values are negative indicating that the kinetic energy transfer is predominantly from the MJO scale to the small scale, rather than the converse.

Since the pattern in Fig. 8 is noisy, it is necessary to test whether the energy conversion is significantly larger than the background variation. For each grid point, we define $R = (\langle KE' \cdot KE'' \rangle_{\text{composite}} - \langle KE' \cdot KE'' \rangle_{\text{mean}}) / \langle KE' \cdot KE'' \rangle_{\text{std}}$, where $\langle KE' \cdot KE'' \rangle_{\text{composite}}$ is the mean energy conversion between $KE'$ and $KE'' (\langle KE' \cdot KE'' \rangle)$ over all MJO days, $\langle KE' \cdot KE'' \rangle_{\text{mean}}$ is the climatological mean of $\langle KE' \cdot KE'' \rangle$ over all days from 1979 to 2002 and $\langle KE' \cdot KE'' \rangle_{\text{std}}$ is the standard deviation (STD) of $\langle KE' \cdot KE'' \rangle$ over all days from 1979 to 2002. In Fig. 8, the white contours mark the regions where $R$ is smaller than -1, since most $\langle KE' \cdot KE'' \rangle$ is negative. It is clear that most regions of negative energy conversion between $KE'$ and $KE''$ during the composite MJO event are statistically significant. Therefore, based on current calculations, the small scale $KE''$ does not contribute significantly to the generation of MJO-scale kinetic energy ($KE'$) during the composite MJO event. Instead, it is a sink for the kinetic energy associated with the MJO. In some theoretical studies (Biello and Majda 2005; Biello et al. 2007) upscale momentum transfer from synoptic-scale disturbances to the MJO scale plays an important role, e.g., in generating the westerly wind burst. Our results do not necessarily rule out such theories entirely, but do indicate that such upscale transfers are not responsible for the overall generation of vertically- and meridionally-integrated kinetic energy on the MJO scale in most events, to the extent that the reanalysis can address the question. Another multiscale process of potential importance is convective momentum transport (CMT). The convective- and mesoscale motions presumably responsible for CMT are not resolved in either ERA-40 or NCEP,
due to their coarse spatial resolution. Thus, the interaction between $KE'$ and $KE''$ discussed above is not CMT. CMT is represented in this study only as part of a residual, shown below.

The vertical profile of $KE'$ at 80°E averaged between 10°N and 10°S during the composite MJO event is shown in Fig. 9. Day 0 is the day when the OLR anomalies reach their minima at 80°E. Kinetic energy has maxima at two levels. One is around 800 hPa on Day 5, due to the westerly wind burst after the convection center. The other one is around the tropopause, due to the easterly winds there. As discussed above, the major energy source for $KE'$ is conversion from $PE'$, which occurs a few days before the convection center on Day 0 (white contours in Fig. 10). This conversion is greatest between 500 hPa and 200 hPa, the levels with largest convective heating (e.g., Houze 1982; Schumacher et al. 2004). However, these levels are not the same as those with the greatest $KE'$ in Fig. 9. Thus, the energy obtained from $PE'$ needs to be transferred by PGF as shown with the color shades in Fig. 10. Around 400 hPa and a few days before the convection, one can see negative work done by PGF, which compensates the energy gained from $PE'$. PGF transfers the energy both upward to 200 hPa and downward to ~700 hPa, which are the levels of greatest $KE'$. Since the other terms in the energy budget for the composite MJO event are much smaller than the two terms shown in Fig. 10, they are not shown here. The total energy source ($TS$ defined in Eq. 3) is shown in the upper panel in Fig. 11. In most of the domain, the residuals are very small, indicating the approximate closure of resolved terms in the reanalysis products. However, there are large residuals in the upper troposphere (around 200 hPa), which are centered on a few days following the convection. These positive residuals are assumed to be compensated by the residual term $u'X' + v'Y'$ (lower panel in Fig. 11), which is mainly composed of cumulus friction (Tung and Yanai 2002; Lin et al. 2005; a.k.a. convective momentum transport). Large cumulus friction in the upper troposphere is consistent with the
results of the zonal momentum budget presented by Lin et al. (2005). Nevertheless, to the best of
our knowledge, the influence of the significant cumulus friction during the composite MJO event
has not been studied intensively thus far and it should be an interesting question for future study.

4.2 Energy budget using decomposition in time rather than space

Equation 2 for the kinetic energy budget at the MJO scale is obtained with the variables
decomposed in the zonal direction (see Appendix B for details). The variables can also be
decomposed with respect to time. We follow a three-scale framework analogous to that used for
the spatial decomposition above. The first component is the slowly-varying one, defined by the
use of a low-pass filter (Butterworth digital filter) with a cut-off period of 100 days; the second is
the intraseasonal component, with periods between 20 and 100 days; and the third one is the high
frequency component with a period shorter than 20 days. The validity of the scale separation is
tested with normalized power spectra of major dynamic and thermodynamic variables (Fig. 1b).
The zonal wind at 850 hPa (the green line in Fig. 1b), for example, is averaged between 10°N
and 10°S. The power spectrum thus calculated at each longitude is averaged between 50°E and
180°E (over the whole Indian Ocean and the western Pacific Ocean) and the averaged spectrum
is normalized by the corresponding 95% significance level (the degree of freedom is 80).
Intraseasonal peaks for various variables are clearly seen in Fig. 1b. The frequency of the
intraseasonal peak for temperature (the blue line in Fig. 1b) is higher than we might normally
consider for the MJO, but still within the intraseasonal band. The spectra with respect to time are
calculated using the reanalysis products for all days and no a priori band-pass filtering is applied;
in particular, they are not averaged only for the days during MJO events, as was done for the
spatially-filtered analysis above (i.e., Fig. 1a).
Taking $u$ for example, the variable can be written as $u = \bar{u} + u' + u''$, using the same notation for the three scales as in the spatial decomposition, though the overbar refers to low frequencies, the single prime to intraseasonal frequencies, and the double prime to high frequencies. Following a similar mathematical procedure to that in Appendix B, we obtain the kinetic energy budget equation at the intraseasonal scale,

$$\frac{\partial KE'}{\partial t} = -\bar{u} \cdot \nabla KE' - \left[ u' \cdot \nabla \left( u'' \cdot \bar{U}' \right) + v' \cdot \nabla \left( v'' \cdot \bar{U}' \right) \right] - \left[ u' \cdot (\bar{U}' \cdot \nabla \bar{u}) + v' \cdot (\bar{U}' \cdot \nabla \bar{v}) \right] - \nabla \left( \bar{U}' \Phi' \right) - \frac{R \omega' T'}{p} + (u'X' + v'Y') \cdots (5),$$

In its compact form, Eq. (5) has exactly the same form as Eq. (2). The physical interpretations of the terms in Eq. (5) are also the same as the corresponding terms in Eq. (2). However, the specific forms have differences. For example, $\frac{\partial \bar{u}}{\partial x}$ in $\nabla \bar{u}$ for the decomposition in space is zero since the zonal mean is independent of $x$, while with the time mean, this term is not zero. Nevertheless, the above conclusions drawn from the spatial decomposition remain valid for the decomposition in time. For example, the vertical profile of the energy conversion between $PE'$ and $KE'$ is shown with white contours in Fig. 12 and the vertical profile of the work done by PGF is shown with color shading in Fig. 12. The general patterns in Fig. 12 are very similar to the patterns in Fig. 10, which shows the counterpart for the decomposition in the zonal direction. Again, the dominant kinetic energy source at the MJO scale is from $PE'$, which ranges from 500 hPa to 300 hPa. The kinetic energy gained from $PE'$ is redistributed by the work done by PGF, which shifts the energy both upward and downward and also shifts the energy to a few days after Day 0 when the OLR anomalies reach their minima. Figure 13 shows the kinetic energy exchange between $KE'$ and $KE''$ (obtained with the temporal decomposition rather than spatial decomposition), which is very similar to Fig. 8. The kinetic energy is transferred one way from
the intraseasonal band to the high-frequency band to the west of the convection center during the whole lifetime of the composite MJO event. The similarity between the decomposition with two different ways demonstrates the robustness of our conclusions.

5. Case studies

As stated in the introduction, various mechanisms for the MJO have been proposed, focusing on various physical processes. In the previous section, we perform the analysis on the composite MJO event and show that the dominant terms in the kinetic energy budget are \([KE' \cdot PE']\), \(-\nabla \left( \bar{U}' \phi' \right)\), and cumulus friction. If one examines all MJO events listed in Table 1, this balance is valid for most of the individual MJO events, but the relative importance of different terms varies with specific MJO events. For some events, the dominant balance can be different from the one in the composite MJO event. The kinetic energy budget during a few events is dominated by the energy gain from the mean flow via the term \([KE' \cdot \bar{KE}]\) and a few events are significantly influenced by the sub-tropics. In Fig. 14 (with both ERA-40 and NCEP reanalysis), major terms in the energy budget (averaged horizontally within 10°N – 10°S and 50°E – 180°E, vertically from 1000 hPa to 100 hPa, and between Day -10 and Day 25 in time) are shown for the MJO events listed in Table 1. For most MJO events, the energy gain from \(PE'\) (thin black dash lines in Fig. 14) is the major energy source and it is compensated by the work done by PGF (thick black solid lines in Fig. 14), which is mainly negative. The sum of these two dominant terms (dash-dot lines in Fig. 14) is very close to the total energy source (thick black solid lines in Fig. 14), which confirms again that this balance is the dominant balance during most MJO events. Nevertheless, there are several MJO events for which this balance does not hold. For example, \(-\nabla \left( \bar{U}' \phi' \right)\) is positive during MJO events No. 14 and No. 15, while the energy gained from \(PE'\) is very small.
Another example of the exception is MJO event No. 19, in which the energy gain from mean kinetic energy via \[ K E' \cdot \overline{KE} \] (thick gray solid lines in Fig. 14) dominates. These two kinds of special cases are analyzed separately in the following two sub-sections.

5.1 MJO events with significant extra-tropical influence

The influence from extra-tropics has been examined in many previous studies, such as Hsu et al. (1990), Matthews and Kiladis (1999), and Weickmann and Berry (2009). With the data obtained from the TOGA COARE Intensive Observing Period (during Event 20 in Table 1), Yanai et al. (2000) reported strong horizontal energy convergence from the extra-tropics. Most recently, Ray and Zhang (2010) examined the extra-tropical influence in detail by picking up only one MJO event, viz. Event 27 in Table 1. As shown in Fig. 14, the dominance of kinetic energy input from the extra-tropics is not common for most MJO events. Thus, such events are somewhat special.

One can integrate the term \(-\nabla \left( \overline{U'} \phi' \right)\) over a large three dimensional domain, i.e.,

\[
\int \int \int \left( -\frac{\partial u' \phi'}{\partial x} - \frac{\partial v' \phi'}{\partial y} - \frac{\partial \omega' \phi'}{\partial p} \right) dx dy dp. 
\]

The assumption that the vertical velocities are very small at the bottom (1000 hPa) and the top of the troposphere (100 hPa) is well satisfied in the reanalysis products (not shown). Thus, the vertical integration of \(\frac{\partial \omega' \phi'}{\partial p}\) is very close to zero. Integrating along any latitude around the globe, \(\oint \frac{\partial u' \phi'}{\partial x} dx = 0\) is also well satisfied, since this integral is along a closed circle. Therefore, the integration of \(-\nabla \left( \overline{U'} \phi' \right)\) is determined by the meridional integration from the northern boundary to the southern boundary, i.e.,

\[
\int_{SB}^{NB} \frac{\partial v' \phi'}{\partial y} dy = v' \Phi'|_{SB} - v' \Phi'|_{NB}, 
\]

where \(NB\) and \(SB\) represent the northern and the southern
boundaries of the integration domain (e.g. at 10°N and 10°S). Physically, 
represents the influence of higher latitudes on the tropics. Fig. 14 shows that MJO events No. 14, 
No. 15, No 27 (which was discussed in Ray and Zhang, 2010), and a few other events have a 
significant positive energy source from the extra-tropics. Some events of this kind also have 
relatively small energy gains from PE’, e.g., MJO events No. 14 and No. 15 (Table 1).
Accordingly, the negative work done by the PGF, which is needed to compensate \([KE' \cdot PE']\), is 
also small. However, the significant influence from the extra-tropics leads to positive values of 
\(-\nabla (\bar{U}' \phi')\) and accounts for a major portion of TS (Eq. 3) during these two events. The vertical 
profiles of mean \(-\frac{\partial \psi' \phi'}{\partial y}\) averaged between 10°N and 10°S at 80°E are shown in Fig. 15. The 
profiles during the composite MJO event are shown in Fig. 15a and the averaged profiles during 
these two MJO events (No. 14 and No. 15 in Table 1) are shown in Fig. 15b. These two profiles 
share some common features, such as the negative values in the upper troposphere (\(~200\) hPa) 
and at the lower troposphere (between 800 hPa to 600 hPa), which are suggestive of wave 
radiation to higher latitudes. The distinct difference resides below 800 hPa. There are 
pronounced positive values in these two specific MJO events. However, during the composite 
MJO event, the meridional average of \(-\frac{\partial \psi' \phi'}{\partial y}\) between the northern and the southern boundaries 
is close to zero. Therefore, during these two specific MJO events, the extra-tropical influence in 
the lower troposphere contributes to the generation of KE’. On the contrary, for the composite 
MJO event, the extra-tropical influence is not significant.

5.2 MJO events in which energy source from KE dominate
As shown in Fig. 14, for a few MJO events, the energy gain from the mean kinetic energy via $[KE' \cdot \overline{KE}]$ dominates, such as No. 4 and No. 19 (comparing the thick gray solid lines and the thick black solid lines in Fig. 14). The explicit form of $[KE' \cdot \overline{KE}]$ is $-u' \cdot (\mathbf{U}' \cdot \nabla \mathbf{u}) - v' \cdot (\mathbf{V}' \cdot \nabla \mathbf{v}).$ Thus, the energy conversion between $KE'$ and $\overline{KE}$ is closely related to both horizontal and vertical shears of the background winds. Note that $\frac{\partial \mathbf{u}}{\partial x}$ is zero for the decomposition in the zonal direction, but it is not zero for the decomposition in time. The energy conversion from zonal mean wind shear is associated with barotropic instability or non-modal growth. These mechanisms are usually hypothesized to be relevant to synoptic-scale tropical disturbances (e.g., Nitta and Yanai 1969; Webster and Chang 1988; Ferreira and Schubert 1997; Sobel and Bretherton 1999; Maloney and Hartmann 2001), rather than the MJO per se. Current analysis focuses on the region from the mid-Indian Ocean to the western Pacific Ocean (from 80°E to 130°E in the Hovmöller diagram such as Fig. 7). A detailed study on the ingredients in $[KE' \cdot \overline{KE}]$ (e.g., the relative importance of $\partial \mathbf{u}/\partial x$ and $\partial \mathbf{u}/\partial y$ along the lines of the discussion in Maloney and Hartmann (2001)) will be carried out in the future. Nevertheless, the possibility that the barotropic instability is critical for the special events listed above can be tested by examining the necessary condition for the barotropic instability, i.e., $\frac{d q}{d y} = \beta - \left| \frac{\partial^2 \overline{\mathbf{u}}}{\partial y^2} \right|$ where $q$ is the quasi-geostrophic potential vorticity (QGPV), $\beta$ is the meridional gradient of the Coriolis parameter, and $\overline{\mathbf{U}}$ is the background mean flow, should change sign in the study area. Between 5°N and 10°N, the meridional shear in zonal winds is enhanced from Aug 15, 1991 to Sep 18, 1991 (Fig. 16a). The large meridional gradient of the background flow overcomes the $\beta$-effect and leads to negative $\frac{d q}{d y}$ during this MJO event (Fig. 16b). Therefore, it is possible that the necessary condition for barotropic instability is satisfied in this region. Whether any unstable waves
potentially generated may contributed on the MJO scale requires further study. Nevertheless, considering the energy conversion calculated with the energy budget equation above, we can speculate that barotropic instability may contribute to the accumulation of $KE'$ during this MJO event. For comparison, one can check another MJO event, for example, No. 9, in which the energy gain from $\bar{KE}$ is not pronounced (Fig. 14). As shown in Fig. 16c, during this MJO event, the meridional gradients of zonal winds are much smaller than they are in Fig. 16a. As a result, positive $\frac{dq}{dy}$ is found everywhere over all regions from Jan 21, 1985 to Feb 22, 1985, indicating that the meridional gradient of zonal winds is too weak to overcome $\beta$ and thus, barotropic instability cannot occur. For this class of MJO events, since the energy is mainly obtained from the mean kinetic energy, large gradients of horizontal winds are a necessary ingredient.

6. Conclusions and Discussion

In this study, the MJO is examined from the kinetic energy point of view. For kinetic energy at the MJO scale ($KE'$), the major balance is between the energy gain from the potential energy at the same scale ($PE'$) and the work done by PGF, which serves to move $KE'$ away from the region of generation to nearby regions. This balance is consistent with the classical linear Gill model. Additionally, a considerable amount of kinetic energy dissipates near the top of troposphere due to cumulus friction. Kinetic energy exchanges between different scales are not the dominant components during the composite MJO event. In most cases, the energy gain from the mean kinetic energy ($\overline{KE}$) is not significant for the composite MJO event. But it could be dominant in a few specific events possibly due to barotropic instability induced by the large zonal wind shear during the monsoon season. The kinetic energy at the small scale ($KE''$) is basically a sink of $KE'$. $KE''$ absorbs kinetic energy from $KE'$ a few days after the deep
convection during MJO events, thus it does not contribute to the accumulation of $KE'$. Note that $KE''$ is at the spatial scale of thousand kilometers, which is the scale of the planetary wave rather than the meso-scale convection. In the vertical profile, the conversion from $PE'$ to $KE'$ occurs between 500 hPa and 200 hPa, which is consistent with the release of latent heat in the upper troposphere. The obtained kinetic energy is shifted both upward to above 200 hPa and downward to the lower troposphere by PGF. Although most MJO events share a common kinetic energy balance, the relative importance of different energy components varies from one event to the next. Some MJO events can have different balances in the kinetic energy budget from the balance for the composite MJO event. A few events mainly depend on the energy source from the background winds while others are significantly influenced by the energy input from the extra-tropics. Since individual MJO events can display distinct properties, the kinetic energy budget can be a useful tool to evaluate the relative importance of various processes during a specific MJO event.

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Appendix A: Decomposition of advection terms

Take the following simple zonal momentum equation for an example,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v = -\frac{\partial \Phi}{\partial x} \cdots (A1),$$

where all notations are the same as used in Eq. (1). If all variables are decomposed into two components, i.e., the mean and the perturbation, Eq. (A1) becomes

$$\frac{\partial (\bar{u} + u')}{\partial t} + (\bar{u} + u') \frac{\partial (\bar{u} + u')}{\partial x} + (\bar{v} + v') \frac{\partial (\bar{u} + u')}{\partial y} - f (\bar{v} + v') = -\frac{\partial (\bar{u} + \Phi')}{\partial x} \cdots (A2).$$

The equation for the mean part is

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{v}' \frac{\partial \bar{u}}{\partial y} - f \bar{v} = -\frac{\partial \Phi'}{\partial x} \cdots (A3).$$

The difference between Eq. (A2) and Eq. (A3) gives the perturbation equation,

$$\frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} + u' \frac{\partial \bar{u}}{\partial x} + v \frac{\partial u'}{\partial y} + v' \frac{\partial \bar{u}}{\partial y} - f v' = -\frac{\partial \Phi'}{\partial x} \cdots (A4).$$

The products of two perturbation terms, i.e., $u' \frac{\partial u'}{\partial x}$ and $v' \frac{\partial u'}{\partial y}$ which represent the interactions between the mean and the perturbation, belong to the equation at the background scale which is larger than the perturbation scale.

For the decomposition with three scales, taking $v \partial u/\partial y$ for example, one obtains

$$\bar{v} + v' + v'' \frac{\partial (\bar{u} + u' + u'')}{\partial y} \cdots (A5).$$

Taking the zonal average, since $\bar{v}' = 0$ and $\bar{v}'' = 0$, the non-zero terms (which are also the terms belonging to the zonal mean momentum equation) are

$$\bar{v} \frac{\partial \bar{u}}{\partial y} + v' \frac{\partial \bar{u}}{\partial y} = v'' \frac{\partial \bar{u}}{\partial y} \cdots (A6).$$

Then, subtract Eq. (A6) from Eq. (A5), one has

$$\bar{v} \frac{\partial u'}{\partial y} + \bar{v} \frac{\partial u''}{\partial y} + v' \frac{\partial \bar{u}}{\partial y} + v'' \frac{\partial \bar{u}}{\partial y} + v'' \frac{\partial \bar{u}}{\partial y} + v'' \frac{\partial \bar{u}}{\partial y} \cdots (A7).$$
Note that the nonlinear term $v'' \frac{\partial u''}{\partial y}$ also has a projection at the zonal mean scale. Therefore, strictly speaking, it should be $v'' \frac{\partial u''}{\partial y} - v'' \frac{\partial u''}{\partial y}$ in Eq. (A7). However, one can check with reanalysis products that $v'' \frac{\partial u''}{\partial y}$ is much smaller than $v'' \frac{\partial u''}{\partial y}$ (not shown). Physically, it is equivalent to assuming that the energy cascade is continuous and the energy cannot transfer from one scale (e.g., the small scale with a double prime) to another scale (e.g., the zonal mean scale with a bar) which is not adjacent to the former one. Therefore, $v'' \frac{\partial u''}{\partial y}$ is neglected in Eq. (A7).

Taking the average of Eq. (A7) at the MJO scale, which is denoted as $\langle \cdots \rangle_M$, since $\langle (\cdots)'' \rangle_M = 0$, the non-zero terms, which belong to the momentum equation at the MJO scales, are

$$\left[ v \frac{\partial u'}{\partial y} \right] + \left[ v' \frac{\partial u}{\partial y} \right] + \left[ v'' \frac{\partial u''}{\partial y} \right] \cdots (A8).$$

Finally, the remaining terms, which belong to the momentum equation at the small scale, are

$$\bar{v} \frac{\partial u''}{\partial y} + v' \frac{\partial u''}{\partial y} + v'' \frac{\partial u''}{\partial y} + v'' \frac{\partial u'}{\partial y} \cdots (A9).$$

Therefore, the decomposition of the advection terms with three scales can be assumed to be (taking $v \frac{\partial u}{\partial y}$ for example):

(1) Equation at the background scale includes $\bar{v} \frac{\partial u}{\partial y}$ and $v' \frac{\partial u'}{\partial y}$;
(2) Advection at the MJO scale includes $\bar{v} \frac{\partial u'}{\partial y}$, $v' \frac{\partial u}{\partial y}$, and $v'' \frac{\partial u''}{\partial y}$;
(3) Small scale advection includes $\bar{v} \frac{\partial u''}{\partial y}$, $v'' \frac{\partial u''}{\partial y}$, $v' \frac{\partial u''}{\partial y}$, and $v'' \frac{\partial u'}{\partial y}$.

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Appendix B: Derivation of Equation 2

After decomposition, the governing equations (Eq. 1) for the MJO scale can be written as

\[
\begin{align*}
\frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} + u'' \frac{\partial u''}{\partial x} + \bar{v} \frac{\partial u'}{\partial y} + v'' \frac{\partial u''}{\partial y} + \bar{\omega} \frac{\partial u'}{\partial p} + \omega' \frac{\partial u'}{\partial p} + \omega'' \frac{\partial u''}{\partial p} - \beta y v' &= - \frac{\partial \Phi'}{\partial x} + X' \cdots (A10a), \\
\frac{\partial v'}{\partial t} + \bar{u} \frac{\partial v'}{\partial x} + u'' \frac{\partial v''}{\partial x} + \bar{v} \frac{\partial v'}{\partial y} + v'' \frac{\partial v''}{\partial y} + \bar{\omega} \frac{\partial v'}{\partial p} + \omega' \frac{\partial v'}{\partial p} + \omega'' \frac{\partial v''}{\partial p} + \beta y u' &= - \frac{\partial \Phi'}{\partial y} + Y' \cdots (A10b), \\
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial \omega'}{\partial p} &= 0 \cdots (A10c), \\
\frac{\partial \Phi'}{\partial p} &= - \frac{\rho v'}{p} \cdots (A10d).
\end{align*}
\]

Applying \( u' \times \) Eq. (A10a) + \( v' \times \) Eq. (A10b) and with the help of continuity equation, one can obtain

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + \bar{U} \cdot \nabla \right) \left( \frac{u'^2 + v'^2}{2} \right) + \left[ u' \cdot \nabla \left( u'' \cdot \bar{U}'' \right) + v' \cdot \nabla \left( v'' \cdot \bar{U}'' \right) \right] + \left[ u' \cdot \left( \bar{U}'' \cdot \nabla \bar{u} \right) + v' \cdot \left( \bar{U}'' \cdot \nabla \bar{v} \right) \right] = -u' \frac{\partial \Phi'}{\partial x} - v' \frac{\partial \Phi'}{\partial y} + u' X' + v' Y' \cdots (A11).
\end{align*}
\]

Adding the equation \(- \Phi' \frac{\partial u'}{\partial x} - \Phi' \frac{\partial v'}{\partial y} - \Phi' \frac{\partial \omega'}{\partial p} = 0\) to Eq. (A11) and using Eq. (A10d), one can obtain Eq. (2) in the main text, i.e., the kinetic energy equation at the MJO scale.
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Captions

Table 1 MJO events defined based on the MJO index of Wheeler and Hendon (2004) and the selection criteria in the main text.

Table 2 Parameters for the numerical solutions to the Gill-type model in a rectangular domain, using the code of Bretherton and Sobel (2003). The Rayleigh damping rate and the Newtonian cooling rate are normalized by $\sqrt{\beta c}$, where $c = 50$ m s$^{-1}$. The heating has the form $M = \cos\left(\frac{x-x_0}{2L_x}\pi\right) \cdot \exp\left[-\frac{(y-y_0)^2}{2L_y^2}\right]$, where $x_0 = 0$, $y_0 = 0$. When $x - x_0 > L_x$ or $x - x_0 < -L_x$, $M = 0$. The length scale for $X$ domain, $Y$ domain, $L_x$, and $L_y$ is $\sqrt{\beta c}$.

Figure 1 (a) Wavenumber spectra of zonal winds at 850 hPa, air temperatures at 850 hPa, geopotential heights at 850 hPa, and total column water mass. All variables are averaged between 10$^\circ$N and 10$^\circ$S before the spectra are calculated. All spectra are normalized by the corresponding 95% confidence level. (b) Power spectra of the above variables, which are also normalized by the corresponding 95% confidence level.

Figure 2 Hovmöller diagram of $KE'$ (averaged between 10$^\circ$N and 10$^\circ$S, from 1000 hPa to 100 hPa) during the composite MJO event. In the left panel, Day 0 is that of minimum OLR anomaly at 80$^\circ$E; while in the right panel, Day 0 is that of minimum OLR anomaly at 130$^\circ$E. See the text for further details. The thin contours are from 10 J kg$^{-1}$ to 60 J kg$^{-1}$ with an interval of 5 J kg$^{-1}$. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.

Figure 3 Same as Fig. 2 but for the total energy source ($TS$ in Eq. 3). The interval for the thin contours is 3 J day$^{-1}$ kg$^{-1}$. The gray contours represent zero. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.

Figure 4 Same as Fig. 2 but for the energy conversion between $KE'$ and $PE'$ via $[KE' \cdot PE']$. The interval for the thin contours is 5 J day$^{-1}$ kg$^{-1}$. The gray contours correspond to zero. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.
Figure 5 Same as Fig. 2 but for the term $-\nabla \left( \overline{U^T \Phi'} \right)$. The interval for thin contours is 5 J day$^{-1}$ kg$^{-1}$. The gray lines are zero contours. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.

Figure 6 Energy budget in the Gill-type model of Bretherton and Sobel (2003). (a) Kinetic energy (the black contours) and $M \cdot \phi - \frac{\partial (u \phi)}{\partial x} - \frac{\partial (v \phi)}{\partial y}$ (white contours and gray shades), where $M$ is the heating and $\phi$ is the geopotential. (b) $M \cdot \phi$ (the black contours) which is the energy source for the potential energy and $- \frac{\partial (u \phi)}{\partial x} - \frac{\partial (v \phi)}{\partial y}$ (white contours and gray shades) which redistributes the energy obtained from potential energy.

Figure 7 Same as Fig. 2 but for $KE' \cdot \overline{KE'}$. The interval for the thin contours is 0.4 J day$^{-1}$ kg$^{-1}$. Zero is shown with gray contours. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.

Figure 8 Same as Fig. 2 but for $[KE' \cdot KE'']$. Negative values means energy is transferred from $KE'$ to $KE''$. Thick gray contours correspond to zero. The interval of thin black contours is $1 \times 10^{-5}$ W kg$^{-1}$. White contours mark the region where the energy conversion is significant ($R < -1$; see the text for the definition of $R$). The thick black contours show negative OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.

Figure 9 Vertical profile of composite $KE'$ at 80°E averaged between 10°N and 10°S. Day 0 is the day when the OLR anomalies reach the minimum at 80°E. The unit is J kg$^{-1}$.

Figure 10 Same as Fig. 9, but for $[KE' \cdot PE']$ with white contours and for $-\nabla \left( \overline{U^T \Phi'} \right)$ with color codes. The unit is J day$^{-1}$ kg$^{-1}$.

Figure 11 Same as Fig. 9, but for $TS$ (Eq. 3) in the upper panel and for $u'X' + v'Y'$ in the lower panel. The unit is J day$^{-1}$ kg$^{-1}$.

Figure 12 Same as Fig. 10, but for the decomposition with respect to time (Eq. 5). $[KE' \cdot PE']$ with white contours and for $-\nabla \left( \overline{U^T \Phi'} \right)$ with color codes. The unit is J day$^{-1}$ kg$^{-1}$.
Figure 13 Same as Fig. 8, but for $[KE' \cdot KE'']$ with the decomposition in time. The interval of thin black contours is $1 \times 10^{-5}$ W kg$^{-1}$.

Figure 14 (a) Major terms in the kinetic energy budget calculated with ERA-40. All terms are averaged horizontally within 10°N – 10°S and 50°E – 180°E, vertically from 1000 hPa and 100 hPa, and between Day -10 to Day 25 in time. Budget denotes $TS$ (Eq. 3), KpKm denotes $[KE' \cdot \bar{KE}]$, KpKpp denotes $[KE' \cdot KE'']$, KpPp denotes $[KE' \cdot PE']$, and Pres denotes $-\nabla \left( \frac{\partial U' \phi'}{\partial y} \right)$. (b) the same as (a) but it is obtained with the NCEP reanalysis.

Figure 15 Same as Fig. 9, but for $-\frac{\partial U' \phi'}{\partial y}$ averaged between 10°N and 10°S. The upper panel is obtained for the composite MJO event. The lower panel is the average of the MJO events No. 14 and No. 15 in Table 1. The unit is J day$^{-1}$ kg$^{-1}$.

Figure 16 (a) and (c): $|\bar{U}_{yy}|$ at 850 hPa during two MJO events. (b) and (d): $\beta - |\bar{U}_{yy}|$ during these two MJO events. The unit is $10^{-11}$ m$^{-1}$ s$^{-1}$. Positive values in (b) are shaded. All values in (d) are positive.
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**Table 1** MJO events defined based on the MJO index of Wheeler and Hendon (2004) and the selection criteria in the main text.
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</tbody>
</table>

Table 2 Parameters for the numerical solutions to the Gill-type model in a rectangular domain, using the code of Bretherton and Sobel (2003). The Rayleigh damping rate and the Newtonian cooling rate are normalized by $\sqrt{\beta c}$, where $c = 50$ m s$^{-1}$. The heating has the form $M = \cos \left( \frac{x-x_0}{2L_x} \pi \right) \cdot \exp \left[ -\frac{(y-y_0)^2}{2L_y^2} \right]$, where $x_0 = 0$, $y_0 = 0$. When $x - x_0 > L_x$ or $x - x_0 < -L_x$, $M = 0$. The length scale for X domain, Y domain, $L_x$, and $L_y$ is $\sqrt{c/\beta}$. 


Figure 1 (a) Wavenumber spectra of zonal winds at 850 hPa, air temperatures at 850 hPa, geopotential heights at 850 hPa, and total column water mass. All variables are averaged between 10°N and 10°S before the spectra are calculated. All spectra are normalized by the corresponding 95% confidence level. (b) Power spectra of the above variables, which are also normalized by the corresponding 95% confidence level.
Figure 2 Hovmöller diagram of $KE'$ (averaged between 10°N and 10°S, from 1000 hPa to 100 hPa) during the composite MJO event. In the left panel, Day 0 is that of minimum OLR anomaly at 80°E; while in the right panel, Day 0 is that of minimum OLR anomaly at 130°E. See the text for further details. The thin contours are from 10 J kg$^{-1}$ to 60 J kg$^{-1}$ with an interval of 5 J kg$^{-1}$. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$. 

Figure 3 Same as Fig. 2 but for the total energy source (TS in Eq. 3). The interval for the thin contours is 3 J day$^{-1}$ kg$^{-1}$. The gray contours represent zero. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$. 
Figure 4 Same as Fig. 2 but for the energy conversion between $KE'$ and $PE'$ via $[KE' \cdot PE']$. The interval for the thin contours is 5 J day$^{-1}$ kg$^{-1}$. The gray contours correspond to zero. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$. 
Figure 5 Same as Fig. 2 but for the term $-\nabla \left( \overline{U'} \Phi' \right)$. The interval for thin contours is 5 J day$^{-1}$ kg$^{-1}$. The gray lines are zero contours. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$. 
Figure 6 Energy budget in the Gill-type model of Bretherton and Sobel (2003). (a) Kinetic energy (the black contours) and $M \cdot \phi - \frac{\partial (u \phi)}{\partial x} - \frac{\partial (v \phi)}{\partial y}$ (white contours and gray shades), where $M$ is the heating and $\phi$ is the geopotential. (b) $M \cdot \phi$ (the black contours) which is the energy source for the potential energy and $- \frac{\partial (u \phi)}{\partial x} - \frac{\partial (v \phi)}{\partial y}$ (white contours and gray shades) which redistributes the energy obtained from potential energy.
Figure 7 Same as Fig. 2 but for $KE' \cdot \overline{KE}$. The interval for the thin contours is 0.4 J day$^{-1}$ kg$^{-1}$. Zero is shown with gray contours. The thick black contours show negative intraseasonal OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$.
Figure 8 Same as Fig. 2 but for $[KE' \cdot KE'']$. Negative values means energy is transferred from $KE'$ to $KE''$. Thick gray contours correspond to zero. The interval of thin black contours is $1 \times 10^{-5}$ W kg$^{-1}$. White contours mark the region where the energy conversion is significant ($R < -1$; see the text for the definition of $R$). The thick black contours show negative OLR anomalies, starting from -10 W m$^{-2}$ with an interval of 10 W m$^{-2}$. 
Figure 9  Vertical profile of composite $KE'$ at 80°E averaged between 10°N and 10°S. Day 0 is the day when the OLR anomalies reach the minimum at 80°E. The unit is J kg$^{-1}$. 
Figure 10 Same as Fig. 9, but for \([KE' \cdot PE']\) with white contours and for \(-\mathbf{\nabla} \left( \mathbf{U}' \Phi' \right)\) with color codes. The unit is J day\(^{-1}\) kg\(^{-1}\).
Figure 11 Same as Fig. 9, but for $TS$ (Eq. 3) in the upper panel and for $u'X' + v'Y'$ in the lower panel. The unit is J day$^{-1}$ kg$^{-1}$. 
Figure 12 Same as Fig. 10, but for the decomposition with respect to time (Eq. 5). $[KE' \cdot PE']$ with white contours and for $-\nabla \left( \bar{U}' \Phi' \right)$ with color codes. The unit is J day$^{-1}$ kg$^{-1}$. 
Figure 13 Same as Fig. 8, but for $[KE' \cdot KE'']$ with the decomposition in time. The interval of thin black contours is $1 \times 10^{-5}$ W kg$^{-1}$. 
Figure 14 (a) Major terms in the kinetic energy budget calculated with ERA-40. All terms are averaged horizontally within 10°N – 10°S and 50°E – 180°E, vertically from 1000 hPa and 100 hPa, and between Day -10 to Day 25 in time. Budget denotes $TS$ (Eq. 3), KpKm denotes $[KE' \cdot KE]$, KpKpp denotes $[KE' \cdot KE'']$, KpPp denotes $[KE' \cdot PE']$, and Pres denotes $-\nabla \left( \bar{U}' \Phi \right)$. (b) the same as (a) but it is obtained with the NCEP reanalysis.
Figure 15  Same as Fig. 9, but for $-\frac{\partial u' v'}{\partial y}$ averaged between $10^\circ$N and $10^\circ$S. The upper panel is obtained for the composite MJO event. The lower panel is the average of the MJO events No. 14 and No. 15 in Table 1. The unit is J day$^{-1}$ kg$^{-1}$. 
Figure 16 (a) and (c): $|\bar{U}_{yy}|$ at 850 hPa during two MJO events. (b) and (d): $\beta - |\bar{U}_{yy}|$ during these two MJO events. The unit is $10^{-11} \text{ m}^2 \text{ s}^{-1}$. Positive values in (b) are shaded. All values in (d) are positive.