Department of Applied Physics and Applied Mathematics
Columbia University

APPH E4210. Geophysical Fluid Dynamics

Spring 2005

Problem Set 4

(Due Feb 28, 2008)

1. *Waves on a β-plane*. Consider the shallow water equations on a β-plane. In class we showed that the dispersion relation for this system is a cubic equation in ω. A graphical analysis revealed that two of the roots correspond to the gravity wave solution, namely Poincare waves, but slightly modified by the presence of β. The third root, corresponding to low-frequency motions, is the Rossby wave.

   (a) For the gravity wave solutions, quantify the deviation of the dispersion relation from that with $\beta = 0$ by deriving the lowest order *correction* to the dispersion relation. (One way to approach this is by Taylor-expanding about the $\beta = 0$ solution.) Give a numerical estimate of this correction for a mid-latitude (45°N) baroclinic plane wave of zonal wavenumber $k\lambda_d = 3$, and meridional wavenumber $l\lambda_d = 0$. Assume a first baroclinic phase speed of 3 m s^{-1}.

   (b) For the Rossby wave solution, derive expressions for the group velocity and zonal phase speed (ω/k). For $l = 0$, make a plot of ω as a function of k. (Nondimensionalize axes in a sensible manner.) Indicate on the figure the direction in which phase and energy propagate. Assuming a mean stratification of $N = 2 \times 10^{-3}$ and a depth of 4300 m, provide numerical estimates of the zonal group and phase velocity for mid-latitude (45°) first baroclinic mode Rossby waves in the ocean.

2. *Doppler shifted dispersion relation*. In the presence of a mean zonal current (or wind) U, the governing equation (linearized about the background flow) for low frequency motions is given by:
$$(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x})(\nabla^2 - \frac{1}{\lambda_d^2})\eta + \beta\frac{\partial \eta}{\partial x} = 0.$$
   (This is the equation one gets if the momentum equations are linearized, not about a state of rest as we have done until now, but about a zonal background flow.)

   (a) Derive and physically interpret the dispersion relation for small amplitude waves governed by this equation.

(b) The figure below shows a longitude-time plot of 500 mbar geopotential height anomalies averaged over 45-50°N. The data are for a 25 day period beginning on January 1, 1997. The mean zonal wind at this latitude and for that period was roughly 20 m s^{-2}. Suppose we were to interpret the pattern of height anomalies as that associated with barotropic Rossby waves. Based on the results of problem 1 (b) above, what is the most striking feature of this plot? Use part (a) above to interpret these observations. A qualitative explanation will do, but feel free to plug in some numerical values to see if your qualitative argument make quantitative sense. (You can make your own plots and animations here: `http://www.cdc.noaa.gov/map`.)