**UserCalc: a web-based Uranium Series Calculator for magma migration problems**

M. Spiegelman

Lamont-Doherty Earth Observatory of Columbia University, Palisades, New York; mspieg@ldeo.columbia.edu

**Abstract.** Measured departures from secular equilibrium in the Uranium-series decay chains provide important constraints on the rates of recent mantle and crustal processes. Nevertheless, the inferences drawn from these observations depend upon the models used to interpret them. While several useful models for U-series exist, they are not all equally easy to use, and it is often the ease of use, rather than the utility of the model that determines which models are applied. The purpose of this paper is to level the playing field by making some of the more general U-series transport models accessible to a wider community through a web-site at www.ldeo.columbia.edu/~mspieg/UserCalc/. These models calculate the effects of both melting and melt transport on two U-series chains ($^{238}U \rightarrow ^{230}Th \rightarrow ^{226}Ra$ and $^{235}U \rightarrow ^{231}Pa$) and allow the user to input their own petrological insight and data to explore the behaviour of this model. The purpose of the site is to provide a tool that can spur additional collaborations between geophysicists and geochemists as well as drive more rapid exploration/evolution of U-series models.

**Introduction**

Radioactive decay chains such as the two Uranium series $^{238}U \rightarrow ^{230}Th \rightarrow ^{226}Ra$ and $^{235}U \rightarrow ^{231}Pa$ are unique among commonly measured geochemical tracers in their ability to place constraints on the rates of recent mantle processes. Because the longer lived daughter nuclides in this series have half-lives ranging from 1600–245,500 years, they are particularly sensitive to processes such as the rate of mantle melting or the flow rates of convection or melt transport. In conjunction with stable trace elements, major elements and long-lived isotopic systems (e.g. Sm/Nd, Rb/Sr…), the U-series nuclides provide important complimentary information for inferring the properties and processes occurring in the Earth’s mantle. Nevertheless, like most proxy measurements, the actual inferences drawn from the observations depend significantly on the models used to interpret them.

For the U-series, there are several standard models available for modeling the observed departures from secular equilibrium due to mantle melting. The simplest models are batch melting models [e.g. Allègre and Condomines, 1982; Sims et al., 1995] which assume that the maximum variations from secular equilibrium are due solely to chemical fractionation. These models may work for very small degrees of melting (e.g. beneath ocean islands [Sims et al., 1995, 1999]) but are problematic for degrees of melting $F$ much larger than the partition coefficients of the U-series nuclides ($D \approx 0.001$) because solutions with $F \gg D$ produce negligible chemical fractionation. More complex “dynamic melting” ingrowth models [e.g. McKenzie, 1985; Williams and Gill, 1989; Qin, 1992; Richardson and McKenzie, 1994; Iwamori, 1994] assume slow, near-fractional melting and instantaneous melt extraction. These models produce excess daughter products by increasing the residence time in the solid of the parent relative to the daughter during melting. These models place constraints on melt generation rates and the volume of equilibration, but not on the actual melt transport process because they do not include the mechanics of melt transport. The most general U-series models to date [Spiegelman and Elliott, 1993] [see also Lundstrom et al., 1995; Iwamori, 1994], however, include the coupled processes of adiabatic melting and melt extraction and calculate the consequences of both processes for U-series excesses.

All of these models are useful; however, they are not all equally easy to use. Both batch melting and the simplest dynamic melting models have analytic solutions and are easily calculated on a spreadsheet. The transport models, however,
require accurate numerical integration of ordinary differential equations (ODE’s). Good ODE integrators can be found in many packages but require a bit more work to implement. Unfortunately, the extra effort required to implement these models is usually sufficient to deter all but the most eager investigators with the result that potentially useful models don’t get used. This is a disservice to users of models because it limits the tools that are available to them, and it is also a disservice to the modeler because it limits the population of people who can test and evaluate the worth of the models. Fortunately, given modern distributed computing, it is now relatively straightforward for modelers to make their models accessible and available to the entire community. The purpose of this paper and web-site (www.ldeo.columbia.edu/~mspieg/UserCalc) is to implement this approach to community models in solid earth science by making available an extended model for U-series in mantle melting based on the models of Spiegelman and Elliott [1993]. The hope is that improved access will increase the body of users of the model, begin a better dialog between producers and users of models, and lead to a better integration of theory and observation. If this exercise proves useful, additional models can readily be added to the site. The remainder of this paper will describe the underlying mechanics of the model and its implementation through the UserCalc web-site.

The Model

The basic model is an extension of the equilibrium transport model of Spiegelman and Elliott [1993] and the derivation follows from that paper. As implemented here, the model calculates the concentrations and activities of the two U-series chains $^{238}$U → $^{234}$Th → $^{228}$Ra and $^{235}$U → $^{231}$Pa for a one dimensional upwelling column undergoing melting by adiabatic decompression. Melt extraction is governed by a simplified form of Darcy’s law and the melt is assumed to behave as a one dimensional upwelling column undergoing melting by adiabatic decompression. Melt extraction is governed by a simplified form of Darcy’s law and the melt is assumed to

$$\frac{\partial}{\partial z} \rho_f \phi w = \Gamma$$

$$\frac{\partial}{\partial z} \rho_s (1 - \phi) W = -\Gamma$$

$$\frac{\partial}{\partial z} \left[ (\rho_f \phi w + \rho_s (1 - \phi) W) D_i c_i^f \right] = \lambda_{i-1} \rho D_i c_{i-1}^f - \lambda_i \rho D_i c_i^f$$

where $\rho_f, \rho_s$ are melt and solid densities, $w, W$ are melt and solid velocities, $\phi$ is the porosity and $\Gamma$ is the “melting rate” or rate of mass transfer from solid to liquid. In Eq. (3), $D_i$ is the local solid concentration is therefore $c_i^s = D_i c_i^f$. $\lambda_i$ is the decay constant for element $i$ and

$$\overline{\rho D_i} = \rho_f \phi + \rho_s (1 - \phi) D_i$$

such that $\overline{\rho D_i} c_i^f$ is the total mass per unit volume of component $i$ in the melt plus solid. All variables subscripted $i - 1$ imply the parent element up the chain. For elements with no parents (i.e. $^{238}$U, $^{235}$U), $\lambda_{i-1} = 0$.

For this model, the melting rate $\Gamma$ is chosen for a 1-D adiabatic melting column such that

$$\Gamma = \rho_s W_0 \frac{\partial F}{\partial z}$$

where $W_0$ is the solid upwelling velocity at the base of the melting column ($z = 0$), and $F$ is the degree of melting and is a function of height in the melting column.$^1$

Equations (1)–(2) and (5) govern the total conservation of mass for melt and solid phases while Eq. (3) governs conservation of mass for each component $i$ in the bulk equilibrium system of melt plus solid. Equation (3) allows for both radioactive production and decay. Given just conservation of mass and composition, Equations (1)–(5) can be rewritten into a system of ordinary differential equations for the concentration of each nuclide in the melt. The basic derivation is given in Spiegelman and Elliott [1993] with the result

$$\frac{dc_i^f}{dz} = \frac{-c_i^f}{F + (1 - F) D_i} \frac{d}{dz} \left[ F + (1 - F) D_i \right] + \frac{\lambda_i \rho D_i c_i^f}{\rho_s W_0 (F + (1 - F) D_i)}$$

where now both $D_i$ and $F$ are allowed to be variable functions of height $z$. Equation (6) governs changes in concentration of element $i$ in the melt with height and has two terms. The first term governs changes in concentration due to melting and changes in partition coefficient. This term is identical to that expected for true batch melting and is independent of porosity and upwelling rate [see Ribe, 1985; Spiegelman and Elliott, 1993]. The second term, however, is the “ingrowth” term which controls changes in concentration due to the balance of radioactive production and decay.

$^1$In their simplest models Spiegelman and Elliott [1993] assumed that $F$ was a linear function of height ($F = F_{max} z / d$), however, the appendix of Spiegelman and Elliott [1993] provides the derivation for general $F(z)$. 

\[ \]
This term is sensitive to both $\phi$ and $W_0$ as these affect the relative residence times of parent and daughter elements in the melting column.

Equation (6) is actually solved in a more numerically tractable form by defining

$$U_i^f = \ln \frac{c_i^f}{c_i^0}$$

(7)

where $c_i^f = c_i^{i_0}/D_i^0$ is the concentration of the first instantaneous melt at the base of the melting column. Here $c_i^{i_0}$ is the concentration of element $i$ in the unmelted solid and $D_i^0$ is the partition coefficient at $z = 0$. Noting that

$$\frac{dU_i^f}{dz} = \frac{1}{c_i^f} \frac{dc_i^f}{dz}$$

(8)

Equation (6) can be divided by $c_i^f$ and rewritten as

$$\frac{dU_i^f}{dz} = \frac{-1}{F + (1 - F) D_i} \frac{d}{dz} \left[ F + (1 - F) D_i \right] + \frac{\lambda_i}{w_{eff}^i} \left[ R_i^{-1} \exp[U_i^f - U_i^f] - 1 \right]$$

(9)

where

$$w_{eff}^i = \frac{\rho_f \phi w + \rho_s (1 - \phi) D_i W}{\rho_f \phi + \rho_s (1 - \phi) D_i}$$

(10)

is the “effective velocity” of element $i$ which varies from $w$ when $D_i = 0$ to $W$ when $D_i \to \infty$. The effective velocity can also be rewritten using Eqs. (1), (2) and (5) as

$$w_{eff}^i = \frac{\rho_s W_0 [F + (1 - F) D_i]}{\rho_f \phi + \rho_s (1 - \phi) D_i}$$

(11)

Finally,

$$R_i^{-1} = \alpha_i \frac{D_i^0}{D_i^{(i-1)}} \frac{\rho D_i^{-1}}{\rho D_i^{(i-1)}}$$

(12)

is the ingrowth factor for elements $i - 1$ and $i$. Here

$$\alpha_i = \frac{(\lambda_i - 1) c_i^{i_0}}{\lambda_i c_i^{i_0}}$$

is the activity ratio of parent to daughter elements in the initial unmelted solid. Spiegelman and Elliott [1993] assumed that all elements were in secular equilibrium before melting (i.e. $\alpha_i = 1$ for all elements). The latest version of UserCalc, however, allows for more general initial conditions. This has been prompted by work in subduction zones where it is likely that large influxes of Uranium have been added from the slab before significant wedge melting begins [e.g. Newman et al., 1984; Gill and Williams, 1990; McDermott and Hawkesworth, 1991; Elliott et al., 1997].

Given $F(z)$ and $D_i(z)$ as input, we only need to specify the porosity $\phi(z)$ to close Eq. (9). For a 1-D steady-state column, Eqs. (1) and (5) can be integrated with boundary conditions $\phi(0) = F(0) = 0$ to show that

$$\frac{w}{W_0} = \frac{\rho_s F}{\rho_f \phi}$$

(13)

so that the melt velocity relative to the solid upwelling velocity is controlled by the ratio of degree of melting to the porosity. Thus any $\phi(z) \leq \rho_s F(z)/\rho_f$ is a valid solution and could be imposed entirely in an ad hoc manner. However, it is somewhat more physical to have the melt flow and porosity be governed by a simplified version of Darcy’s law for a permeable medium

$$\phi(w - W) = k_0 \Delta \rho g (1 - \phi)$$

(14)

where

$$k_0 = k(z) \phi^n$$

(15)

is the permeability which is assumed to be a power-law in porosity with a spatially varying pre-factor $k(z)$; $\Delta \rho g$ is the buoyancy difference between solid and melt. Integrating Eqs. (1), (2) and (5) with boundary conditions $\phi(0) = F(0) = 0$, and $W(0) = W_0$ and substituting into Eq. (14) yields an implicit polynomial relationship for $\phi(z)$.

$$\left[ \frac{\rho_s F}{\rho_f (1 - \phi)} \right] = A(z) \phi^n (1 - \phi)$$

(16)

where

$$A(z) = k(z) \frac{\Delta \rho g}{W_0}$$

(17)

If $k(z)$ were known, Eq. (16) could be solved (numerically) for $\phi$ immediately, however, as permeability is usually not well known, it is more convenient to calibrate $A$ by setting the porosity at the top of the column ($z = d$) to some reference porosity $\phi_0$ which fixes $A$ at $z = d$ to

$$A_d = \left[ \frac{\rho_s F_{max}}{\rho_f} - \frac{\phi_0 (1 - F_{max})}{(1 - \phi_0)} \right] / \phi_0^n (1 - \phi_0)$$

(18)

Where $F_{max}$ is the porosity at the top of the melting column. We can then define the “relative permeability function” as

$$k_r(z) = k(z) / k(d)$$

(19)

which gives the spatially varying component of permeability relative to its value at $z = d$. With these definitions, the polynomial for $\phi(z)$ that is zero at $z = 0$, $\phi_0$ at $z = d$ and satisfies both mass conservation and Darcy’s law is

$$k_r(z) A_d \phi^n (1 - \phi)^2 + \phi \left[ 1 + F \left( \frac{\rho_s}{\rho_f} - 1 \right) \right] - \frac{\rho_s}{\rho_f} F = 0$$

(20)

2 $\Delta \rho = \rho_s - \rho_f$ and $g$ is the acceleration due to gravity

3 All the solutions in Spiegelman and Elliott [1993] assume that $k_r = 1$ everywhere.
For highly permeable systems ($A_d$ large and $\phi \ll F \ll 1$) the porosity is approximately

$$\phi(z) \approx \frac{\rho_s F}{\rho_f k_r A_d}^{1/n} \quad (21)$$

Given $\phi_0$, $W_0$, $F(z)$ and $k_r(z)$ the porosity $\phi(z)$ is determined everywhere in the melting column.

**Implementation**

Equations (9) and (20) form a closed system of ordinary differential equations that can be solved for any number of decay chains within the same physical system. These equations are solved numerically using a 5th order Runge-Kutta Cash-Carp ODE solver with adaptive stepping. A Brent algorithm is used to find the roots of Eq. (20) [for both see Press et al., 1992]. Initial conditions are $U_j^f = 0$ for all elements and there is a single input tolerance (default $10^{-6}$) which controls the relative accuracy of the ODE solvers. These routines are implemented in an optimized fortran program. The UserCalc web-site forms a wrapper for controlling the input of this program as well as providing graphical and textual output.

As implemented, UserCalc solves for the concentrations and activity ratios of the two decay chains $^{238}\text{U} \to ^{230}\text{Th} \to ^{226}\text{Ra}$ and $^{235}\text{U} \to ^{231}\text{Pa}$ given $W_0$ (in cm/yr), $\phi_0$, permeability exponent $n$ and initial relative activities $a_j$ for each element. In addition, the model requires input for the continuous functions $F(z)$, $k_r(z)$, and $D_i(z)$ which are input in tabular form as a spreadsheet and then interpolated using cubic splines. Since it is more common in petrology to give melting functions in terms of pressure, the actual spreadsheet is a seven column, tab (or space) separated spreadsheet with columns

<table>
<thead>
<tr>
<th>$P$ (kb)</th>
<th>$F(P)$</th>
<th>$k_r(P)$</th>
<th>$D_U$</th>
<th>$D_{Th}$</th>
<th>$D_{Ra}$</th>
<th>$D_{Pa}$</th>
</tr>
</thead>
</table>

where the pressure $P$ is converted to depth by inputting a single constant pressure gradient $\partial P/\partial z$ in kb/km.

This spreadsheet can be calculated in any manner the user chooses and simply passed to the web-site. For convenience, however, UserCalc also provides a Spreadsheet Calculator which provides spreadsheets in the proper format for a simple 2-layer model (see Figure 1). In this model (which might be appropriate for melting columns that span the garnet-spinel transition) the lower layer extends from $P_{\text{max}}$ to $P_1$ and $F(P)$ increases linearly from zero to $F_1$. The upper layer extends from pressure $P_1$ to $P_{\text{min}}$ and $F(P)$ increases linearly from $F_1$ to $F_{\text{max}}$. Each layer can have a different partition coefficient for each element and the lower layer can have a different intrinsic relative permeability $k_r$ than the upper layer (which has $k_r = 1$ by definition).

With these inputs, the web-site implements the intrinsic model in two formats: a single 1-D column model showing the changes with height and as contour plots showing the pattern of activity ratios at the top of a set of columns that use the same spreadsheet but span a two-dimensional parameter space in upwelling rate $W_0$ and maximum porosity $\phi_0$. The best approach is probably just to go to the web-site and start playing (all forms have reasonable default values to start). If you prefer to read the manual however, each module is described here in more detail.

**Column models**

The direct output of the column model is a comma-separated spreadsheet with columns | Pressure(kb) | depth(km) | $F$ | $\phi$ | $^{(230)}\text{Th}/^{(238)}\text{U}$ | $^{(226)}\text{Ra}/^{(230)}\text{Th}$ | $^{(231)}\text{Pa}/^{(235)}\text{U}$ | $^{(235)}\text{U}$ | $^{(230)}\text{Th}$ | $^{(238)}\text{U}$ | $^{(230)}\text{Ra}$ | $^{(235)}\text{Pa}$ | $D_{U}$ | $D_{Th}$ | $D_{Ra}$ | $D_{Pa}$ | $z'$ where $z'$ is the dimensionless height of the column ($0 \leq z' \leq 1$). The output form also provides tables showing activity ratios at the top of the column, the ratio of melt velocity to solid velocity ($w_0/W_0$) at the top of the column and some other information about the run (there is also a glossary of terms on the site to explain specific items).

In addition to numeric output, however, the site also provides graphics. Figure 2 shows some of the graphical output of the 1-D column models which compares the vertical structure of the porosity $\phi$ to the degree of melting $F$, the partition coefficient structure with height and the activity ratios for the three daughter/parent pairs ($^{230}\text{Th}/^{238}\text{U}$), ($^{226}\text{Ra}/^{230}\text{Th}$), and ($^{231}\text{Pa}/^{235}\text{U}$). These figures are supplied as gif images and as either encapsulated postscript files (EPS) or PDF files depending on user choice.

**Contour plots**

The 1-D column models are useful for understanding the effects of the input parameters on the vertical structure of concentration and porosity within the melting column. However, it can be tedious to use this version of the model to explore parameter space. For this reason, the second implementation of the model is in the form of contour plots which allow exploration over a 2-D parameter space of porosity and upwelling velocity. The spreadsheet and other input variables are identical to the 1-D column models, this module...
simply runs a series of 1-D columns over a 2-D parameter space of \( W_{\text{min}} \leq W_0 \leq W_{\text{max}} \) and \( \phi_{\text{min}} \leq \phi_0 \leq \phi_{\text{max}} \). The parameter space is sampled logarithmically at \( m \times n \) points (where \( m \) and \( n \) can be chosen each to vary from 3–30 points in each dimension). The output in this models is a 2-D grid of concentrations and activity ratios at the top of the columns. Numerical output can be requested in ASCII, GMT (.grd) or matlab (.mat) formats and will be returned as a compressed tar file. Graphical output is contour plots of activity ratios as a function of porosity and upwelling rate for the three daughter/parent pairs (\(^{230}\text{Th}/^{238}\text{U}\)), (\(^{226}\text{Ra}/^{230}\text{Th}\)), (\(^{231}\text{Pa}/^{235}\text{U}\)) and are provided as gif files and EPS or PDF files. Figure 3 shows some of the contour plots. Given a solution, additional reploting of contours and annotation is available.

In addition to simply contouring the output of the model, this module also includes the ability to overlay target data onto the solutions to compare data to models. This target data could be actual measurements from single analyses or composite ranges of activity ratios for a region. The site doesn’t know or care, you just put in three target values for (\(^{230}\text{Th}/^{238}\text{U}\)), (\(^{226}\text{Ra}/^{230}\text{Th}\)), (\(^{231}\text{Pa}/^{235}\text{U}\)) and an optional set of range parameters \( \sigma_i \). If this feature is selected, three extra contours (\( \alpha_i - \sigma_i, \alpha_i, \) and \( \alpha_i + \sigma_i \)) are added to each contour plot (see Fig. 3). In addition a fourth plot superposes the three target contours on the same \( \phi_0, W_0 \) diagram for direct comparison. Valid solutions of the model that are consistent with the data occur in regions where the three contours intersect. The potentially strong constraints from coupled U-series models arise from the significantly different structures of contours for different nuclide pairs. The behaviour of the contour plots and their relationship to the underlying physical parameters are discussed in detail in Spiegelman and Elliott [1993]. The principal results, however are that \(^{230}\text{Th}\) has a long enough half-life such that excesses produced in the bottom of the column can be preserved at the surface if mantle upwelling is sufficiently rapid. In this regime, the activity ratio (\(^{230}\text{Th}/^{238}\text{U}\)) is most sensitive to the upwelling rate \( W_0 \). Very short-lived nuclides such as \(^{226}\text{Ra}\) however, only see the top of the column and are principally sensitive to the porosity \( \phi_0 \) so that these contours can readily intersect the (\(^{230}\text{Th}/^{238}\text{U}\)) contours. \(^{231}\text{Pa}\) however, with its intermediate half-life, but very small partition coefficient behaves somewhere between Th and Ra. Finding intersections of all three systems can be challenging (and is left as an exercise for the reader).
Figure 2. Example graphical output of the 1-D column models. The first panel shows porosity $\phi$ and degree of melting $F$ as a function of pressure (note the two different horizontal scales for $\phi$ and $F$. In general $\phi \ll F$). The second panel shows bulk partition coefficients as a function of pressure $D(P)$ (which are different from those in Fig. 1). The third panel shows calculated activity ratios vs. pressure for the three principal daughter/parent pairs. All of this data (and more) is also provided numerically as a comma-separated spreadsheet.

Discussion

The principal purpose of this contribution is to begin exploring a new mode of model accessibility for which $G^3$ is ideally suited. The models in the UserCalc web-site are extended from those of Spiegelman and Elliott [1993] but are not radically new. What is new, is their accessibility and distribution to a much larger audience of potential users. The benefits of wider accessibility should be apparent. First, this web-site provides a relatively flexible framework for both geophysicists and geochemists to explore the quantitative (and potentially observable) consequences of their own ideas about mantle melting on U-series. In particular, the new models provide significant latitude to vary parameters such as partition coefficients and permeabilities. By increasing the number of users, this approach allows parallel exploration of parameter space driven by real data and scientific agendas beyond that of the model provider. This, in turn, should provide more efficient verification/rejection of these models. Finally, by providing a common platform for spurring collaboration and discussion, this approach could lead to a more efficient evolution and extension of these sorts of models driven by an appropriate balance of community interest, data and the current state of the art in modeling.

The potential benefits of this approach are clear — as are the obvious pitfalls. Recognizing the difference between private “research” code and robust “public” code, the immediate danger of releasing useful but (possibly) buggy code to the public is that I could easily spend significant time tweaking and fixing the site at the expense of other science. More interesting but equally serious is that, if the site is actually useful, I could also spend significant time helping users to understand the output of the models and the inferences that could/should be drawn from them. In general this is good as the principal purpose of the site is to build interesting scientific collaborations. Nevertheless, there is an unstated issue of etiquette that comes with giving away any intellec-
Figure 3. 2-D contour plot output. The black lines in the upper three figures show contours of activity ratios calculated for the three daughter/parent pairs \((230\text{Th}/238\text{U}), (226\text{Ra}/230\text{Th}), (231\text{Pa}/235\text{U})\), plotted in \(\log \phi_0, \log W_0\) space. These figures contour the results of \(10 \times 10\) 1-D columns distributed evenly across the 2-D parameter space \(\log \phi, \log W_0\). In addition, colored contours are shown for the three target values \((230\text{Th}/238\text{U}) = 1.15 \pm 0.01, (226\text{Ra}/230\text{Th}) = 1.7 \pm 0.1\) and \((231\text{Pa}/235\text{U}) = 2.1\). The lower figure superposes the three target data contours on a single \(\phi_0,W_0\) diagram. Valid model solutions that are consistent with the target data occur where all three contours interact.

Spiegelman: UserCalc: July 14, 2000

1.05
1.1
1.15
1.2
1.25

GMT
20:31 Oct 29 1999

(230Th/238U)
(226Ra/230Th)
(231Pa/235U)

Upwelling Velocity
10
1
0.1
0.01
0.001

Porosity
0.001
0.01
0.1

(226Ra/230Th)
(231Pa/235U)

Upwelling Velocity
10
1
0.1
0.01
0.001

Porosity
0.001
0.01
0.1

6 If you find this site useful in your work, the appropriate citation is Spiegelman, M. (2000), UserCalc: a web-based Uranium series calculator for magma-migration problems, G\(^3\) (submitted).

7 In particular, excess requests for my time could result in co-authorship.

8 Lord Kelvin and the age of the earth comes to mind.
ful enough to explore the consequences of this model beyond the model itself. It is the gap between model and reality that leads to understanding and better models. I hope this is the just the beginning of that evolution.

Acknowledgments. Many thanks to Tim Elliott, Rebecca Thomas, Marc Hirschmann, Ken Sims, Matthew Jull and Ken Rubin for comments on this paper and various versions of UserCalc. Funding for this work is provided by NSF grant OCE96-18706. This is LDEO contribution 6096

References


M. Spiegelman, Lamont-Doherty Earth Observatory of Columbia University, Palisades, NY 10964. (email: mspeig@ldeo.columbia.edu)

This preprint was prepared with AGU’s LATEX macros v5.01, with the extension package ‘AGU+’ by P. W. Daly, version 1.6b from 1999/08/19.