Motion of mass on spring (no gravity)

Reference state

\[ \begin{align*}
M & \quad \text{x} \\
\text{spring} & \quad \text{mass}
\end{align*} \]

Deformed state

\[ \begin{align*}
K & \quad \text{x+u} \\
\text{mass} & \quad \text{spring}
\end{align*} \]

\( u = \text{displacement} \)

Newton's force law: \( F = ma \)

Hooke's law: \( F = -k u \)

\( k = \text{spring constant} \)

Connects force and acceleration

\[ M \frac{d^2 u}{dt^2} = -K u \]

\( u'' = -\frac{K}{m} u \)

Oscillatory motion

Amplitude \( C \), freq. \( \omega \)

\[ u = C \sin(\omega t) \]

\[ -MC\omega^2 \sin(\omega t) = -\frac{KC}{m} \sin(\omega t) \]

\( C = \text{anything} \)

\( \omega = \sqrt{\frac{K}{m}} \)
VIBRATIONS IN AN IDEAL GAS

"Sound"

1. Ideal gas law: \( PV = nRT \)
   - \( n \) moles in here
   - \( P \) pressure, \( T \) temp, \( V \) volume

2. Convert from volume - "Bulk property" to density - "Point property"
   - \( n \) moles in here
   - \( \rho \) density

\[ M = mass = \rho V \]
\[ n = \frac{M}{m} \]
\[ = \frac{\rho V}{m} \]
\[ P = \frac{RT}{m} \]
\[ PV = nRT = \rho VRT/m \]

Let \( K = \frac{RT}{m} \)
\[ P = \rho K \]

Nitrogen:
\[ R = 8.3 \ \text{m}^2\text{kg} \text{s}^{-2} \text{K}^{-1} \text{mol}^{-1} \]
\[ T = 300 \ \text{K} \]
\[ m = 0.028 \ \text{Kg/mol} \]
\[ \rho = \frac{RT}{m} = (298 \text{ m/s})^2 \]
1-D deformation in a gas

Reference state

Deformed state

Deformation related to displacement \( u \).
Old position, \( x \) New position \( x + u(x) \)
\( x + \Delta x \)
\( x + \Delta x + u(x+\Delta x) \)

1-D means \( u \) is horizontal

Reference volume \( V_0 = A \Delta x \)
Deformed volume \( V = V_0 + A (u(x+\Delta x) - u(x)) \)

\[ \Delta V = V - V_0 = A (u(x+\Delta x) - u(x)) = A \frac{du}{dx} \Delta x = V_0 \frac{du}{dx} \]

\[ \frac{V - V_0}{V_0} = \frac{\Delta V}{V_0} = \text{fractional change in volume} = \frac{du}{dx} = \text{volumetric strain} \]
convert volume to density
\[ V_0 = \frac{M}{S_0} \quad \frac{\Delta V}{V_0} = \Delta \frac{1}{S_0} = \frac{1}{S_0} \Delta \frac{1}{S_0} \]

but \( \Delta \frac{1}{S_0} = -S_0^{-2} \Delta S \) so

\[ \frac{\Delta V}{V_0} = -\frac{\Delta S}{S_0} = \frac{du}{dx} \]

By Ideal gas law \( p = kS \) so \( \Delta p = k \Delta S \)

and \( \frac{\Delta V}{V_0} = -\frac{\Delta p}{p_0} \)

\( \Box \) Newton's Law \( F = Ma \)

\[ F = -A \left[ p(x+dx) - p(x) \right] = -A dx \frac{dp}{dx} \]

\( M = \int_0^x A dx \)

\( a \approx \ddot{u}(x) \) sloppily!

\[ F = Ma \]

\[ -A dx \frac{dp}{dx} = \int_0^x H dx \ddot{u} \quad \text{or} \quad \int_0^x u \ddot{u} = -\frac{dp}{dx} \]
let $P = P_0 + \Delta P$

$S = S_0 + \Delta S$

with reference state constant in $x$, $t$

so $\frac{dP}{dx} = 0 + \frac{d\Delta P}{dx}$

$\frac{dP}{dx} = 0 + \frac{d\Delta P}{dx}$

now differentiate w.r.t. $x$

$\frac{d^2}{dx^2} \frac{d^2}{dx^2} \Delta P = -\frac{dP}{dx}$

so $\frac{d^2}{dx^2} \frac{d^2}{dx^2} \Delta P = -\frac{d^2}{dx^2} \frac{d^2}{dx^2} \Delta P$

$-\frac{d^2}{dx^2} \frac{d^2}{dx^2} \Delta P = -\frac{d^2}{dx^2} \frac{d^2}{dx^2} \Delta P$

$\Delta P = \sin \left( \frac{x - vt}{K} \right)$

$C \sin (x - vt)$

$C = \text{anything}$

$\frac{v^2}{K} = 1 \text{ or } v = \sqrt{K} \approx 300 \text{ m/s for air}$