\[ u_i = A_i \cos (k \hat{n} \cdot x - \omega t) \]

\[ \frac{\omega}{K} = \text{velocity} \quad \hat{n} = \text{direction} \quad \theta = \text{angle} \]

\[ \mathbf{p} = \text{slowness vector} \]

\[ = A_i \cos \left( w \left( \frac{K \hat{n}}{\omega} \right) \cdot x - \omega t \right) \]

\[ \mathbf{p} = \frac{K}{\omega} \mathbf{n} = \frac{n}{V} \quad \text{(Vbig, Psmall)} \]

so \( \mathbf{p} \) called slowness

If \( \mathbf{n} = \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} \) then \[ \mathbf{p} = \frac{1}{V} \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} \]

This intersection point moves right at slowness \( p_x = \frac{\sin \theta}{V} \)

at apparent velocity \( v_x = V \)

This intersection point moves down at slowness \( p_z = \frac{\cos \theta}{V} \)

at apparent velocity \( v_z = V \)

Note \( (p_x, p_y, p_z) \) is a vector; \( (v_x, v_y, v_z) \) is not a vector.
Suppose wavefront crosses origin at \( t = 0 \).

Then it crosses a station at \((x, y)\) at a time \( \frac{x}{V_x} + \frac{y}{V_y} \).

So you can estimate \( P_x, P_y \) from any time at 3 stations, \( A, B, C \):

\[
T_B = T_A + P_x (x_B - x_A) + P_y (y_B - y_A) \\
T_C = T_A + P_x (x_C - x_A) + P_y (y_C - y_A),
\]

2 equations in 2 unknowns:

\[
\begin{pmatrix}
(x_B - x_A) & (y_B - y_A) \\
(x_C - x_A) & (y_C - y_A)
\end{pmatrix}
\begin{pmatrix}
P_x \\
P_y
\end{pmatrix} =
\begin{pmatrix}
T_B - T_A \\
T_C - T_A
\end{pmatrix}
\]
Issues

$$(\text{lon}, \text{lat}) \rightarrow (x, y) \quad \text{flat earth approx}$$

$$x = 111.12 \cdot \text{lon} \cdot \cos(\text{typical latitude})$$

$$y = 111.12 \cdot \text{lat}$$

Invert 2x2 matrix:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad M^{-1} = \frac{1}{AD - BC} \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}$$

Azimuth from north from $Px, Py$:

$$\text{azimuth} = \tan^{-1}\left(\frac{Px}{Py}\right)$$

Horizontal slowness in direction of azimuth $\sqrt{P_x^2 + P_y^2}$

Horizontal apparent velocity $\sqrt{P_x^2 + P_y^2}$

Vertical slowness:

Use $P_x^2 + P_y^2 + P_z^2 = \frac{1}{V^2}$

$$P_z^2 = \frac{1}{V^2} - P_x^2 - P_y^2$$

$$P_z = \sqrt{\frac{1}{V^2} - P_x^2 - P_y^2}$$

Angle of incidence:

Use $P_z = \cos \Theta / V$

$$\Theta = \cos^{-1}\left(\frac{V}{P_z}\right)$$
Two uniform half-spaces in contact

**Medium 1**

Plane P, S waves here with velocity \( \alpha_1, \beta_1 \)

**Medium 2**

Plane P, S waves here with velocity \( \alpha_2, \beta_2 \)

All waves must have this horizontal apparent velocity

Waves on other side of boundary must have this apparent velocity

Displacement, traction continuous across interface \( \Rightarrow \)

Wavefronts must not tear as they go across the interface

**Horizontal apparent velocity (horizontal slowness) must be equal on two sides**

\[
\frac{\sin \Theta_{\text{top}}}{\alpha_{\text{top}}} = \frac{\sin \Theta_{\text{bottom}}}{\alpha_{\text{bottom}}}
\]

Snell's law.
displacement continuous: boundary remains "welded"
tractions continuous: no infinite forces on small
muses near boundary

Six conditions

\[ M_x^{\text{top}} = U_x^{\text{bot}} \]
\[ U_y^{\text{top}} = U_y^{\text{bot}} \]
\[ U_z^{\text{top}} = U_z^{\text{bot}} \]
\[ T_{xz}^{\text{top}} = T_{xz}^{\text{bot}} \]
\[ T_{yz}^{\text{top}} = T_{yz}^{\text{bot}} \]
\[ T_{zz}^{\text{top}} = T_{zz}^{\text{bot}} \]

P input: \( \hat{A}_x^{\text{top}}, \hat{N} \) specified

\[ A = C \hat{n}^{\text{top}} \]

Output: \( \hat{A}_x^{\text{bot}}, \hat{N}^{\text{bot}} \) given by shell's law

One output not enough to satisfy 6 conditions

Need 6 outputs

Note 2: Waves in each layer
SV: Polarization in plane of paper
SH: Polarization L
Special cases:

- **Prop in**: no motion \( \perp \) to page
d

- **Prop out**, **Bot out**

- **Bot out**

- **Top out**

- **Top out**

- **No ** \( S_{\text{Bot out}} \) or \( S_{\text{Top out}} \) due to symmetry.

\[ \text{Note: "S" is always steeper than "P", due to Snell's law:}
\]

\[ \sin \theta_p = \frac{\sin \theta_s}{\alpha} = \frac{p_x}{\beta} \]

\( p_x \) = horizontal slowness

\( \alpha > \beta \)