APPLICATION OF A CEPSTRAL F−STATISTIC FOR IMPROVED DEPTH ESTIMATION

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ABSTRACT

A definitive way to rule out seismic events as possible nuclear tests is to accurately determine the source depth. However, the determination of depth for regionally recorded events remains one of the most challenging problems for regional seismic monitoring. Cepstral techniques are used to detect echoes in a signal (such as the depth phases pP and sP) and have been utilized in the past for seismic depth determination by Alexander (1996), Bennett *et al.* (1989) and Kemerait (1982). To further test the cepstral technique, we formulated a cepstral F−statistic found by using a classical approach to detecting a signal in N stationarily correlated time series (Shumway, 1971). We have also implemented techniques to window and taper the cepstra that decrease significantly the number of false alarms in the F−statistic. To help with the interpretation of the F−statistic, we perform the analysis using data recorded on multiple arrays and look for correlations in peaks of the cepstra.

We first performed the cepstral F−statistic analysis on a set of 32 events from the pIDC REB on 12 Feb 2000 to determine the usefulness of the technique on teleseismic data where the P waves are often less complicated by path effects, focal mechanism, and coda complexity than regional signals. Using the cepstral F−stat, we were able to determine depths for 16 of the 32 events. For eight other events, the SNR was too small for analysis, while the remaining eight events were possibly too shallow for resolution of a depth phase (< 8 km). The technique determined depths ranging from 10 km to 90 km for seven of 17 events that the pIDC had fixed to 0.0 km. For pIDC m_b magnitudes less than 3.8, there is a significant amount of scatter in the depths reported by the pIDC, the USGS, and those calculated by the cepstral analysis. The scatter is reduced at magnitudes greater than 3.8. For events with magnitude greater than 4.0, the average difference between the USGS and Weston cepstral depths is less than 25 km, while the mean difference between the pIDC and Weston depths is approximately 35 km.

We also tested the cepstral F−statistic method using a combination of teleseismic and regional data by determining focal depths for earthquakes in Central America and northern South America using TXAR, PDAR, ILAR, NVAR, and YKA data. The USGS located 29 events within the study region between 05 May 2000 and 15 June 2000, and 21 of the depths were constrained to 33 km. The USGS did not use depth phases for any of the events to determine the focal depth. We were able to determine depths for 26, while two were too small for analysis and a third had two plausible depths. We found that the median depth determined from the cepstral analysis was 31 km which is in agreement with the nominal USGS depth of 33km, however, there were several events in which the USGS and Weston cepstral depths disagreed significantly (e.g. 110 km for one event with a visible pP). We observed that a significant amount of the scatter in the cepstral depths is the result of the analysis at regional distances. By comparing depths calculated by teleseismic arrays separately from depths determined by regional arrays, we observed that the scatter averages approximately 9 km for events with magnitude 4 and decreases about 4 km per magnitude unit up to our maximum magnitude studied of 5.2. Thus, for larger events ($m_b > 5$), the depths obtained by regional and teleseismic analysis should be within 5 km of each other. For small events, which are of particular interest in areas of high monitoring concern, the scatter may prove to be too large for the cepstral F−statistic to be used in an operational capacity on regional data.

Key Words: seismic, depth estimation, cepstral, F−statistic

OBJECTIVE

The primary objective of this research is to establish the operational usefulness of cepstral techniques to determine the depth of a seismic event. To accomplish this objective, we have developed a statistical criterion to accompany cepstral estimates of seismic depth phase delay times. The statistical measure, or cepstral F−statistic, provides a measure of the significance of a particular peak in the cepstrum. We have applied the technique to data from the Prototype International Data Center (pIDC) Reviewed Event Bulletin (REB) and from the United States Geological Survey (USGS) bulletin, and the results show the method is highly successful for teleseismic distances, slightly less reliable at far−regional distances, and problematic at distances less than 10° from the source.

RESEARCH ACCOMPLISHED

Cepstral techniques are used to detect echoes in a signal using simple Fourier transform processing methods. They have been used in such diverse applications as speech analysis, image processing and radar data analysis (Bogert et al., 1963; Childers et al., 1977). The cepstrum is defined as the Fourier transform of the log of the spectrum of a time domain signal (Oppenheim and Schafer, 1975), and was designed to help analyze the periodicity that occurs in the power spectrum when echoes are present in the original signal. In principle, cepstral analysis can be used to detect the periodicity of scalloping in the seismic spectrum that is due to the interference of the direct (*P*) and depth (*pP* and/or *sP*) phases. This periodicity is directly related to the depth phase delay time, which is in turn directly related to focal depth. Cepstral techniques have been utilized in the past for seismic depth determination (Alexander, 1996; Bennett et al., 1989; Kemerait, 1982), but the results have been mixed, and the operational capability of the method has never been established. To improve the method, we have formulated a cepstral F−statistic (Shumway et al., 1998) that attaches statistical significance to peaks in the cepstrum. The F−statistic is found using a classical approach to detecting a signal in N stationarily correlated time series (Shumway, 1971). Testing the hypothesis $S(d) = 0$, where S is the signal transform at delay time d, leads to an F−statistic given by

$$F_{2,2(N-1)}(d) = (N-1)\frac{SCB(d)}{SCE(d)},$$

which can be interpreted as a signal−to−noise ratio. The subscripts refer to an F distribution with 2 and 2(N−1) degrees of freedom. In this instance, the signal consists of the spectrum of the stacked log spectra, or *beam cepstrum (SCB),* defined as

$$SCB(d) = N\left|\overline{Q}(d)\right|^2$$

where

$$\overline{Q}(d) = N^{-1}\sum_{j=1}^{N} Q_j(d),$$

is the mean Fourier transform of the log spectra. The noise is defined as the *error cepstrum (SCE):*

$$SCE(d) = \sum_{j=1}^{N}\left|Q_j(d) - \overline{Q}(d)\right|^2$$

$$= SCT(d) - SCB(d),$$

where $SCT(d)$ is the *total stacked cepstrum,*

$$SCT(d) = \sum_{j=1}^{N} \left| Q_j(d) \right|^2 .$$

The error cepstrum is a measure of the extent to which the individual channel or window transforms differ from the mean transform. The beam and total cepstra are computed from multiple windows of data, which can be extracted from elements in an array. Note that the total stacked cepstrum (*SCT(d)*) is equivalent to the sum−stack proposed by Alexander et al. (1995), which is computed by simply adding up the cepstra of individual windows. Our tests have shown that the sum−stack will not reflect the common signal components as well as either the beam cepstrum or the F−statistic. In the following paragraphs, we describe the methods used to implement this procedure.

Preprocessing. We have found that bandpass filtering the data prior to processing helps accentuate the cepstral peaks. Currently, we employ a 3−pole, Butterworth bandpass filter between 0.6 and 4 Hz to the demeaned data. For arrays such as CMAR and KSAR, high−frequency cultural noise may prompt lowering the hi−cut frequency to 2 Hz for optimum peak enhancement. The improvement in the cepstral peaks comes from the increase in the signal to noise (SNR) ratio brought on by filtering along with the smoothing of the P−wave caused by the relatively narrow band. The filtering also removes long period noise and signals that complicate the cepstra as well.

Cepstral Analysis. The P wave arrival and coda are windowed and the log−power spectrum of each window of data is calculated. The log spectrum is detrended with a cubic spline (Shumway, 1997) to remove the Fourier component of the spectra that is not due to possible depth phases. Reiter and Shumway (1999) noted that noise in the high frequency part of the spectra often obscures peaks in the cepstrum that are due to true signal echoes. To overcome this high frequency noise, we use cosine tapered windows to extract only the signal portion of the spectra. Since most teleseismic signals are in the 1−2 Hz bandwidth, the window usually was chosen between zero and 2−3 Hz. For regional signals, the bandwidth is increased to between 3−4 Hz. The results of this method decrease significantly the number of false peaks seen in the cepstral F−statistic. The cepstra of the raw data are now ready to be calculated by taking the Fourier transform of the windowed and detrended log power spectra. We calculate the following three cepstral quantities.

Beam Cepstra (**SCB**) = cepstra of summed windowed and detrended log spectra
Total Cepstra (**SCT**) = sum of individual cepstra
Cepstral F−statistic = (N−1) **SCB**/(**SCT**−**SCB**)

The results of these calculations on a teleseism recorded at ILAR are shown in Figure 1. For this event, there are four large peaks that are visible in all three cepstra at 5.1, 9.5, 11.8, and 19.5 seconds (the time scale refers to delay times in seconds i.e. pP−P or sP−P). The most obvious choice for the pP phase based upon visual inspection of the seismogram (Figure 2) would be the largest cepstral F−statistic peak at 5.1 seconds. Upon analysis of the cepstral F−stat, caution should be exerted when picking large peaks that may be created by random processes that cause the beam and total cepstra to have similar values at certain points. This results in large amplitude spikes in the F−statistic that are not the result of echos in the signal. Thus the peaks in the F−statistic should be picked while examining the beam and total cepstra for co−located peaks. For the analysis of this example event as well as the others we determined the three largest peaks above the confidence line and recorded for depth analysis.

Peak Correlation. In some cases, only a single peak may exhibit an amplitude above the 99% confidence level. However, in most cases (e.g. Figure 1) there will be numerous peaks that may result from pP or sP depth phases, secondary arrivals not associated with depth phases (e.g. PcP), arrivals from other earthquakes, and/or noise. Thus, we feel it is difficult for an accurate depth determination to be based upon the cepstral F−statistic peak produced by analysis of a single array. Instead, we compute the cepstra for several arrays so that a comparison of F−statistic peaks between arrays can be completed. The analyst will overlay the F−statistics from all arrays and search for correlations in the peaks to determine if more than two arrays are noting similar delays. If so, we use the IASP91 (Kennett and Engdahl, 1991) standard earth model to compute depths based upon the delays. The depths calculated at each array were averaged together to give a source depth that was then compared with the

USGS and pIDC data. The results of our comparisons will be discussed in the following sections.

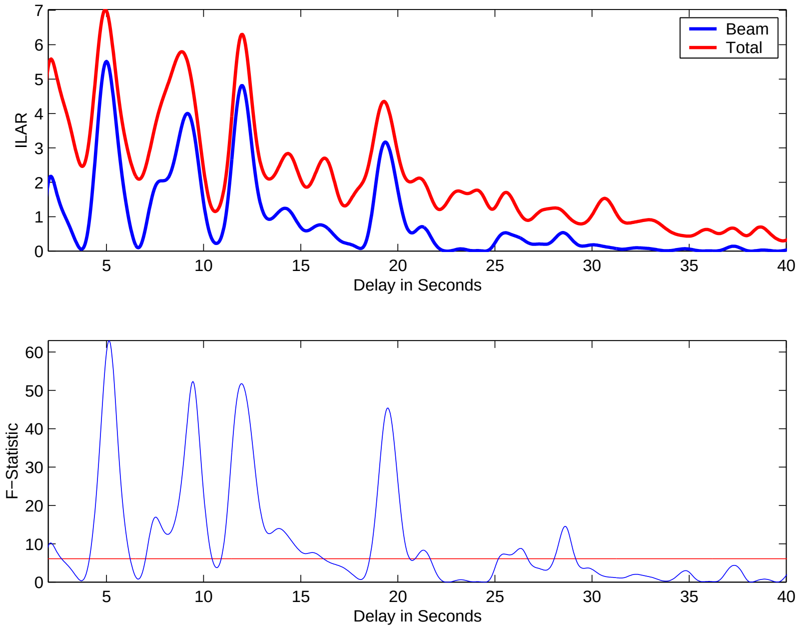

Figure 1. (Upper) Beam and total cepstra for a Central American earthquake recorded at ILAR. (Lower) Cepstral F−statistic with 99% confidence line shown in red.

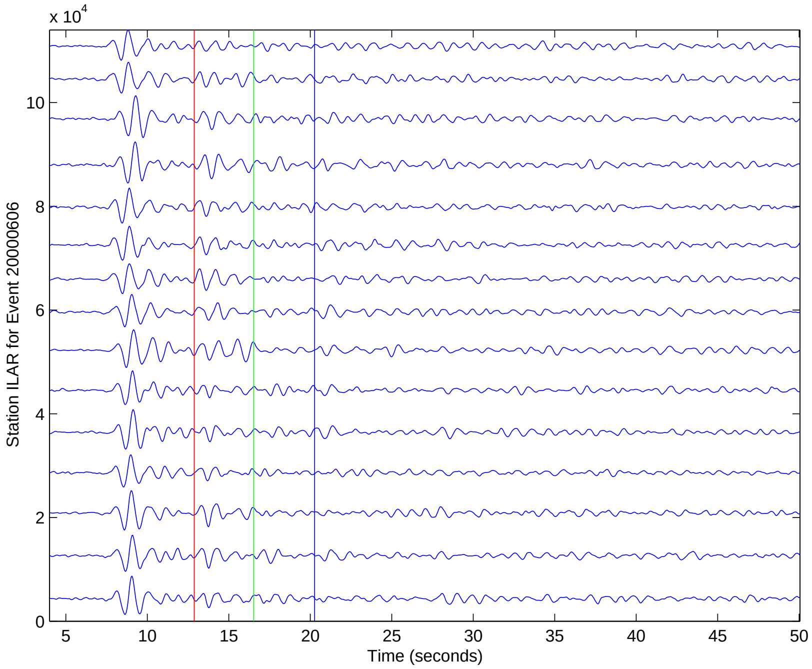

Figure 2. Seismograms recorded at ILAR for an event in central America. The P wave is followed by another arrival approximately 5 seconds later that may be pP based upon the reverse polarity. The red, green, and blue lines mark the largest, second largest, and third largest peaks, respectively, in the cepstral F−statistic.

Application of the Technique At Teleseismic Distances. To examine the usefulness of our technique at teleseismic distances, we determined depths for a number of events using the cepstral F−statistic method (referred throughout as the Weston method) and compared our cepstral depths to depths determined by the pIDC and USGS. We picked February 12, 2000 at random and downloaded the REB from the pIDC. For this day, 32 events were located (Figure 3) by the pIDC while the USGS computed locations for 26 of the 32 events. Because the F−statistic requires beaming individual elements from an array, we downloaded data from CMAR, KSAR, TXAR, NVAR, PDAR, ILAR, and YKA when available. The data was converted from GSE2.0 to SAC and read into memory in SAC2000. Using the new Matlab routine in SAC, we copied the data from the SAC workspace into the Matlab environment where the cepstral analysis was performed. The cepstral analysis requires a signal−to−noise ratio (SNR) greater than 4 to obtain statistically valid peaks that may be correlated between arrays. When the SNR is low, we find many peaks in the cepstral F−statistic that are not associated with depth phases. For the events analyzed thus far, a pIDC mb > 3.8 seems to be sufficient for teleseismic analysis at more than 2 arrays. However, if the event was only recorded at CMAR or KSAR or other arrays where background noise is high, the threshold for cepstral analysis may be raised to mb > 4.0.

Using the Cepstral analysis, we were able to determine depths for 16 of the 32 events. Of the other 16 events, the SNR was too small for analysis for eight events, while the remaining eight were too shallow for a depth phase (< 8 km) or the depth phase was not observed. For the 32 events (note the USGS had only located 26 of the events to date), the pIDC had fixed the depths of 17 of these events to 0.0 km. The Weston cepstral technique obtained depths for seven of these fixed−depth events ranging from 10 to 90 km. Figure 4 shows the depths reported by the USGS and pIDC (with error bars for the pIDC) and the Weston cepstral depths with associated error. Figure 5 is a plot of the residuals (Weston Cepstral Depth−USGS and Weston−pIDC) for the events. For magnitudes less than 3.8, there is a significant amount of scatter in the residuals between the depths calculated by the two groups and the cepstral analysis. The scatter is reduced at magnitudes greater than 3.8. For events with magnitude > 4.0, the average difference between the USGS and Weston cepstral depths is less than 25 km, while the difference between pIDC and Weston depths is near 35 km.

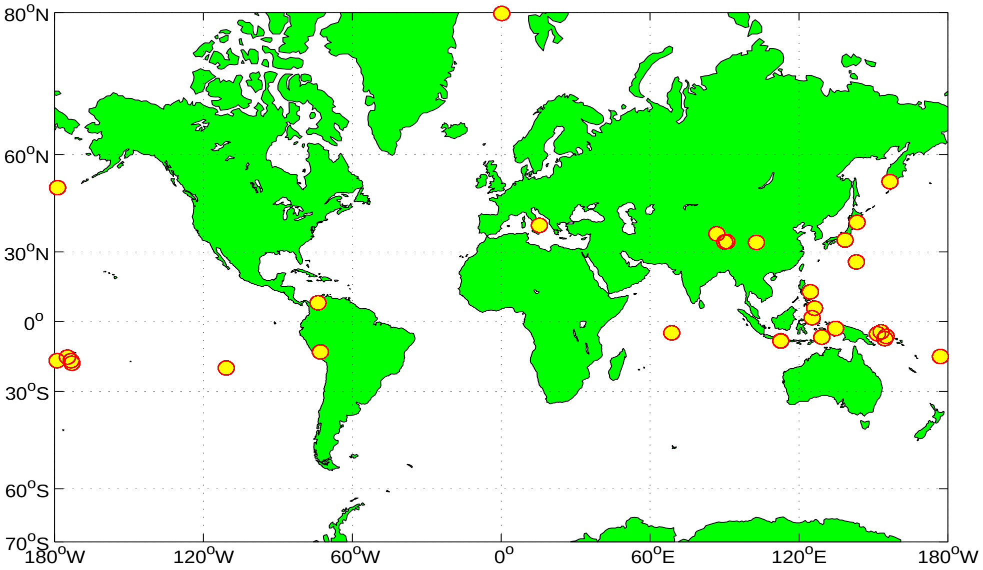

Figure 3. Location of the pIDC REB events for 12 Feb 2000.

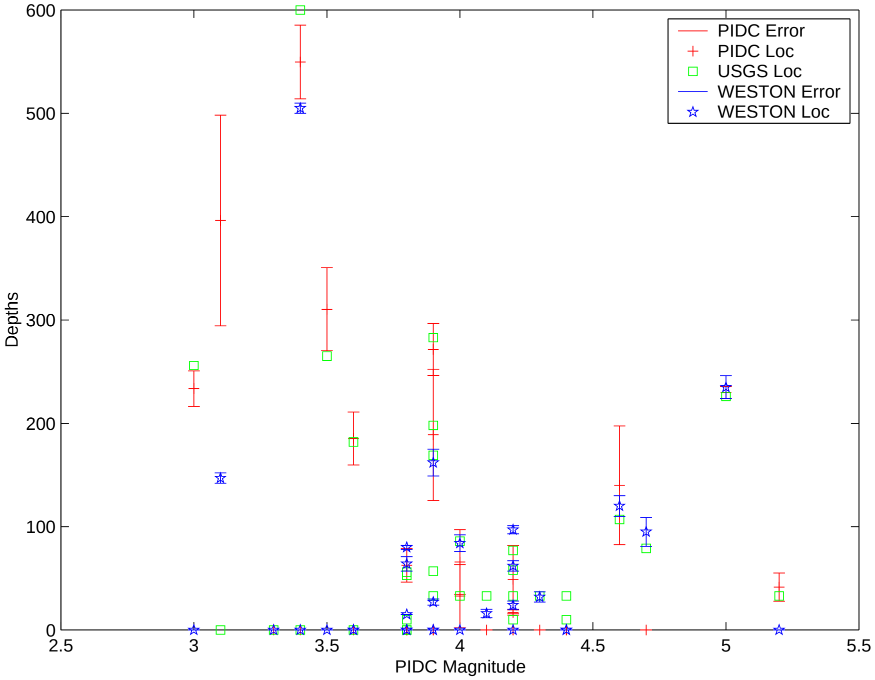

Figure 4. Comparison of depths (and errors) reported by the pIDC and USGS for 32 events on 12 Feb 2000. The depths and scatter as determined by the Weston cepstral F−statistic analysis are also shown.

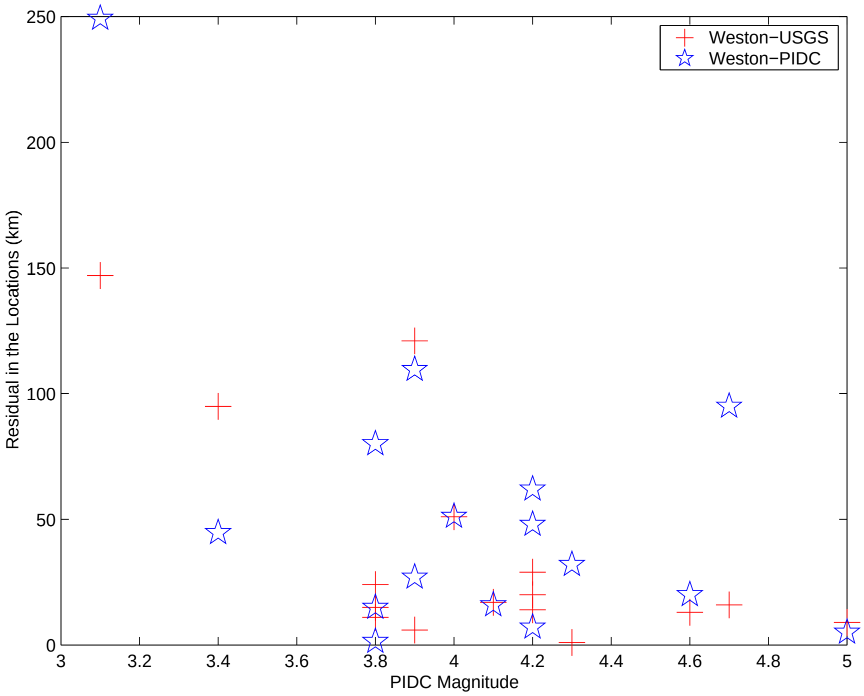

Figure 5. Residuals for depths calculated by the Cepstral technique and the USGS and pIDC reported depths.

Application at Teleseismic and Regional Distances. The cepstral F−statistic technique was applied to 29 events located by the USGS in Central America and northern South America between 05 May 2000 and 15 June 2000. While this region has little or no significance to treaty monitoring, it does have an abundance of earthquakes with a wide range of depths recorded at regional and teleseismic distances. The event characteristics and the results of the analysis are shown in Table I and the locations are shown in Figure 6. For Event No. 20000530, we were not able to determine a reliable depth due to its small magnitude that resulted in poor SNR at TXAR and no signal at NVAR, PDAR, ILAR, and YKA. For events 20000516 and 20000608_3, we were only able to determine depths based upon a single array due to small magnitude and lack of data availability, respectively. Figure 7 shows the Weston depths and error in comparison to the USGS depths. As of the publication of this manuscript, we still had not received the pIDC depths for events located during this time period. Of the 28 events for which Weston depths were calculated, the USGS had fixed the depths of 21 of the events to 33 km. For two additional events, the USGS had constrained the depths to 10 and 20 km. None of the depths reported by the USGS for these events were constructed using depth phases. For 25 of 26 events, we were able to determine a single depth that corresponded to several statistically significant peaks in the cepstral analysis noted on multiple arrays. For Event 20000615, we found two depths (192.5 and 89.25) that were feasible. For 25 of 29 events, TXAR was located at regional distances to the earthquakes (10° < Δ < 20°) allowing for an interesting comparison of the cepstral analysis at regional and teleseismic distances. Of these 25 events, there were six (6) events (24%) for which a stable depth could not be estimated based upon the TXAR analysis. The reasons for the failure of the method included small amplitude Pn arrivals, complicated secondary phases such as Pg, and the interplay of the crossover between the P and Pn phases near approximately 16−20°. When comparing the TXAR depths with the depths calculated from the other arrays, we find that a significant amount of the standard deviation in the Weston cepstral depths comes from the analysis at the regional distances (Figure 8).

Weston Event No.	Year	Month	Data	Origin Time	Lat	Long	Depth	Magnitude	Type		Cepstral Dept	Mean	Med	STD	Range	Min	Max
20000505	2000	5	5	143449	11.93	−86.18	103	4.1			5 (2 sP−P)	64	62	5.1	12	60	72
20000510	2000	5	10	155742.27	16.22	−96.86	33	4.5	mb	GS	10 (6 sP−P)	31.4	29	6.5	19	24	43
20000511_1	2000	5	11	174045.82	16.15	−97.93	33	4.5	mb	GS	7 (2 sP−P)	18	16	4.4	12	12	24
20000511_2	2000	5	11	175826.45	16.28	−97.98	33	4.8	mb	GS	6 (1 sP−P)	18	17.5	2.3	6	16	22
20000513	2000	5	13	112719.52	14.57	−93.29	33	4	mb	GS	8 (3 sP−P)	54.6	53.5	6.2	14	48	62
20000514_1	2000	5	14	90019.88	18.11	−102.7	33	4.8	mb	GS	6 (2 sP−P)	25.3	25	1.9	5	23	28
20000514_2	2000	5	14	152437.13	14.11	−92.76	33	4.4	mb	GS	5	34.2	31	9.8	22	25	47
20000515_1	2000	5	15	72606.61	16.91	−100.6	33	4.2	mb	GS	6 (3 sP−P)	26.3	27	2.5	7	22	29
20000515_2	2000	5	15	104710.76	13.19	−91.11	33	4.3	mb	GS	6 (2 sP−P)	35.2	34	7.3	17	29	46
20000516	2000	5	16	81900.6	11.68	−86.93	20	4.2			1	75?	N/A	N/A	N/A	N/A	N/A
20000519	2000	5	19	224212.06	14.06	−92.49	33	4.3	mb	GS	6	20.5	21	5.8	16	11	27
20000521	2000	5	21	5317.21	16.48	−97.9	33	4.5	mb	GS	5	17.2	17	3.4	10	12	22
20000523	2000	5	23	163645.69	2.06	−78.39	33	5	mb	GS	5 (2 sP−P)	31	32	2.8	6	28	34
20000524_1	2000	5	24	103602.13	2.24	−79.52	33	4.4	mb	GS	5 (1 sP−P)	21.6	22	0.9	2	20	22
20000524_2	2000	5	24	202833.84	13.53	−91.21	33	4.4	mb	GS	5 (1 sP−P)	34.5	33	8.2	19	27	46
20000524_3	2000	5	24	231416.67	14.38	−92.76	79	4.6	mb	GS	4	22.5	22	2.1	5	20	25
20000527	2000	5	27	73500.68	11.67	−86.98	72	5	mb	GS	5	51.4	48	9.4	25	42	67
20000528	2000	5	28	50150.17	11.52	−87.04	33	4.6	mb	GS	5	35.4	35	3.6	9	31	40
20000530	2000	5	30	132918.08	5.7	−77.37	33	4	mb	GS							
20000603_1	2000	6	3	75736.34	16.16	−94.2	33	5	mb	GS	10 (5 sP−P)	112.2	107.5	15.4	46	94	140
20000603_2	2000	6	3	123944.1	15.33	−104.85	10	4.2	mb	GS	7 (2 sP−P)	23.1	22	3.2	10	18	28
20000606_1	2000	6	6	170316.72	16.25	−97.09	33	4.3	mb	GS	8 (4 sP−P)	33.4	30	6.3	16	24	40
20000606_2	2000	6	6	182541.15	19.01	−103.83	125	4.4	mb	GS	7 (4 sP−P)	17.5	16	3.5	13	22	9
20000607	2000	6	7	52018.93	10.56	−86.41	33	4.3	mb	GS	5 (2 sP−P)	35.4	35	5.4	13	29	44
20000608_1	2000	6	8	80810.27	14.3	−92.07	33	5.2	mb	GS	5	24.5	24.5	2.1	5	22	27
20000608_2	2000	6	8	130130.31	14.1	−91.9	33	4.5	mb	GS	5	23.5	22.5	3.1	7	20	27
20000608_3	2000	6	8	153339.59	14.29	−92.06	33	5	mb	GS	1	24?	N/A	N/A	N/A	N/A	N/A
20000613	2000	6	13	171119.9	14.01	−92.27	33	4.4	mb	GS	7 (3 sP−P)	22.3	22	3.1	9	16	25
20000615	2000	6	15	34813.79	12.84	−87.92	209.6	4.6	mb	GS	2	192.5	192.5	23.3	33	176	209
											4 (2 sP−P)	89.25	85.5	10.9	24	81	105

Table I. Depths for USGS located events in central America using the cepstral F−statistic analysis.

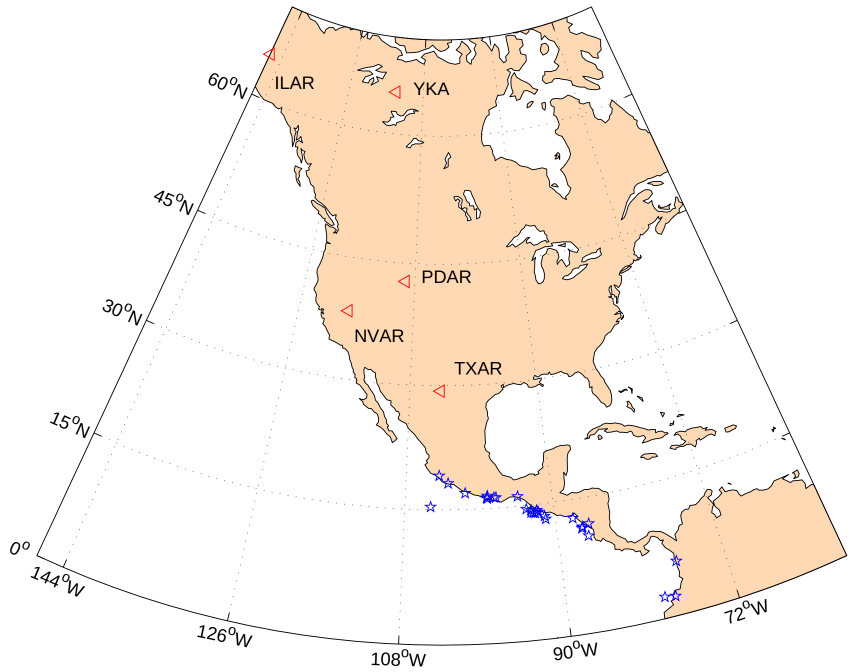

Figure 6. USGS locations of the events in Central America and northern South America that were used to evaluate the cepstral F−statistic method at both teleseismic and regional distances. Also shown are the arrays used in the cepstral F−statistic analysis.

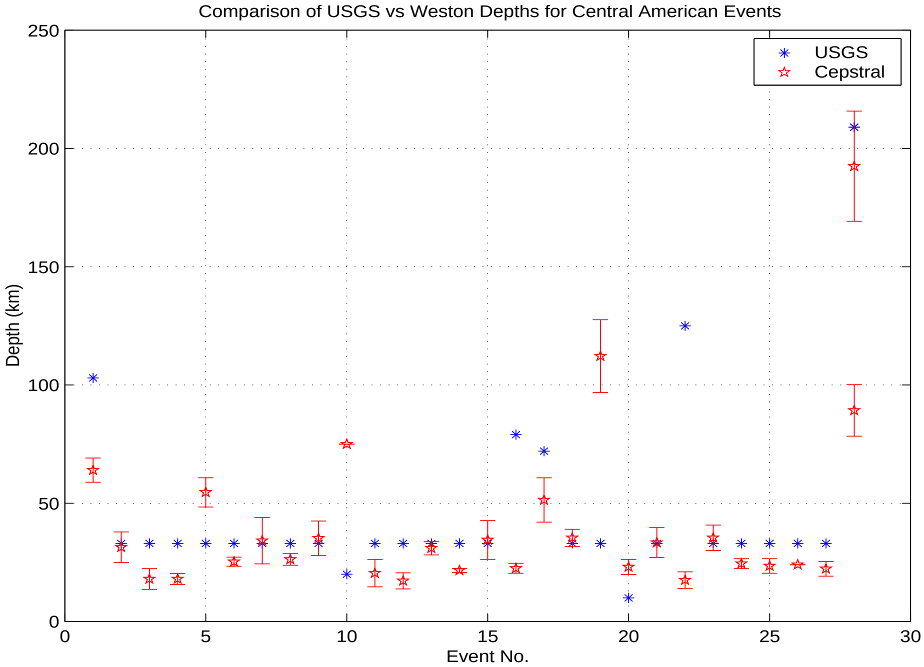

Figure 7. A comparison of the Weston cepstral depths and scatter with the USGS depths for 28 events in Central and northern South America.

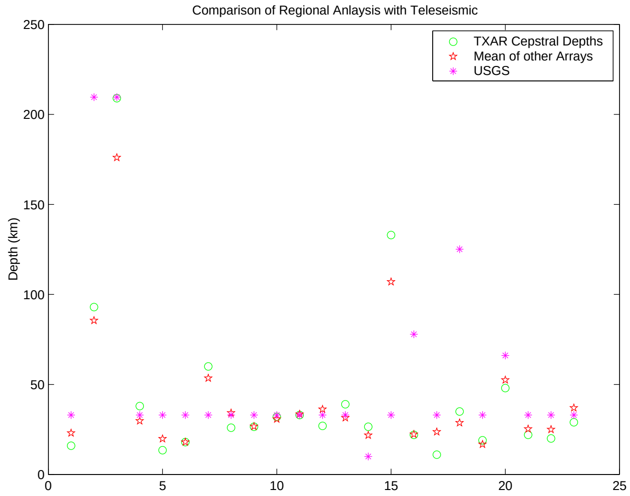

Figure 8. A comparison of the Weston cepstral depths using TXAR only (green circle), using all arrays but TXAR (red stars), and the USGS depths (magenta astericks).

We also note that the difference between the Weston cepstral depths using TXAR only and the depths obtained from the other arrays decreases in general as a function of magnitude (approximately 4 km per unit magnitude). The obvious interpretation for the magnitude dependence is the fact that the Pn and pPn arrivals are larger and more easily identified by the cepstral technique for greater magnitude events. This presents a difficult problem in that a main reason to use the regional arrays would be to determine depths for small regional events since larger events will be recorded at teleseismic distances where the cepstral analysis is more straightforward and successful.

Near−Regional Analysis. Our initial work with near−regional data (< 10°) has shown the analysis is even further complicated by the complexity of the seismograms. As an example, we completed the cepstral F−statistic analysis on copper mining explosions from Arizona as recorded at TXAR and found we were getting statistically valid peaks for surface detonated events. These peaks (Figure 9) were found to be caused by PnPn and Pg arrivals thus indicating the difficulty with using the Weston method at these distances.

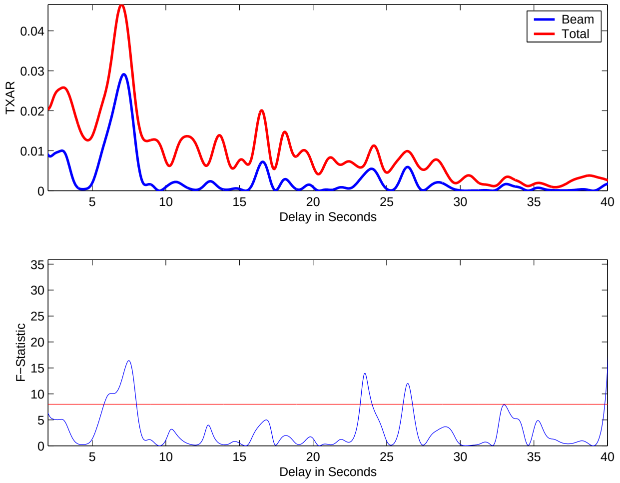

Figure 9. Cepstra for a delay−fired surface explosion in Arizona as recorded at TXAR (Δ = 6.2°). The peaks in the cepstra are the result of secondary P arrivals such as PnPn and Pg, and are not the result of depth phases.

CONCLUSIONS

We have formulated a cepstral focal depth estimation technique that provides a statistical estimate of the significance of the peaks in the stacked cepstrum. This significance measure has been missing in previous cepstral estimates of focal depth. During the past year, we have examined the usefulness of the technique on a variety of different datasets at both teleseismic and regional distances. Our results show that the method is highly successful for teleseismic distances, slightly less reliable at far−regional distances, and problematic at distances less than 10° from the source. The variability in the far−regional cepstral depths seem to be dependent upon signal bandwidth and the magnitude of the event. For small events, which are of particular interest in areas of high monitoring concern, the scatter may prove to be too large for the cepstral F−statistic to be used in an operational capacity on regional data.

The question remains as to "Whose depths are more reliable? The USGS, pIDC, or Weston's cepstral depths? This is very much a valid question, however, we believe that the cepstral F−statistic provides a statistically valid and robust means for determining focal depths. Based upon our procedure that searches for peaks that correlate between arrays, we do not rely on measurements made at a single station, but instead search for agreement amongst all the data. Our future work will focus on detailed datasets where the systematic error in the focal depths is GT5 or better to answer part of this question as to the reliability of the technique. We will continue to develop methods to automate the entire procedure as well as attempt to incorporate additional tests of the validity of the peaks in the cepstral F−statistic. Our continued major focus during the final stages of this project will be on the operational potential of this method for seismic depth estimation.

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