CHARACTERIZATION OF SEISMIC SOURCES

Lane R. Johnson
University of California, Berkeley

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ABSTRACT

The seismic discrimination between weapons tests, industrial explosions, natural earthquakes, and other types of seismic events is limited by an incomplete understanding of the basic physics of the source processes. The problem of quantifying how the generation of elastic waves is related to such factors as mode of energy release, source dimensions, material properties, and stress concentrations is still unsolved for the most part. While a complete solution of this complicated problem may not be possible at the present time, an examination of the similarities and differences between various types of seismic sources could prove useful in the discrimination task. This was one of the objectives of this research effort.

A starting point for the comparison of the different types of seismic sources is the large amounts of empirical data that are summarized in various seismic scaling relationships, such as moment versus yield for contained explosions and stress drop versus moment for natural earthquakes. By converting these scaling relationships for different types of sources to a common form, it is possible to assess whether there are significant differences in the basic source processes. For instance, both contained explosions and natural earthquakes are consistent with a linear relationship between seismic scalar moment and source energy, with the proportionality factor being identical if the efficiency of the earthquake source is taken to be about 0.15%. A recent study of the energy required to generate pseudotachylites suggests that this value of efficiency is not unreasonable. Another approach to the comparison of seismic sources is the consideration of energy density in the source region, which is related to the total energy of the source and its spatial dimensions. Here again recently acquired data suggest that, provided a distinction is made between the initial source dimension and the effective source dimension, the energy densities of different types of sources are more similar than previously thought. This leads to a consideration of such matters as the interaction between the seismic source and the material properties, the degree to which the material properties are modified by the source, and the possibility of secondary sources. These aspects of the problem have been examined by simulating the stress field near different types of seismic sources, comparing the results, and exploring the effect of various source parameters.

Key Words: discrimination, seismic sources, explosions, earthquakes

OBJECTIVES

The scope of this research effort is concerned with the development of an improved understanding of methods for locating and characterizing seismic events in a heterogeneous earth. The general objective is to investigate how improved models for the generation and propagation of elastic waves can help in the evaluation of methods and models currently being used. The work statement for the research contains two primary tasks:

1. Continue the development of analytical and numerical techniques capable of modeling the physical processes that cause the generation of elastic waves by seismic sources. Also continue the development of three-dimensional models of the earth's velocity structure which have sufficient resolution to provide accurate locations of seismic events at regional and teleseismic distances.

2. The modeling capability will be validated against existing data bases and any new data which can be acquired by taking advantage of targets of opportunity to perform broad band recording experiments.

This paper reports on work that has bee done during the past three years on scaling relationships for seismic sources, with particular emphasis on such relationships for small earthquakes. Developing a more complete understanding of these scaling relationships for both contained explosions and natural earthquakes has the potential to provide a more comprehensive theoretical foundation for the methods that are currently being used to monitor a Comprehensive Nuclear-Test-Ban Treaty and perhaps uncover new methods.

RESEARCH ACCOMPLISHED

Haskell (1964) introduced a model of an earthquake as a rupture propagating over a finite section of a fault and provided convenient analytical formulas for the radiated elastic waves. A characteristic of this model in its most simple form is a very uniform release of displacement and stress on the fault surface. Haskell recognized this as a deficiency and suggested that some form of heterogeneity was needed in the distribution of displacement in either time or space. Haskell (1966) and Aki (1967) attempted to model this heterogeneity by describing the acceleration and velocity, respectively, of fault displacement in terms of random functions that were correlated over only limited portions of the fault plane. Brune (1970) considered the time history of displacement on the fault and provided spectral models relating the frequency content of radiated elastic waves to dimensions and stress present on the fault. These models for an earthquake formed the basis for the interpretation of large amounts of observational data in terms of earthquake source parameters, but the emphasis was primarily upon average properties and not much attention was given to the heterogeneity originally explored by Haskell and Aki. Kanamori and Anderson (1975) summarized many of these empirical data and used them to justify scaling laws that related such parameters as fault dimension, stress drop, moment, magnitude, and energy. Emerging from studies of this type was a generally held conviction that the stress drops of most shallow earthquakes was independent of size and generally fell in the range of 10 to 100 bars. Implicit in most of these studies was the assumption that the fault was relatively uniform in terms of material properties and energy release.

A rather different approach to the study of heterogeneity on the fault surface has emerged rather unexpectedly from studies of small repeating earthquakes along the San Andreas fault system in California. Nadeau and Johnson (1998) showed how measurements of moment release rates of these repeating earthquakes could be used to construct scaling relationships between seismic moment, repeat time, fault slip and fault dimension. Their method of interpreting the data does not require the use of any model for the earthquake process and uses the single assumption that slip on the fault at depth is closely related to the measured slip at the surface. The study of Nadeau and McEvilly (1999) provides strong support for this assumption by showing that second order variations of slip in both time and space are strongly correlated between the surface and the depth of the seismogenic zone. The basic scaling relationship obtained by Nadeau and Johnson (1998) relates repeat time of earthquakes T to scalar seismic moment according to the formula

$$T \propto M_o^{\frac{1}{6}} \tag{1}$$

where the fraction $\frac{1}{6}$ is a close approximation to the numerical value of 0.17 which was obtained by regression analysis. This result can also be interpreted in terms of scaling relationships for fault displacement u and fault area A of the form

$$u \propto M_o^{\frac{1}{6}} \tag{2}$$

and

$$A \propto M_o^{\frac{5}{6}} \tag{3}$$

When these relationships are interpreted in terms of stress drops they lead to values in the GPa range, which is not impossible for crustal rocks (Sammis et al., 1999), but much larger than expected values in the 1 to 10 MPa range. This result, together with the fact that the repeating earthquakes occupy a very small fraction of the fault and are surrounded by regions which appear to be creeping and thus quite weak, argues for a fault which is highly heterogeneous in its strength properties. Alternative interpretations have been recently presented (Anooshehpoor and Brune, 1998, 2000; Sammis and Rice, 2000; Beeler, 2000) which attempt to avoid the need for high stress drops and thus do not require the heterogeneous fault surface.

While one of the strengths of the study by Nadeau and Johnson (1998) was that it did not use a source model that presumed either a homogeneous or heterogeneous fault surface, a full interpretation of the results does require that some type of model be used. Here we introduce such a model and use this model to interpret the data in terms of physical properties and processes on the fault surface. The model introduced here begins with the assumption that strength on the fault surface is highly heterogeneous and attempts to show that such a model is consistent with the scaling relationships presented in Nadeau and Johnson (1998). Only the quasi-static part of the earthquake process is considered here, which extends up to the time that rupture begins.

Consider a planar fault surface lying in the xy plane and let the slip across this surface be denoted by

$$\mathbf{d}(x,y) = d_x(x,y)\hat{\mathbf{x}} + d_y(x,y)\hat{\mathbf{y}} \tag{4}$$

Without loss of generality we will assume that the motion on the fault is in response to a loading traction T_x that acts only in the $\hat{\mathbf{x}}$ direction. Let the slip that would occur if the fault were uniformly weak and creeping uniformly be denoted by $\mathbf{d}_c$. Then the displacement deficit on the fault can be defined as

$$\mathbf{u}(x,y) = \mathbf{d}_c - \mathbf{d}(x,y) \tag{5}$$

At points where the fault slip is keeping up with the applied traction the displacement deficit will be zero.

The fault is considered to be sufficiently weak so that it continually creeps as tectonic stress is applied except on isolated patches which we will call asperities. The asperities are small areas of the fault which are much stronger than the surrounding regions and able to withstand the tectonic stress until a stress limit is reached and rupture occurs. These asperities will be assumed to be circular with radius r_o and area $A_o = \pi r_o^2$. In the static problem being considered here we only consider the situation up to the instant just before the asperities rupture, so there will be no movement on these asperities. Thus the displacement deficit $\mathbf{u}$ will be a maximum at the asperities.

Focus now upon a region of the fault where a group of asperities are close enough together so that they strongly interact. Assume that this group of asperities is roughly in the form of a circle with radius r_a and area $A_a = \pi r_a^2$. The displacement deficit will be the same on all asperities and denoted by u_a. The asperities are assumed to be close enough together so that they prevent almost all motion on the intervening areas between the asperities, and thus the displacement deficit is very close to u_a over the entire area A_a. A solution for elastostatic problem just described can be found in Westmann (1968). The stress on the region A_a where the displacement is u_a is given by

$$\sigma_{xz}(r) = \frac{4\mu u_a}{(2-\nu)\pi}\frac{1}{\sqrt{r_a^2 - r^2}} \tag{6}$$

where $r^2 = x^2 + y^2$, μ is the shear modulus, and ν is Poisson's ratio. The stress on the weak part of the fault outside A_a is assumed to be zero in this solution. However, the displacement caused by the group of asperities does extend outside A_a and in the region $r \geq r_a$ is given by

$$u_x(r,\theta) = \frac{2u_a}{(2-\nu)\pi}[(2-\nu)\sin^{-1}(\frac{r_a}{r}) + \frac{\nu r_a}{r^2}\sqrt{r^2 - r_a^2}\cos(2\theta)] \tag{7}$$

$$u_y(r,\theta) = \frac{2u_a}{(2-\nu)\pi}\frac{\nu r_a}{r^2}\sqrt{r^2 - r_a^2}\sin(2\theta) \tag{8}$$

where θ is the polar angle in the xy plane. We see that the group of asperities is surrounded on the fault plane by a displacement shadow that decays as r^{-1} at distances much greater than r_a. Note that the first term in the expression for u_x is about 7 times larger than the second term in u_x and the only term in u_y. Thus it will be sufficient in what follows to assume that the displacement deficit within the shadow is given approximately by

$$u_x(r) = \frac{u_a r_a}{r} \tag{9}$$

Next we will assume that the displacement shadow of the group of asperities is terminated when the displacement u_x falls below a critical level denoted by u_c. One can appeal to state variable friction theories to justify the existence of such a critical level of displacement. The radius of the displacement shadow r_c is then given by

$$r_c = \frac{u_a}{u_c} r_a \tag{10}$$

The geometrical picture of the situation on the fault plane is now complete, and it is shown in Figure 1. It will be assumed that $u_a \gg u_c$ and it follows that $r_c \gg r_a$.

Were all of the asperities to rupture and all of the displacement deficit released, the scalar moment of such an event would be approximately

$$M_o = \mu[2\pi \int_0^{r_a} u_a r dr + 2\pi \int_{r_a}^{r_c} \frac{u_a r_a}{r} r dr] = \pi \mu u_a r_a (2r_c - r_a) \approx 2\pi \mu u_a r_a r_c \tag{11}$$

From the expression for stress on the asperity patch (6) it is clear that this stress in not uniform and is greatest near the edge of the patch. Thus, assuming the individual asperities are similar in their properties, one would expect the failure of the asperity patch to begin with the failure of one of those closest to border r_a. This is consistent with the results of Das and Kostrov (1983, 1986) who found that failure of an asperity began on its border and then progressed toward the center. The average stress on one of these border asperities can be evaluated to yield

$$\bar{\sigma}_{max} = \frac{1}{\pi r_o^2} \int_{A_o} \sigma_{xz}(x, y) dx dy = C \, \frac{u_a}{\sqrt{r_o r_a}} \tag{12}$$

where

$$C = \frac{32\mu}{3\pi^2(2 - \nu)} \tag{13}$$

and it has been assumed that $r_a \gg r_o$ in evaluating the integral. Note that this result is similar to that of Sammis and Rice (2000), who used it to argue for a very weak asperity. Here we use it to argue that when a group of asperities are combined together in close proximity on a fault, they interact in such way as to reduce the stress felt by any individual asperity. In effect, the displacement field is smoothed by the interaction of the asperities, thus reducing displacement gradients, strain, and stress. Assuming that $\bar{\sigma}_{max}$ is a constant that controls the initiation of rupture on the asperity patch, we have

$$u_a = \frac{\bar{\sigma}_{max}}{C} \sqrt{r_o r_a} \tag{14}$$

and then

$$r_c = \frac{\bar{\sigma}_{max}}{C u_c} r_o^{\frac{1}{2}} r_a^{\frac{3}{2}} \tag{15}$$

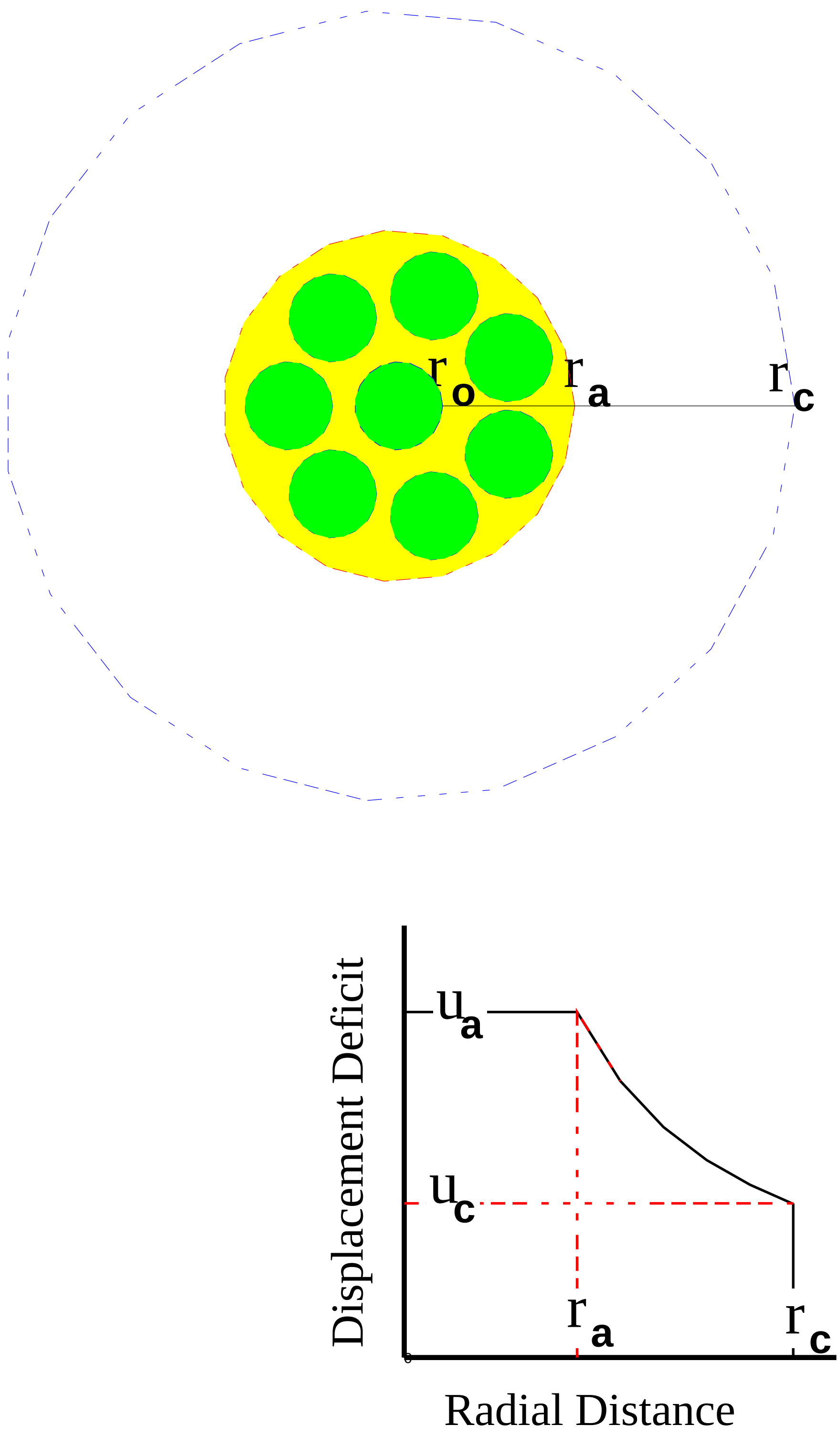

Figure 1: Schematic sketch of the earthquake model consisting of a patch of interacting strong asperities. The upper panel shows a group of asperities having radius r_o with a strong interacting region of radius r_a and surrounded by a displacement shadow that extends out to a radius r_c. The lower panel shows the disiplacement deficit as a function of radial distance from the center of the asperity patch.

The next step is to consider how the results given above change when the size of the asperity patch changes. For this it is convenient to introduce the number n, which is taken to be the number of small asperities in the asperity patch. If the asperities were to add linearly with no interaction, then one would expect that

$$A_a = \pi r_o^2 n \tag{16}$$

which would imply that

$$r_a(n) = r_o n^{\frac{1}{2}} \tag{17}$$

If the asperities were to add with some interaction between them, then we might expect a more general relationship of the form

$$r_a(n) = r_o n^{\kappa} \tag{18}$$

where $\kappa > 1/2$ but is otherwise undetermined at this point.

Now consider what happens when the number of asperities in the patch is increased by one. Then we have

$$r_a(n+1) = (n+1)^{\kappa} r_o \approx r_a(n)(1 + \frac{\kappa}{n}) \tag{19}$$

$$r_c(n+1) \approx r_c(n)(1 + \frac{3\kappa}{2n}) \tag{20}$$

$$u_a(n+1) \approx u_a(n)(1 + \frac{\kappa}{2n}) \tag{21}$$

and finally

$$M_o(n+1) \approx M_o(n)(1 + \frac{3\kappa}{n}) \tag{22}$$

From this last equation we can form the differential equation

$$\frac{d}{dn} M_o(n) = M_o(n) \frac{3\kappa}{n} \tag{23}$$

which has the solution

$$M_o(n) = M_o(1) n^{3\kappa} \tag{24}$$

The same process produces

$$r_a(n) = r_a(1) n^{\kappa} = r_o n^{\kappa} \tag{25}$$

$$r_c(n) = r_c(1) n^{\frac{3}{2}\kappa} = \frac{\hat{\sigma}_{max}}{C} r_o^2 n^{\frac{3}{2}\kappa} \tag{26}$$

$$u_a(n) = u_a(1) n^{\frac{1}{2}\kappa} = \frac{\hat{\sigma}_{max}}{C} r_o n^{\frac{1}{2}\kappa} \tag{27}$$

and from these it is possible to obtain

$$M_o(n) = 2\pi\mu \frac{\hat{\sigma}_{max}^2}{C^2 u_c} r_o^4 n^{3\kappa} \tag{28}$$

These last four equations form a consistent set of parametric equations for all of the geometrical properties of this asperity model of the earthquake process. They are all expressed in terms of the parameter n, which was introduced as the number of asperities involved, but may be considered more generally as a surrogate for the size of the earthquake.

Using the expressions listed above for $r_a(n)$ or $r_c(n)$ one can calculate the areas of the asperity patch or the displacement shadow, respectively. However, neither of these corresponds to the area that was calculated by Nadeau and Johnson (1998). If we define

$$M_o(n) = \mu \tilde{A}(n) u_a(n) \tag{29}$$

then $\tilde{A}(n)$ is an effective area that corresponds to the area calculated by Nadeau and Johnson (1998). It follows that

$$\tilde{A}(n) = A_o(1) n^{\frac{5}{2}\kappa} = 2\pi \frac{\hat{\sigma}_{max}}{C u_c} r_o^3 n^{\frac{5}{2}\kappa} \tag{30}$$

These results for displacement, area, and scalar moment can be expressed in terms of the scaling relationships

$$u_a \propto M_o^{1/6} \tag{31}$$

$$\tilde{A} \propto M_o^{5/6} \tag{32}$$

Assuming that the displacement on the asperities u_a accumulates at a steady rate in time t then we also have

$$t \propto M_o^{1/6} \tag{33}$$

This is the basic observational result presented by Nadeau and Johnson (1998). Note that all of these scaling relationships are independent of the parameter n and the parameter κ, which together control the size of the area where there is strong interaction between the asperities.

Stress drop is a parameter that is commonly used to characterize earthquakes, with conventional thought being that stress drop is independent of earthquake size and in the range of 10 to 100 bars (Kanamori and Anderson, 1975). In the asperity model presented in this paper stress is highly heterogeneous, ranging from infinity (in a mathematical sense) on the border of a single asperity to a very small value in the displacement shadow. This makes it difficult to characterize the stress with a single number, so several available options will be discussed. The only stress that is critical to the theory is the average stress that causes an asperity near the edge of the patch to fail $\bar{\sigma}_{max}$, which was assumed to be a constant material property. This can be compared with the average stress felt by a single isolated asperity

$$\bar{\sigma}_o = \frac{8\mu}{\pi(2-\nu)} \frac{u_a(1)}{r_o} \tag{34}$$

From this we have

$$\bar{\sigma}_{max} = \frac{4}{3\pi} \sqrt{\frac{r_o}{r_a(n)}} \bar{\sigma}_o = \frac{4}{3\pi} n^{-\frac{1}{2}\kappa} \bar{\sigma}_o \tag{35}$$

which shows clearly how interaction of a group of asperities can reduce the maximum stress. Note that it was assumed that $n \gg 1$ in deriving $\bar{\sigma}_{max}$ so this result does not give the correct answer as $n \rightarrow 1$.

Stress drop is commonly calculated using the formula (Kanamori and Anderson, 1975)

$$\Delta\sigma = \frac{7\pi\mu}{16} \frac{\bar{u}}{\bar{r}} \tag{36}$$

where $\bar{u}$ is the average slip over a circular crack of radius $\bar{r}$. Using $u_a(n)$ for $\bar{u}$ and $r_a(n)$ for $\bar{r}$ yields

$$\Delta\sigma_a = \frac{7\pi\mu}{16} \frac{u_a(n)}{r_a(n)} = \frac{21\pi^3(2-\nu)}{512} \hat{\sigma}_{max}\, n^{-\frac{1}{2}\kappa} \tag{37}$$

Still another possibility is to use the formula (Kanamori and Anderson, 1975)

$$\Delta\sigma = \frac{7}{16} \frac{M_o}{\bar{r}^3} \tag{38}$$

Using $r_c(n)$ for $\bar{r}$ yields

$$\Delta\sigma_c = \frac{7}{16}\frac{M_o(n)}{r_c(n)^3} = \frac{28\mu^2}{3\pi(2-\nu)}\,\frac{u_c^2}{\bar{\sigma}_{max} r_o^2}\; n^{-\frac{3}{2}\kappa} \tag{39}$$

The stress drops calculated in Nadeau and Johnson (1998) correspond to taking $u_a(n)$ for $\bar{u}$ and $\sqrt{\tilde{A}(n)/\pi}$ for $\bar{r}$ and this yields

$$\Delta\tilde{\sigma} = \frac{7\pi\mu}{16}\frac{u_a(n)}{\sqrt{\tilde{A}(n)/\pi}} = \frac{7\pi^2\sqrt{3\mu(2-\nu)}}{128}\,\sqrt{\bar{\sigma}_{max}}\; r_o^{-\frac{3}{2}}\; n^{-\frac{3}{4}\kappa} \tag{40}$$

If this result is scaled against moment we obtain

$$\Delta\tilde{\sigma} \propto M_o^{-1/4} \tag{41}$$

which agrees with the empirical result Nadeau and Johnson (1998) obtained.

CONCLUSIONS AND RECOMMENDATIONS

The model of an earthquake that has been presented here is based on the concept of small strong asperities that resist motion on the fault. When a group of these asperities are close enough together so that they can strongly interact, an asperity patch is formed which is stronger than any of the individual asperities and so a greater amount of tectonic displacement can accumulate before failure occurs. The asperity patch is surrounded by a displacement shadow where creep displacement on the weak part of the fault is retarded because of the presence of the asperity patch. The asperity patch ruptures and an earthquake occurs when the stress on one of the asperities, most likely near the border of the patch, reaches its strength limit and fails, which causes a cascade failure of all the other asperities in the patch. At this time the entire displacement deficit associated with the asperity patch, that within both the asperity patch and the surrounding displacement shadow, is released and contributes to the scalar moment of the earthquake. This model depends upon only a few parameters, the radius of the individual asperities r_o, the critical level of displacement that determines the limit of the displacement shadow u_c, the stress level at which an individual asperity will fail $\bar{\sigma}_{max}$, the number of asperities in an asperity patch n, and the exponential factor that controls how the radius of the asperity patch grows with the number of asperities κ. What is rather remakable is that the scaling relationships for displacement, area, and time to failure as a function of moment do not depend upon any of these parameters. They are direct consequences of analytic elastostatic solutions for the exterior crack problem. Another characteristic of this model is that it involves a very heterogeneous stress field that is difficult to characterize in a meaningful way with a single parameter such as stress drop.

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