# CAPTURING UNCERTAINTIES IN SOURCE-SPECIFIC STATION CORRECTIONS DERIVED FROM A 3-D MODEL

N.M. Shapiro,[1] M.H. Ritzwoller,[1] M.P. Barmin,[1] P.B. Weichman,[2] L.F. Tenorio,[3] W.C. Navidi[3]

University of Colorado at Boulder,[1] ALPHATECH Inc,[2] Colorado School of Mines[3]

Sponsored by Defense Threat Reduction Agency

Contract No. DTRA01-99-C-0019

## ABSTRACT

Uncertainties in Source-Specific Station Corrections (SSSCs) arise from several sources. If the SSSCs are constructed directly from empirical travel times, there are errors caused by origin time and hypocenter location errors, measurement errors, and phase misidentification. If a 3-D model is used either to interpolate an empirical surface or to compute the SSSC directly, then uncertainties in the 3-D model will also introduce errors in the SSSC. In order to capture the uncertainties in the SSSC that arise solely from the 3-D model, we trace rays through a large number of random model realizations and catalogue the fluctuations in the computed SSSCs. Each random realization of the 3-D model should fit the data used to construct the model and be drawn from information about the uncertainties at each point in the 3-D model, including both the vertical and horizontal correlation of the model uncertainties. We have made progress in estimating point-wise model uncertainties by performing a Monte-Carlo inversion of broad-band surface wave dispersion. The spatial correlation of uncertainties is more poorly known than the point-wise uncertainties, but we suggest that it is governed largely by the spatial correlation of the Earth itself. Therefore, with reasonable confidence we identify the correlation of the errors with the spatial correlation of the model. We describe the method used to generate the set of random models and report the first results of the application of this method to the CU-Boulder 3D model. Our results show that theoretical estimates of uncertainties are similar to those obtained using empirical information. We also demonstrate that the uncertainties in SSSCs depend strongly on the model properties, in particular the vertical velocity gradient and the horizontal correlation of the model errors.

**KEY WORDS:** seismic location, SSSCs, 3-D models, error estimates, uncertainties

## OBJECTIVES

The principal objective of this paper is to develop a method to estimate uncertainties in Source-Specific Station Corrections (SSSCs) that arise from the uncertainties in a 3-D model and to assess how the uncertainties in SSSCs depend on the vertical velocity gradient and the horizontal correlation of the model errors.

## RESEARCH ACCOMPLISHED

### Introduction

Most earthquake location methods are based on travel times calculated using 1-D velocity models. However, a 1-D model is only a very rough approximation of the real Earth. Therefore, to improve the accuracy of earthquake location, one needs more accurate predictions of the travel times. This has lead to the idea of Source-Specific Station Corrections (SSSCs). The SSSCs could be estimated directly from the observed travel times, but this approach can work only for a limited number of wave paths concentrated in seismically active regions with good station coverage. Therefore, in many cases, it is useful to have SSSCs based on travel times predicted by realistic 3-D earth models. A complete solution of the earthquake location problem includes hypocenter coordinates, an origin time, and estimates of uncertainties in all these parameters. The uncertainties characterize the quality of the reported earthquake location, and require a knowledge of uncertainties in the predicted SSSCs.

Uncertainties in SSSCs would arise both from errors in the empirical information and from errors in the seismic model. In this paper, we consider the uncertainties in the SSSCs that arise from errors in a 3-D model. One way to estimate these uncertainties is to compare model-predicted travel times with observations. In the absence of observations, theoretical methods are needed to compute uncertainties in the SSSCs. We use average empirical errors to establish certain average trends. In order to understand the geographical variability, however, the errors in the SSSCs will have to be based on model errors.

We estimate the errors in SSSCs with a simple Monte Carlo method. The basic idea is to generate a large number of random model realizations based on the statistics of the model errors. For each model realization, we compute travel times and obtain a statistical ensemble of predicted travel times, which is used to characterize the uncertainties in the SSSCs arising from the model. We consider the SSSCs calculated for the seismic phase Pn using the 3-D model of the crust and upper mantle developed at the University of Colorado (CU) 3D model). This model has been obtained by Monte Carlo inversion of broad-band surface wave dispersion. One of the main advantages of this inversion method is that it provides point-wise estimates of model uncertainties (Shapiro and Ritzwoller, 2001).

We begin the discussion with a brief description of the CU 3-D model. Then we estimate average empirical errors in SSSCs predicted by the CU 3-D model by computing the RMS misfit between the travel times predicted by the CU model and the observed Pn travel times reported in the EHB catalog (Engdahl et al., 1998). We then describe the Monte Carlo method used to estimate the theoretical errors in the SSSCs. We apply this method to calculate theoretical errors in travel times predicted by the CU 3-D model in several locations in Eurasia. Our results show that theoretical estimates of uncertainties are similar to those obtained using the average empirical information. We also demonstrate that the uncertainties in SSSCs depend strongly on the vertical velocity gradient and the horizontal correlation of the model errors.

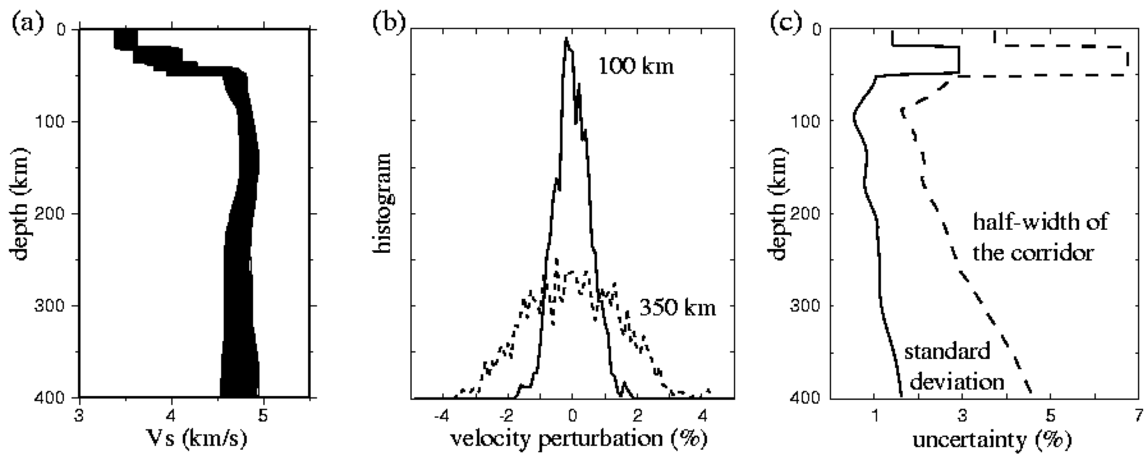


**Figure 1**. a. Ensemble of acceptable 1-D models obtained during the inversion of broad-band surface-wave dispersion data at a point in a cratonic region, the East European Platform (54N 30E). b. Histograms of velocity perturbations at two depths: 100 km (solid line) and 350 km (dashed line). c. Estimates of uncertainty obtained using the ensemble of acceptable models. Standard deviation of velocity at each depth is shown with the solid line. The half-width of the corridor of acceptable values is shown with the dashed line.

**The CU 3-D Model**

The 3-D model developed at CU is obtained from inversion of broad-band surface-wave dispersion data. The details of the measurements and the inversion have been described in previous papers (Ritzwoller and Levshin, 1998; Barmin et al., 2001; Shapiro and Ritzwoller, 2001). Here we summarize briefly the procedure of model construction emphasizing the features relevant to error analysis.

The inversion procedure is divided into two steps: (1) surface-wave tomography (e.g. Ritzwoller and Levshin, 1998; Barmin et al., 2001), which is the construction of 2-D maps characterizing the geographical distribution of surface-wave phase and group velocities, and (2) the inversion of these maps for the velocity structure of the crust and upper mantle. As described by Barmin et al. (2001), the first step (surface-wave tomography) is a nearly linear inverse problem from which we can estimate at each geographical location four dispersion curves: Rayleigh and Love phase and group velocities. In the second step, these dispersion curves at each geographical location are inverted for a local 1-D S-wave velocity model of the crust and upper mantle to depth of 400 km. The P-wave velocity model is obtained by applying a simple scaling relationship:

$$d\ln V_P = 0.5 d\ln V_S \tag{1}$$

The inversion is performed using a Monte Carlo method that provides a local ensemble of acceptable 1-D models at each spatial node. This ensemble characterizes the statistical properties of the model. Local ensembles of acceptable 1-D models obtained for points located in the East European Platform and in the Caucasus are shown in Figures 1 and 2, respectively.

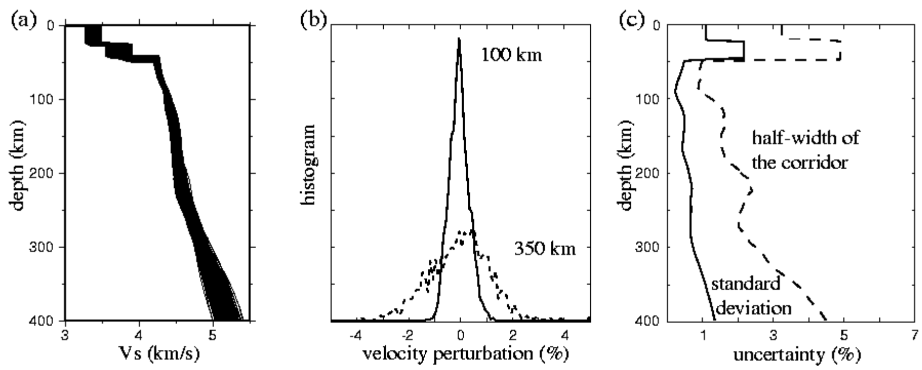


**Figure 2**. Similar to Figure 1, but for a tectonically deformed region, a point in the Caucasus (40N 44E).

The ensemble of acceptable models at each spatial node is used to define a local average model and model uncertainties. The average model is defined as the median of the velocity distribution at each depth. However, the statistical properties of this distribution are not uniquely defined but depend on multiple choices made during the inversion (i.e. parameterization, acceptance criteria, model space sampling algorithm). As a consequence, the standard deviation of the velocity at each depth underestimates the model uncertainty. A more adequate representation of this uncertainty is given by the half-width of the corridor of acceptable values. A systematic comparison of the standard deviation and the corridor half-width (Figures 1c and 2c) shows that the half-width is about three times larger. Therefore, a useful definition of the model uncertainty is to set it equal to three times the standard deviation of the ensemble of acceptable
1-D models at each depth.

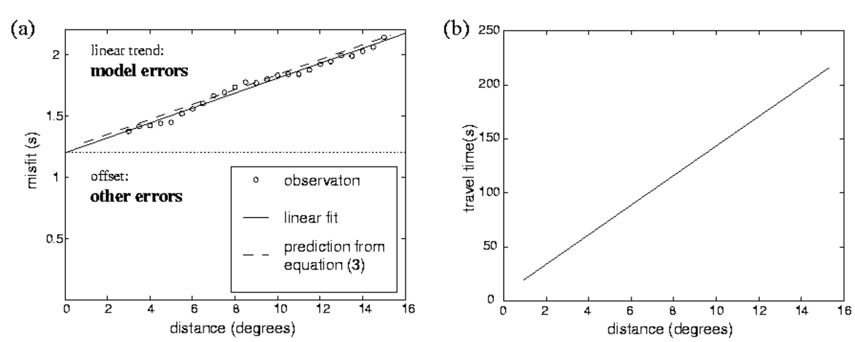


**Figure 3**. a. Average RMS misfit between Pn travel times predicted by the CU 3-D model and Pn arrival times reported in the EHB catalog (circles).

In Figure 3, the solid line shows the results of linear regression (equation 2 with $E_0 = 1.2s$ and $C_d = 0.061 s/degree$). The dashed line shows the values obtained with equation (3) and $C = 0.0044$; b. Pn travel times predicted by the IASPI91 model.

**Average empirical errors in SSSCs for reference**

In order to characterize average empirical errors in Pn-wave SSSCs computed using the CU 3-D model, we compare Pn travel times calculated using the CU 3-D model with the travel times reported in the EHB catalog (Engdahl et al., 1998). We consider distances between 3 and 15 degrees where the first arrival is normally identified as the Pn wave. Average empirical errors $E$ at different distances $\Delta$ are calculated as the RMS misfit between observed and predicted travel times. The resulting function $E(\Delta)$ is shown in Figure 3a. It can be approximated by a linear regression:

$$E(\Delta) = E_0 + C_d \qquad (2)$$

This equation shows that the average empirical error in SSSCs can be subdivided into two parts: (1) an offset $E_0 \approx 1.2s$ and (2) a linear trend with $C_d \approx 0.061\,s/degree$. We propose a simple model of the errors in which the linear trend results from the error in the model, and the offset $E_0$ is a combination of errors in travel time picking, phase misidentification, event mislocation, and errors in station corrections.

In Figure 3b, we show Pn travel times predicted by the IASPI91 model (Kennett and Engdahl, 1991). Between 3 and 15 degrees, this is a linear function of distance $t(\Delta)$. Considering this linear behavior of average travel time and equation (2), we can approximate the errors in SSSCs resulting from errors in the 3D model as a linear function of travel time $t$

$$E_m(t) = C\,t \qquad (3)$$

where $C \approx 0.0044$. This coefficient $C$ is called the "relative travel-time error". Analysis of the RMS misfit has, therefore, established that the relative travel-time error resulting from model uncertainties will be about $0.5$ percent, on average. This is a reference value that theoretical estimates should fit.

**Theoretical errors in predicted travel times: The method**

Uncertainties in a 3-D model will result in uncertainties in predicted travel times. However, the travel-time errors are not controlled by the point-wise model uncertainties alone but also depend on the spatial correlation of these uncertainties and on characteristics of the model itself. We propose to estimate the uncertainties in the SSSCs that arise from the 3-D model with a Monte Carlo approach. This approach is based on ray tracing through a large number of random model realizations followed by a statistical analysis of the fluctuations in the computed travel times. Each random realization of the 3-D model should fit the data used to construct the model and be drawn from information about the uncertainties at each point in the 3-D model, including both the vertical and horizontal correlation of the model uncertainties.

We consider here an application of this approach to the CU 3-D model. The average CU 3-D model is obtained as a combination of the local average 1-D models at all nodes on a $2^\circ \times 2^\circ$ geographical grid. This model is used to calculate the average values of predicted travel times and to construct the SSSCs. The uncertainties in the SSSCs are estimated by considering all local 1-D acceptable models found during the inversion at all locations.

Each random realization of the 3-D model is obtained by selecting one of the members of the ensemble of 1-D acceptable models at each geographical location. This approach allows us to satisfy two conditions: (1) the random model realization fits the data, and (2) the vertical correlation of velocity perturbations in the 3-D model realizations is drawn from the vertical correlation of the model errors. The condition of the horizontal error correlation is more difficult to satisfy. The main problem is that this correlation is not estimated during the inversion. We suggest that it is governed largely by the spatial correlation of the Earth itself. Therefore, with reasonable confidence, we can identify the correlation of the errors with the spatial correlation of the model. In the examples shown below, we demonstrate that taking this horizontal error correlation into account can significantly affect the estimated uncertainty in predicted travel times.

We trace rays through all random 3-D model realizations and obtain a statistical distribution of predicted travel times for all distances. From this distribution we estimate the standard deviation of the predicted travel times. However, this standard deviation underestimates the uncertainty. Following the approach established during the analysis of the ensemble of 1-D acceptable models, we define the uncertainty in predicted travel times to be three times their standard deviations.

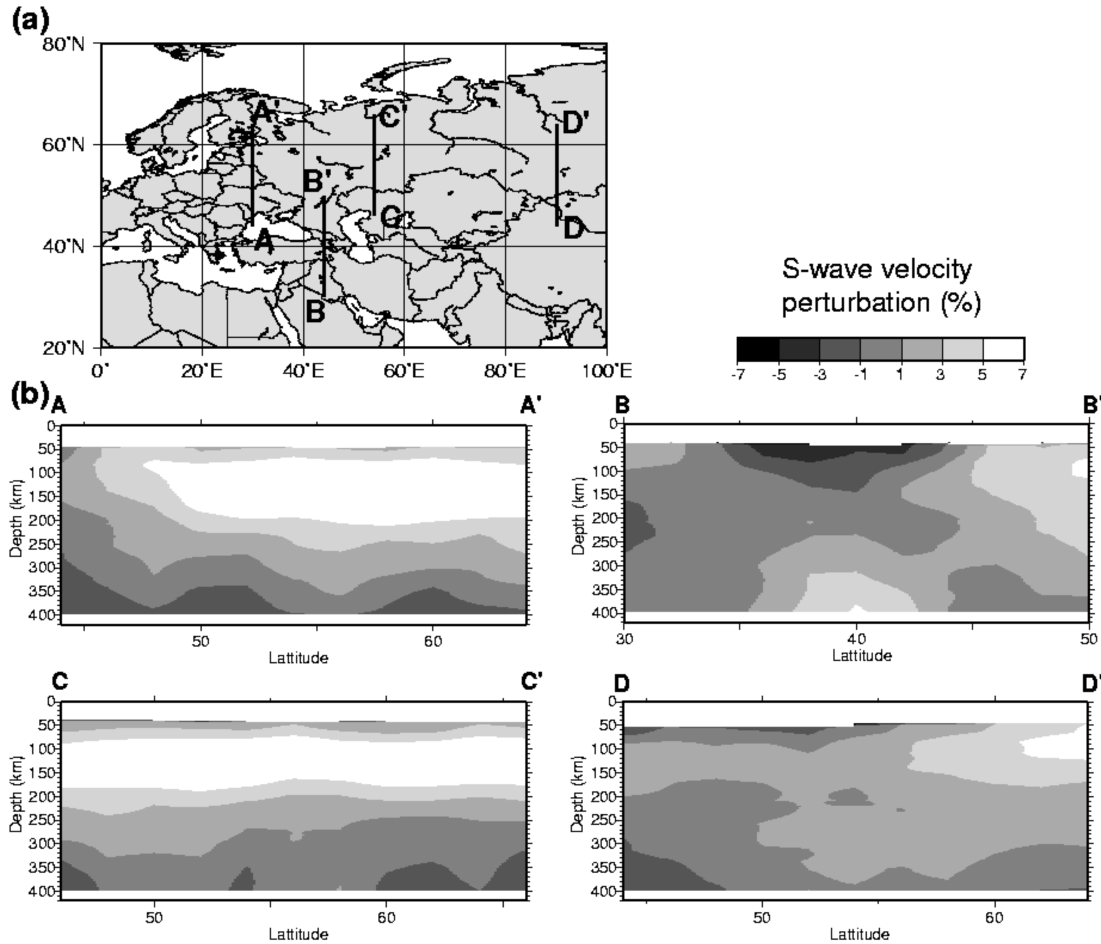


**Figure 4**. a. Map of West Central Eurasia showing the location of four profiles used in calculation of the uncertainties in model-predicted travel times. All profiles are $20^{\circ}$ in length. b. Vertical cross-sections of the S-wave velocity structure beneath the four profiles shown in (a).

**Theoretical errors in predicted travel times: Preliminary results**

We use the Monte Carlo method to calculate the errors in travel times for four 2-D profiles shown in Figure 4. Two profiles cross the East European Platform (profiles A-A' and C-C'); the other two profiles cross tectonically active regions (profile B-B' goes from Iraq across Eastern Turkey and the Caucasus, profile D-D' goes from the Junggar Basin to Central Siberia crossing the Altay and Sayan mountains). The length of each profile is set to $20^{\circ}$, a typical distance used to calculate the SSSCs for regional phases.

There are important differences in the velocity structures of these four profiles in terms of both vertical velocity gradient and horizontal correlation. The profiles crossing the East European platform are horizontally well correlated (especially profile C-C') and are characterized by a thick, high-velocity lithosphere. The horizontal correlation is weak for the two profiles crossing the tectonically active regions. The lithosphere is much less prominent beneath profile D-D' and completely disappears beneath profile B-B', which is characterized by a very prominent positive vertical velocity gradient.

We consider two end-member algorithms for generating random model realizations of 2-D profiles. In the first case, at each geographical location on a two-degree grid along each profile, we randomly select one of the members of the local ensemble of acceptable 1-D models. The local models are selected independently at each point and, therefore, the resulting 2-D model realizations are horizontally uncorrelated.

The second algorithm produces maximally horizontally correlated 2-D model realizations. To describe how this algorithm works, remember that each member of the ensemble of 1-D acceptable models is described by a simple function *c(z),* where *c* is seismic velocity and *z* is depth. At each location, we also define an average model $c_a(z)$. Each individual model can also be described in terms of perturbations relative to this average model, or in terms of errors $e(z) = c(z) - c_a(z)$. The construction of a maximally correlated 2-D model realization begins with the selection of a reference point in the middle of each profile. At this point, we randomly select a local model $e_r(z)$. Then, at all other points, the local models $e_i(z)$ are selected to minimize the one-norm difference with respect to the local model at reference point $e_r(z)$.

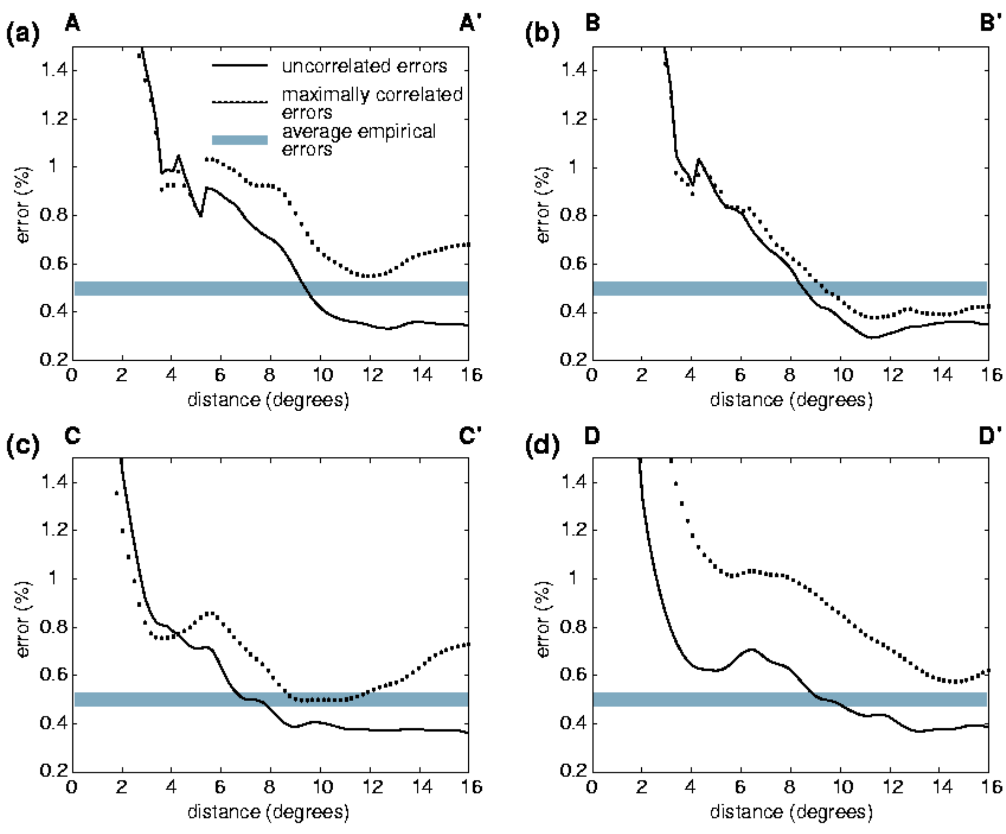


**Figure 5.** Relative errors in predicted travel times. Solid and dotted lines show the results obtained with completely uncorrelated and perfectly correlated model realizations, respectively. Gray lines show the average relative empirical errors (~0.5%). a. Results for profile A-A'. b. Results for profile B-B'. c. Results for profile C-C'. d.

Results for profile D-D'.

Using the described algorithms, we generate uncorrelated and maximally correlated ensembles of 2-D model realizations for all four profiles. Then, we trace rays through all model realizations and estimate uncertainties in Pn travel times for each of the eight ensembles. The results are shown in Figure 5.

Relative errors predicted by the spatially correlated ensembles are systematically larger than those predicted by the uncorrelated ensembles. This result is not surprising. Correlated velocity perturbations result in larger fluctuations of travel times and, as a consequence, in larger uncertainties. However, the difference between the uncertainties resulting from correlated and uncorrelated models can be very small, as it is for profile B-B'.

We conclude that the horizontal correlation of model errors affects differently the travel times calculated for different velocity models. For example, in the case of profile C-C' that crosses the cratonic area, most of the rays are concentrated in the high velocity continental lithosphere (Figure 6a) and Pn waves propagate quasi-horizontally over long distances. In this case, the horizontal correlation of the model errors significantly increases the travel-time uncertainties. Note that the lithospheric layer is also present beneath profiles A-A' and D-D'. Profile B-B' crosses the tectonically active region characterized by a prominent positive vertical velocity gradient (Figure 2a). In this case, the distribution of rays is much more homogeneous vertically. Seismic waves do not propagate quasi-horizontally, as in the case of the well-developed lithosphere, but dive to greater depths (Figure 6b). Therefore, even when there is a strong horizontal correlation of model errors, the along-ray velocity structure is not correlated and the resulting uncertainty does not increase substantially.

For all four profiles, the distance dependence of the errors in travel times can be roughly approximated by two asymptotics. At short distances ( $<4^{\circ}$ ), the travel-time errors are significantly affected by large uncertainties in the crustal structure. Therefore, the errors are large but rapidly decrease with distance. At long distances, the relative errors are approximately constant. This long-distance asymptotic behavior is similar to the behavior of the average empirical travel-time error (*C* in equation 3). Moreover, the predicted values of these relative errors are close to the observed value of 0.5%. We also note that the prediction obtained using correlated ensembles of 1-D models tends to underestimate the empirical uncertainty, while the perfectly correlated ensemble tends to overestimate it.

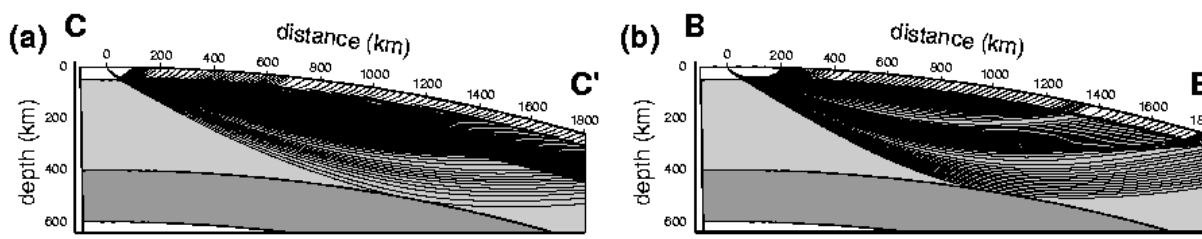


**Figure 6.** Configuration of Pn rays. The Pn phase is considered to be formed by rays with turning points in the upper mantle (light gray) above the transition zone (dark gray). The ray-tracing algorithm accounts for the sphericity of the Earth. a. Results for the average model beneath profile C-C'. b. Results for the average model beneath profile B-B'.

**Discussion**

Our results show that the horizontal correlation of the model errors can significantly increase travel-time uncertainties. We have considered here two extreme cases of the behavior of the horizontal correlation of the model errors: maximally spatially correlated and completely uncorrelated models. The spatial correlation of the model uncertainties is more poorly known than the point-wise uncertainties, but we suggest that it is governed largely by the spatial correlation of the Earth itself. Therefore, with reasonable confidence we can identify the correlation of the errors with the spatial correlation of the model. Following this hypothesis, we expect that the approximation of maximally correlated errors is more appropriate for stable cratonic areas where the structure is relatively homogeneous. In

tectonically active areas, the structure is more heterogeneous and the model errors are expected to be poorly correlated. Therefore, surprisingly, we expect the errors in predicted travel times to be larger in stable regions than in tectonically active regions.
In the future, we plan to test the effects of the error correlation more rigorously. Toward this goal, the horizontal model correlation has to be characterized quantitatively to allow us to construct new ensembles of random 3-D model realizations, where the error correlation is governed by the 3-D model correlation.

To quantify the spatial correlation or, more exactly, the similarity of the model at different spatial locations, we introduce the "similarity function" defined as the normalized one-norm difference between two local 1D models:

$$S_{ij} = \int \frac{|m_i(z) - m_j(z)|}{\sigma(z)} dz \qquad (4)$$

where $c_i(z)$ and $c_j(z)$ are the seismic velocities at locations $i$ and $j$ and $\sigma(z)$ is the average model uncertainty. The similarity function is zero when the local models at two points are identical. The difference between two models is weak if the value of $S$ is less than 1. A value of $S$ larger than 1 means that the difference between two models is significant.

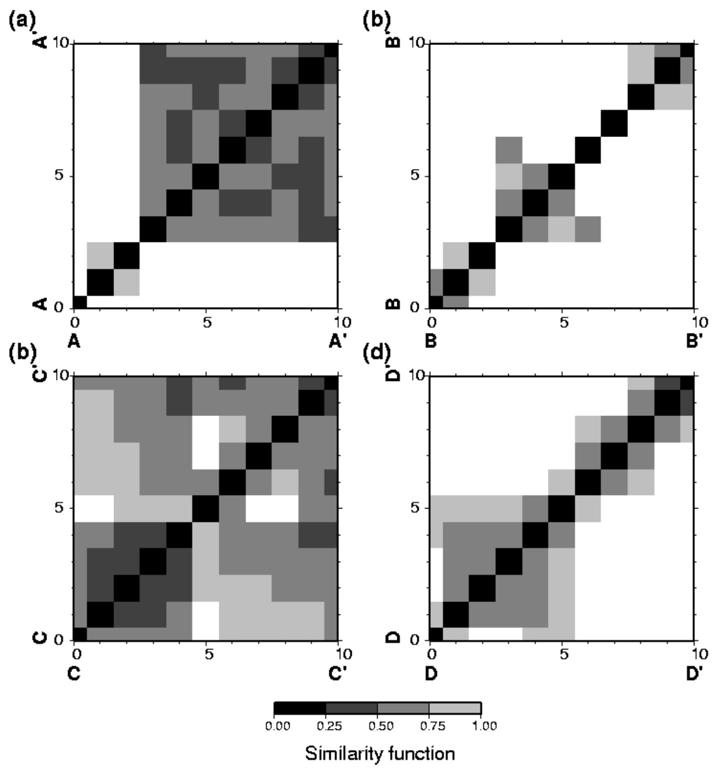


**Figure 7.** Similarity matrices calculated for the profiles shown in Figure 4. *Similarity function* is defined by equation 4. Dark shades denote high similarity between the 1-D models at the corresponding spatial locations.

The CU 3-D model is defined on a $2^{\circ} \times 2^{\circ}$ grid. Therefore, each of the 20-degree length profiles shown in Figure 4 is fully described by 11 local 1-D models. For each 2-D profile, the values of the similarity function calculated between the 11 points form a 11x11 "similarity matrix". The similarity matrices computed for the four 2-D profiles are shown in Figure 7. For profiles A-A' and C-C' the spatial correlation is high and most of the off-diagonal elements of the similarity matrices are less than 1. For the weakly correlated profiles B-B' and D-D', most of the off-diagonal elements are larger than 1.

In the future, we will use these similarity matrices to construct random model realizations with realistic spatial correlation. The main idea is to modify the algorithm used to produce the random model realizations so that the model errors will be governed by the similarity matrices.

## CONCLUSIONS AND RECOMMENDATIONS

We presented the first estimates of errors in SSSCs that arise from the errors in 3-D velocity models. This work remains in its early stages. However, several important results are clear:

- We developed a Monte Carlo method to estimate errors in SSSCs that arise from the errors in 3-D velocity models.
- The estimated theoretical travel-time errors are similar to the average empirical travel-time errors.
- Strong horizontal correlation of the model errors increases the errors in the SSSCs.

The errors in SSSCs depend strongly on the vertical velocity gradient in the model.

In the future, we plan to improve the algorithm for generating of 3-D model realizations by using a more realistic estimate of the spatial correlation of the model errors based on the correlation of the model itself.

## REFERENCES

Barmin, M.P., M.H. Ritzwoller, and A.L. Levshin (2000), A fast and reliable method for surface wave tomography, *Pure and Appl. Geophys.*, in press.

Engdahl, E.R., R. van der Hilst, and R. Buland (1998), Global teleseismic earthquake relocation with improved travel times and procedures for depth determination, *Bull. Seism. Soc. Am.*, **88**, 722 - 743.

Kennett, B.L.N. and E.R. Engdahl (1991), Travel times for global earthquake location and phase identification, *Geophys. J. Int.*, **105**, 469 - 465..

Ritzwoller, M.H., and A.L. Levshin (1998), Eurasian surface wave tomography: Group velocities, *J. Geophys. Res.*, **103**, 4839 - 4878.

Shapiro, N.M. and M.H. Ritzwoller (2001), Monte-Carlo inversion of broad-band surface wave dispersion for a global shear velocity model of the crust and upper mantle, *Geophys. J. Int.*, submitted.