BAYESIAN TOMOGRAPHY APPLIED TO SEISMIC EVENT IDENTIFICATION PROBLEMS

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ABSTRACT

The advantages of a Bayesian approach to tomography are that large-scale and high-resolution tomographic models available from other well-accepted studies can be used as prior background models. The resulting refined tomographic model blends into these prior background models. Moreover, the error budget is well established in a Bayesian framework. We assume a general linear Gaussian (least squares) model, where the covariance matrix is partitioned into data and prior model components. Uncertain data are naturally down weighted and certain model components will be subject to small perturbations.

Bayesian attenuation tomography (Tarantola, 1987) is being incorporated into the Magnitude and Distance Amplitude Correction (MDAC) methodology used for amplitude corrections to seismic discriminants (Taylor *et al.,* 1999). Because a signal-to-noise criterion is used to select amplitudes for the inversion, the data are left-censored and the resulting Q_0 models will be biased high. We examine the utility of the Expectation Maximization (EM) algorithm to the left-censored data problem (Dempster *et al.*, 1977). In this case, we can define only an upper bound to measured amplitudes based on pre-phase noise. The EM algorithm can be used to fill in the missing data values with their conditional expectation based on the relationship of amplitudes with other related completely observed variables. We have initially chosen a bivariate Gaussian model for filling in missing amplitudes based on the relationship between amplitude and pre-phase noise. The methodology can be extended to the multivariate case by the incorporation of other variables such as magnitude and distance, background noise, and Pn velocity tomography.

We apply the technique to eastern Asia for Pn, Pg, Sn, and Lg signals at 1 Hz using data from 1651 earthquakes recorded at 12 stations. Tomographic patterns correlate well with those expected from geophysical considerations (e.g. high attenuation in Tibet and low attenuation up into the stable regions of Kazakhstan). Interestingly, many of the large cratonic basins surrounding the Tibet Plateau (e.g. Tarim, Junggar) show reduced attenuation. Application of the EM algorithm tends to increase attenuation in Tibet (particularly western Tibet) and the cratonic basins to the north of but not to the east of Tibet.

We are refining regionalized surface slowness models in a number of regions for surface wave magnitude measurements to be used for m_b — M_s discriminants. Regional M_S can be measured at lower magnitudes through the application of phase-match filters. The filters can be constructed from tomographic group velocity maps from which path-specific dispersion curves can be calculated. Global and regional group velocity maps are available from previous studies (e.g., Stevens and Adams, 2000; Levsin et al., 2000) and Bayesian slowness tomography provides a framework for utilizing existing global and regional models as priors on which to improve resolution in specific regions.

Tomography provides only an approximation to the propagation effects a seismic wave may experience. For example, phase blockage can occur over relatively short distances in the absence of any anelastic effects. Station-centric kriged amplitude or slowness correction surfaces on top of the tomographic models may assist in quantifying departures from the smooth tomographic model (Pasyanos, 2000).

KEY WORDS: seismic, attenuation tomography, Bayesian, surface waves, magnitude, discrimination

OBJECTIVE

Our work to date involving MDAC (Magnitude and Distance Amplitude Corrections; Taylor and Hartse, 1998; Taylor *et al.,* 1999) corrections to seismic discriminants has combined simplified one-dimensional propagation models with kriged surface corrections. As pointed out by Taylor and Hartse (1998) the amplitude residuals to the 1D propagation models can be spatially averaged in some way to account for deviations (*e.g.* unmodeled lateral variations in Q and/or geometrical spreading). Bayesian kriging (Schultz *et al.,* 1998; Phillips, 1999; Rodgers *et al.,* 1999) provides a useful way for computing amplitude correction surfaces and propagating errors. However, kriging is a purely empirical approach and provides little or no information in data-poor regions. Thus, it is necessary to refine the 1D models to more realistic, physically based, spatially varying propagation models. One approach we discuss in this report is the incorporation of phase amplitude attenuation tomography into the MDAC procedure.

Attenuation tomography is a well-established method that has been used for many years (e.g. Young and Ward, 1980; Singh and Herrmann, 1983; Mitchell *et al.,* 1997). Phillips *et al.,* (1999) have recently investigated the utility of attenuation tomography for correcting seismic discriminants using 1 Hz Pg/Lg amplitude ratios. In subsequent work, we have tested 1 Hz Lg amplitude tomography (Phillips *et al.,* 2000) with promising results. In this report, we discuss an extension of the amplitude tomography method using a Bayesian framework (Tarantola, 1987). As will be discussed further in the methodology section, Bayesian methods are more general than standard least squares methods. An advantage of the Bayesian approach to tomography is that the posterior model will blend into the prior background model. The prior model can be a well-accepted global model that can contain constraints from detailed studies in specific regions. For example Xie (pers. comm., 2001), included results from PASSCAL experiments within Tibet that suggest attenuation within the plateau is greater than that calculated using data crossing the entire plateau (which may be due to a data censoring problem discussed below).

Importantly, the error budget is very well defined in a Bayesian approach. The covariance matrix is partitioned into data and prior model components. Uncertain data are naturally down weighted and certain model components will be subject to small perturbations. For example, matrix-conditioning problems are handled by inclusion of a prior model and associated uncertainties in the model and data (as opposed the relying on the selection of a poorly constrained ridge or damping parameter).

Because a S/N criterion is used to select paths used in the inversion, the data are left-censored. This means that the points below a certain S/N noise threshold are eliminated from the inversion causing the resulting Q_0 values to be biased high. The data censoring may be due to effects such as phase blockage, phase conversion (e.g. Lg to Sn scattering in a basin) amplitudes below detection thresholds from magnitude and distance effects, or transmission through high attenuation paths. In this study, we examine the utility of the Expectation Maximization (EM) algorithm to the left-censored data problem (Dempster *et al.*, 1977). In this case, we can only define an upper bound to measured amplitudes based on pre-phase noise. The EM algorithm can be use to fill in the missing data values with their conditional expectation based on the relationship of amplitudes with other related completely-observed variables.

In this report, we first describe the Bayesian approach to attenuation tomography where the EM algorithm is used to fill in missing amplitudes with their conditional expectations based on pre-phase noise measurements. We then discuss application to regional phases in central Asia.

RESEARCH ACCOMPLISHED

Bayesian Tomography

The tomographic equations to be solved are given by

$$D_{ij} = S_i + P_j - k\sum_m \alpha_m r_{ijm} \qquad (1)$$

where $D_{ij} = A_{ij} — G(,r_0)$ is the amplitude from source *i* to receiver *j* corrected for geometrical spreading, *k* is a constant, α_m is the attenuation coefficient and r_{ijm} is the normalized chord length in cell *m* used to parameterize the imaged area. Equation (1) forms a linear system of equations that can be solved for the attenuation coefficients, source, and site terms. To solve the system of equations (1) we use the Bayesian approach of Tarantola (1984).

Application to Eastern Asia

The Baysian tomographic inversion was applied to a data set of earthquakes and explosions in eastern Asia (Figure 1). RMS measurements from regional seismograms were computed using the processing described in Hartse *et al.,* (1997). We converted the RMS measurements to pseudo displacement spectra using a technique described in Taylor *et al.,* (2001) based on Parseval s Theorem. We preformed the inversion on Pn, Pg, Sn, and Lg signals at 1 Hz and will discuss the Lg results in this report. Data were selected from 12 stations AAK, BRVK, ENH, KURK, LSA, LZH, MAKZ, NIL, TLY, ULN, WMQ, and XAN.

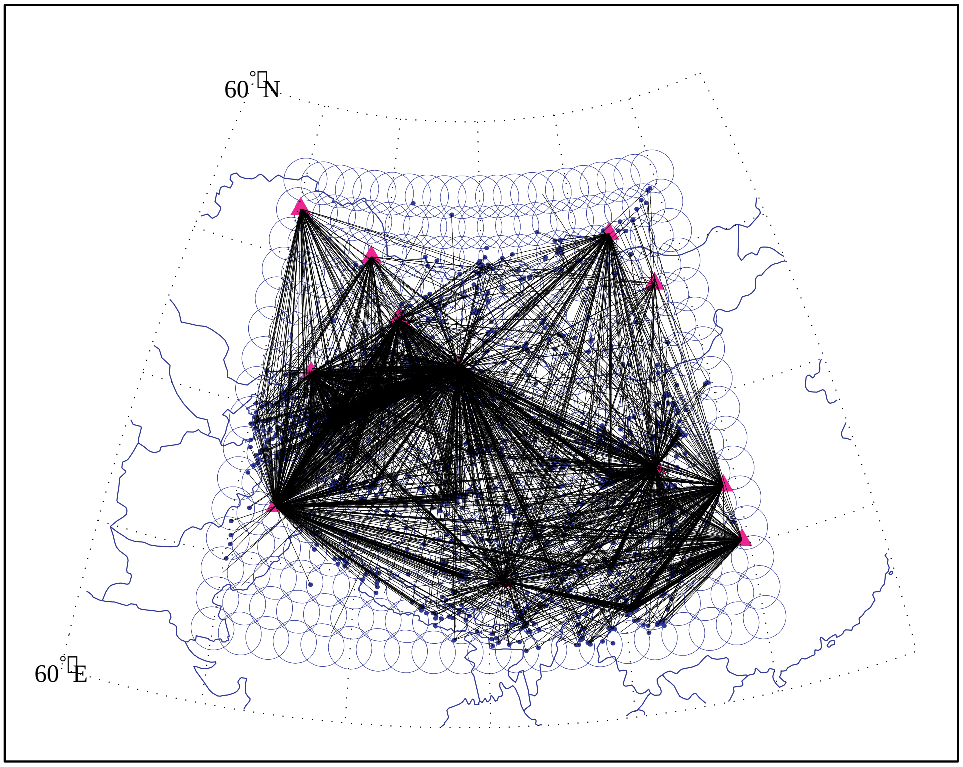

Figure 1. Map showing event-station paths and grid used in tomographic inversion.

Using pre-phase noise, we included spectral values having a signal-to-noise (S/N) ratio greater than 0. This resulted in a total number of 2855 paths from 1651 events shown in Figure 1. The region is parameterized into a grid of small circles evenly spaced at 2-degree increments. To insure complete coverage, the radius of each small circle is given by the grid spacing times $\sqrt{2}/2$. Based on our work in Phillips *et al.,* (2000) we used a 2° grid spacing.

We used the 3° Eurasia Lg coda Q model and associated errors of Mitchell *et al.,* (1997) as the prior attenuation and covariance model. The priors for the site terms, P_j, were set to zero. The prior errors on the site terms can be used as a way to damp the solution. For example, the prior errors on the site terms can be set to zero to fix them. For the source terms, it would be most desirable to have regional seismic moments for each event. Because the moments were not available, we used a regional moment-magnitude scale for Pn of Patton (1999).

As with the site terms, prior errors can be specified for the source terms. Setting the errors to zero will tie directly to m_b. A direct tie to m_b in regional attenuation studies (e.g. Phillips *et al.,* 2000) is undesirable because upper mantle bias may be mapped into the tomographic models. By specifying source term errors, we can allow the source terms to adjust which can absorb some of the problems associated with m_b bias and source scaling. Unaccounted source scaling (e.g. corner frequency) effects can be absorbed in the source terms without having to specify a given source model or scaling relationship. The data errors are used to control the importance (weighting) of each data value. Initially, we have chosen an ad hoc approach to weight the data based on S/N. A more rigorous statistical formulation will be used in future work.

The tomographic map for Lg attenuation is shown in Figure 2. We obtained a variance reduction of 27% over the prior model. The model is similar in many respects to that shown in Phillips *et al.,* (2000). The main differences are on the edges of the model where the Bayesian tomography blends into the background model of Mitchell *et al.,* (1997; Figure 3). As pointed out by Phillips *et al.,* (2000), the tomographic patterns correlate well with that expected from geophysical considerations (e.g. high attenuation in Tibet and low attenuation up into the stable regions of Kazakhstan). Interestingly, many of the large cratonic basins surrounding the Tibet Plateau (e.g. Tarim, Juggar) show reduced attenuation. As discussed by Whitted *et al.,* (1999), it may be that some of these large basins are thought to represent regions of stronger lithosphere relatively undeformed by the Indo-Asian continental collision and thus are regions of lower attenuation.

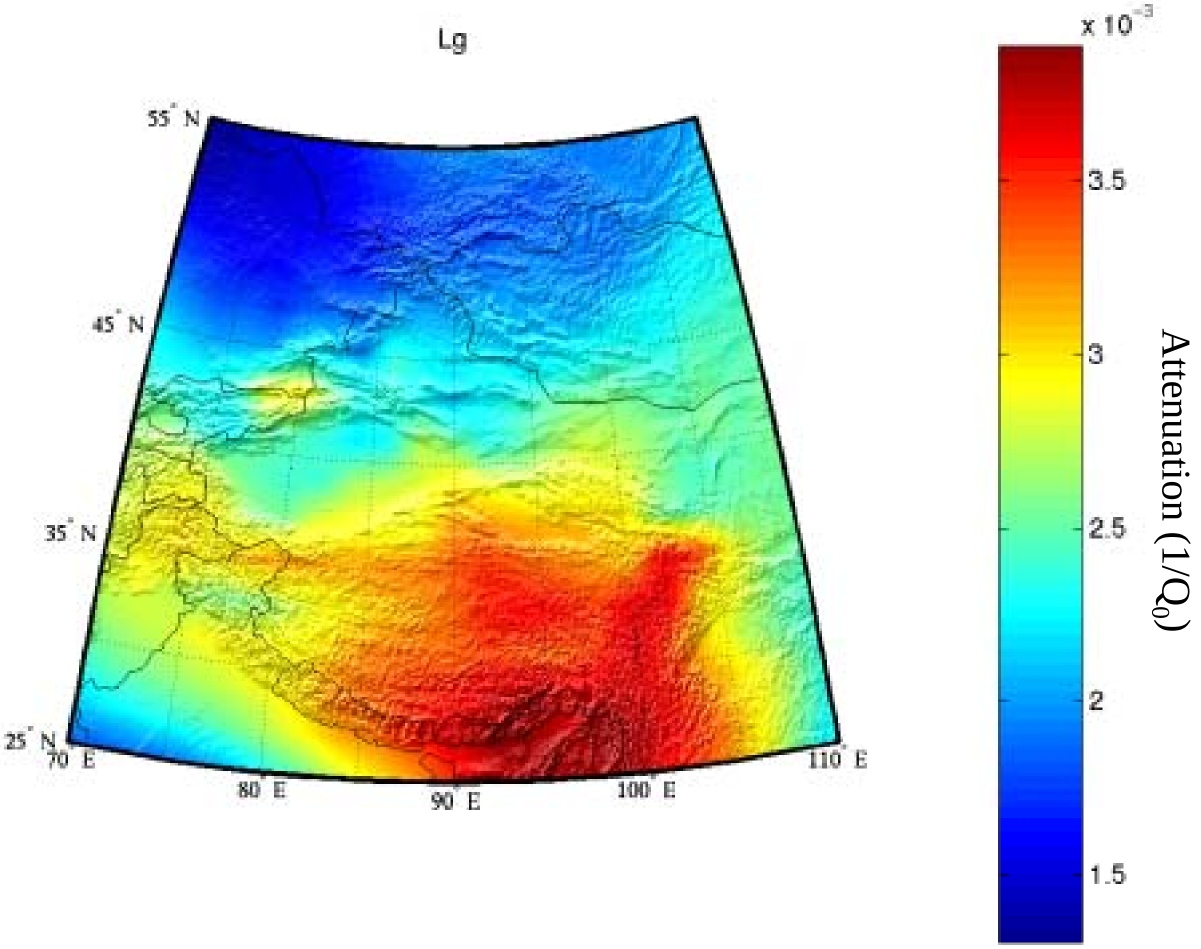

Figure 2. Attenuation tomography map for Lg. Red indicates high attenuation and blue low attenuation.

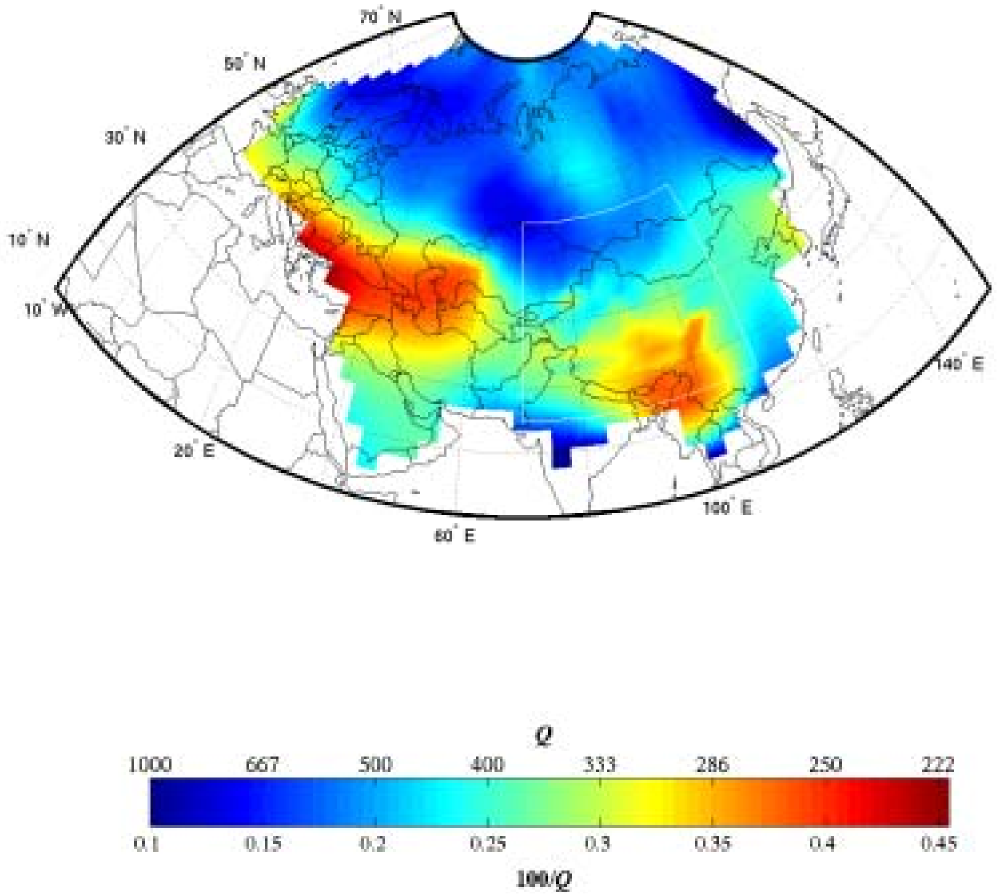

Figure 3. Two degree attenuation tomography map for 1 Hz Lg of Figure 2 (white outline) blending into 3 degree Lg coda Q map of Mitchell *et al.* (1997).

Figure 4 shows Kriged tomographic amplitude residuals at station WMQ using the Bayesian kriging algorithm of Schultz *et al.* (1998). These maps can be used as station-centric amplitude correction surfaces to the tomographic model and can be used to identify phase blockages. A number of interesting features can be observed from these maps. For example, the strong effects of the Tibetan Plateau on Lg can be observed. Also, a region of enhanced Lg amplitudes is observed to the east of the Tsaidam Basin.

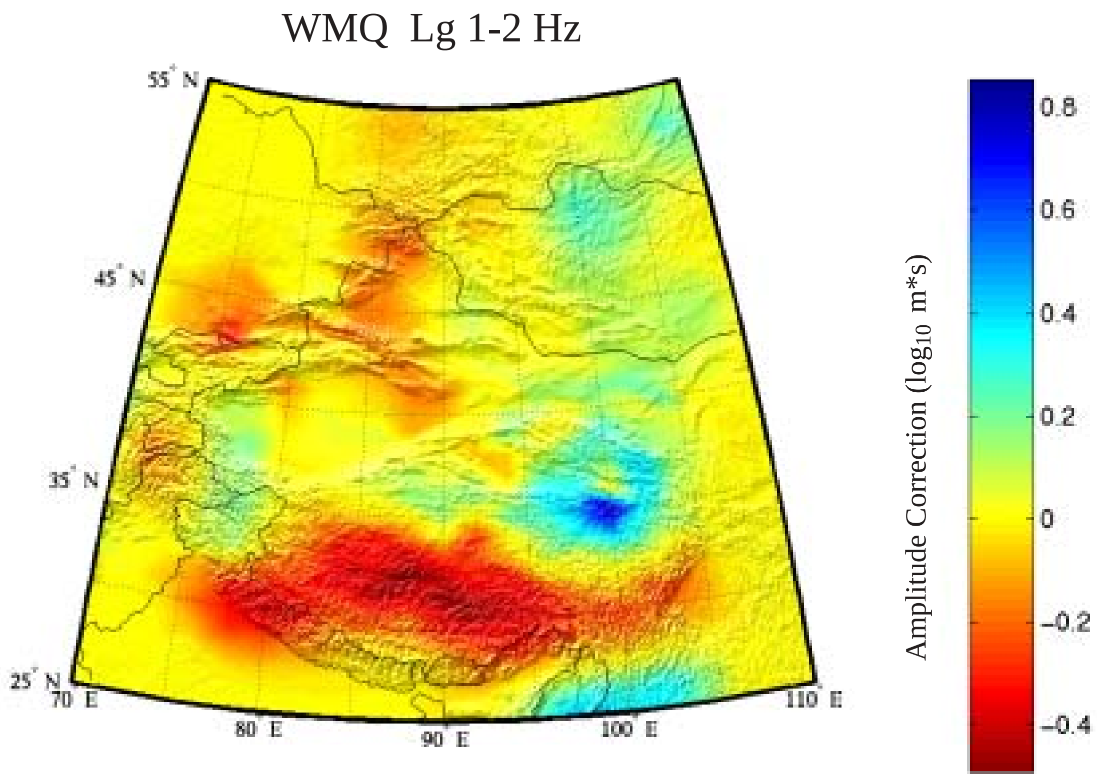

Figure 4. Kriged tomographic residuals for WMQ.

Application of the Expectation Maximization Algorithm to Left-Censored Data

Because a S/N criterion is used to select paths used in the inversion, the data are left-censored. This means that the points below a certain S/N noise threshold are eliminated from the inversion causing the resulting Q_0 values to be biased high. The data censoring may be due to effects such as phase blockage, phase conversion (e.g. Lg to Sn scattering in a basin) amplitudes below detection thresholds from magnitude and distance effects, or transmission through high attenuation paths. In this study, we examine the utility of the Expectation Maximization (EM) algorithm to the left-censored data problem (Dempster *et al.*, 1977). In this case, we can only define an upper bound to measured amplitudes based on pre-phase noise. The EM algorithm can be use to fill in the missing data values with their conditional expectation based on the relationship of amplitudes with other related completely-observed variables. In this example we choose a bivariate Gaussian model for filling in missing amplitudes based on the relationship between amplitude and pre-phase noise. The methodology can be extended to the multivariate case by the incorporation of other variables such as magnitude and distance, background noise, and Pn velocity tomography. An advantage of EM is that the an estimate of the censored data and associated errors can be made prior to the inversion, so that the Bayesian methodology discussed above need not change.

Figure 5 shows the difference in Q_0 for a tomographic inversion with an EM correction and for one without (Figure 2). The Q_0 values are observed to decrease in Tibet and particularly in western Tibet. Also, the Q_0 values decrease in the Tarim Basin while they are relatively unchanged in the Dzungarian Basin and in the large basins to the east of Tibet (such as the Tsaidam and Szechwan Basins).

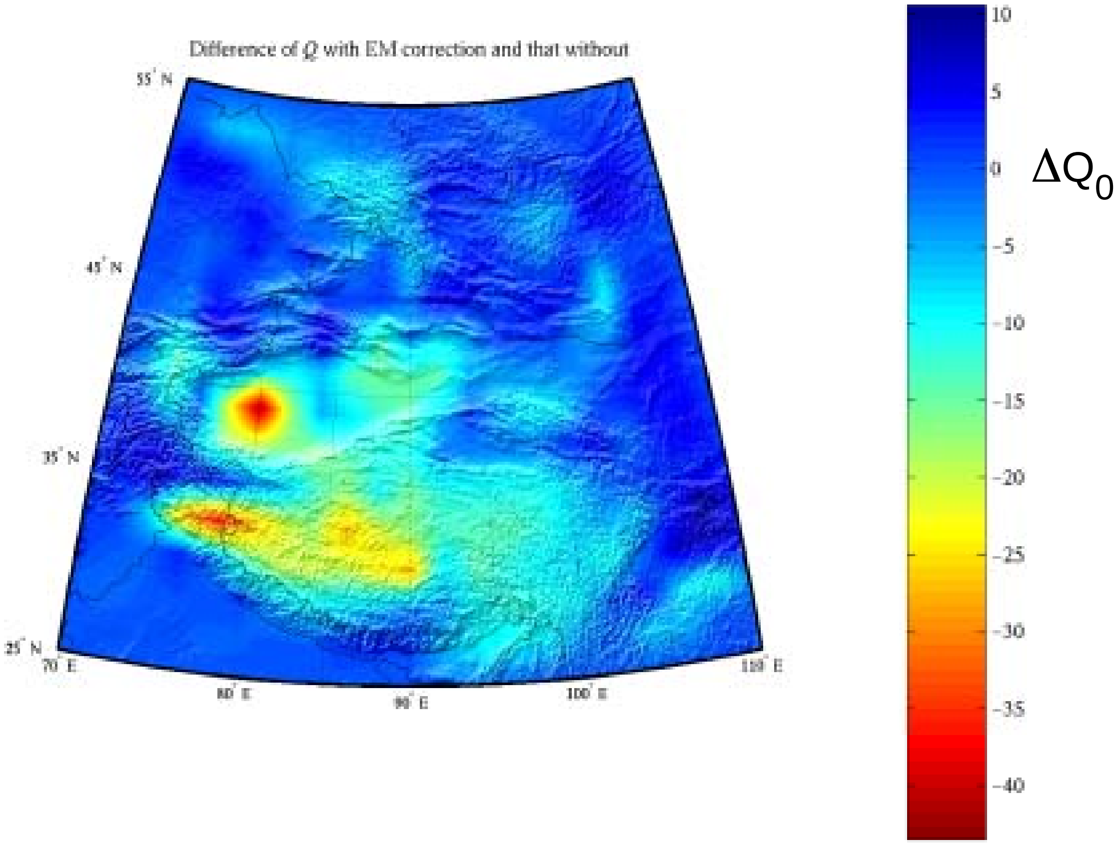

Figure 5. Difference in Q_0 between tomography with EM correction and without (Figure 2).

CONCLUSIONS AND RECOMMENDATIONS

We have begun examining the utility of Bayesian tomography to regional phase attenuation and surface slowness models. The Bayesian approach allows for refinement of available prior models in selected regions. The refined models smoothly blend into the background models. The Bayesian approach also tracks model and data errors very efficiently. We are also investigating the utility of the Expectation-Maximization algorithm for handling censored data in an effort to reduce bias in tomographic models.

REFERENCES

Dempster, A.P., N.M. Laird, and D.B. Rubin, Maximum likelihood from incomplete data via the EM algorithm, *J. Roy. Statist. Soc.,* B39, 1-38, 1977.

Hartse, H.E., S.R. Taylor, W.S. Phillips, and G.E. Randall, Regional event discrimination in central Asia with emphasis on western China, *Bull. Seism. Soc. Am.*, 87, 551-568, 1997.

Levsin, A. L., M. H. Ritzwoller, M. P. Barmin, and J. L. Stevens, Assessing abilities to estimate short period group velocities, in *Proceedings of the 22nd Annual DoD/DOE Seismic Research Symposium*, vol. 1, 67-76, 2000.

Mitchell, B.J., Y.Pan, J. Xie, and L. Cong, Lg coda Q variation across Eurasia and its relation to crustal evolution, *J. Geophys. Res.,* 102, 22,767-22,779, 1997.

Patton, H.J., Regional magnitude scaling, transportabiliy, and Ms:mb discrimination at small magnitudes, Los Alamos National Laboratory, LAUR-99-6705, submitted to PAGEOPH, 1999.

Phillips, W.S., G.E. Randall, and S.R. Taylor, Regional phase path effects in central China*, Geophys. Res. Lett*., 25, 2729-2732, 1998.

Phillips, W.S., Empirical path corrections for regional-phase amplitudes, *Bull. Seism. Soc. Am.,* 89, 384-393, 1999.

Phillips, W.S, H.E. Hartse, S.R. Taylor, A.A. Velasco, and G. E. Randall, Regional phase amplitude ratio tomography for seismic verification, Los Alamos National Laboratory, LAUR-99-1904, submitted to PAGEOPH, 1999.

Phillips, W.S, H.E. Hartse, S.R. Taylor, 1 Hz Lg Q tomography in central Asia, *Geophys. Res. Lett.,* 20, 3425-3428, 2000.

Rodgers, A.J., T. Lay, W.R. Walter, and K.M. Mayeda, Comparison of regional phase amplitude ratio measurement techniques, *Bull. Seis. Soc. Am*., 87, 1613-1621, 1999.

Rodgers, A.J., W.R. Walter, C.A. Schultz, S.C. Myers, and T. Lay, A comparison of methodologies for representing path effects on regional P/S discriminants, *Bull. Seism. Soc. Am.,* 89, 394-408, 1999.

Schultz, C.A., S.C. Myers, J. Hipp, and C. Young, Nonstationary Bayesian Kriging: A predictive technique to generate spatial corrections for seismic detection, location, and identification, *Bull. Seism. Soc. Am.,* 1275-1288, 1998.

Stevens, J. L, and D. A. Adams, Improved surface wave detection and measurement using phase-match filtering and improved regionalized models, in *Proceedings of the 22nd Annual DoD/DOE Seismic Research Symposium*, vol. 1, 145-154, 2000.

Tarantola, A., Inverse Problem Theory, Elsevier Science, Amsterdam, The Netherlands, 1987.

Taylor, S.R., and H.E. Hartse, An evaluation of generalized likelihood ratio outlier detection to identification of seismic events in western China, *Bull. Seis. Soc. Am*., 87, 824-831, 1997.

Taylor, S.R., and H.E. Hartse, A procedure for estimation of source and propagation amplitude corrections for regional seismic discriminants, *J. Geophys. Res.,* 103, 2781-2789, 1998.

Taylor, S.R., A.A. Velasco, H.E. Hartse, W. S. Phillips, W.R. Walter, and A.J. Rodgers, Amplitude corrections for regional seismic discriminants, *Los Alamos National Laboratory, Los Alamos, NM*, LAUR-99-3040, accepted PAGEOPH, 1999.

Xie, J. and H.J. Patton, Regional phase excitation and propagation in the Lop Nor region of central Asia and implications for P/Lg discriminants, *J. Geophys. Res.,* 104, 941-954, 1999.