DETECTION AND LOCATION CAPABILITIES OF MULTIPLE INFRASOUND ARRAYS

Robert H. Shumway

University of California, Davis

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ABSTRACT

In the first phase of this contract, we have developed an integrated approach to using wavenumber parameters and their covariance properties from a collection of local arrays for estimating location, along with an uncertainty ellipse. Hypothetical wavenumber estimators and their uncertainties are used as input to a Bayesian nonlinear regression that produces fusion ellipses for event locations using probable configurations of detecting stations in the proposed global infrasound array. The network capability is characterized as a function of separate local-array characteristics, including signal-to-noise ratios, bandwidth, array geometry, local correlation and coherent interfering signals. A summary map displays the average areas of the 90% posterior probability ellipses for each hypothetical location, assuming a random configuration of detecting stations.

In the second phase of the project, we are developing local-array parameters that will be used as input for estimating the capabilities of the global International Monitoring System (IMS). A small-array theory has been given in previous work that characterizes the detection probabilities and large sample variances of the local-array optimal maximum likelihood detectors. We are working on assessing the local-array performance of the multiple signal F-statistic as well as those of alternative high resolution detectors produced by Capon (1969) and the multiple signal classification (MUSIC) algorithm proposed by Schmidt (1979). For pure and mixed infrasound signals from two explosions, we find that all statistics have comparable resolution. The F-statistic retains a number of theoretical advantages, namely (1) a known large sample distribution that yields detection and false alarm probabilities (2) direction of arrival (DOA) estimators with means and covariance matrix determined by the Cramer-Rao lower bound and (3) easily estimated signal-to-noise ratios. Hence, the maximum likelihood procedure produces the input necessary for evaluating the global location performance of the IMS.

With the above in mind, current efforts are focused on analyzing events in the infrasound database located at the Center for Monitoring Research. We are planning to develop from the CMR R&D Test Bed Infrasound Waveform Library information on detection probabilities and estimated variances for real IMS type arrays as a function of bandwidth, signal-to-noise and geometry. These characteristics are needed for input into the multiple station global location simulation.

KEY WORDS: maximum likelihood, Bayes, multiple signal estimation and detection, nonlinear regression, F-Statistics, high resolution, MUSIC

OBJECTIVE

Monitoring explosive events using a collection of small infrasound arrays can lead to improved detection performance and to predictive uncertainty regions for the location of an explosive event. This project seeks to determine the detection performance of small infrasound arrays and to estimate the variances associated with the estimated wavenumber parameters that are directly related to velocity and azimuth. These characteristics are then used as input to a nonlinear regression program for location and will determine the overall uncertainty ellipse for locating a given event. In the early phase of this project, we have developed a methodology for integrating wavenumber estimators from detecting local arrays into an overall estimator for location and its associated posterior probability region. Global capability is expressed as a contour map showing areas of 90% posterior probability ellipses as a function of the expected configuration of detecting stations (see, for example, Shumway, 2000).The expected configuration is based on the signal detection capabilities of the given collection of sub-arrays and the variance properties of the maximum likelihood estimators of velocity and azimuth for each sub-array. The contour map is based on simulations involving 500 hypothetical events originating from each point on a 5-degree grid covering the surface of the earth. Each event is detected at a random configuration of International Monitoring Stations (IMS).

In our current research efforts, we are focusing on the problems involved in evaluating the local sub-array input parameters that determine location uncertainty and the average areas mentioned above. The characteristics of interest are sub-array detection probabilities and estimators for the variance of the wavenumber estimates corresponding to the velocity and azimuth of the propagating signal. In previous reports and papers (Shumway et al, 199, Shumway 1999 and Shumway, 2000), we have characterized small array performance for the optimum single signal detector in terms of bandwidth, signal-to-noise ratio, array configuration, and signal decorrelation. We are currently applying this technique to a number of events contained in the infrasound database located at the Center for Monitoring Research, referred to in the sequel as the CMR R&D Test Bed Infrasound Waveform Library. The database contains earthquakes, explosions, bolides and missile launches. There is a meteorite recorded at six IMS type stations that could provide useful input parameters for evaluating global location capabilities.

A second objective of this next phase is investigate the effects of interfering signals and (or) correlated noise on the conventional F detector, which is based on the assumptions that signals are perfectly correlated and that noise is spatially white. We also consider the performance of alternative estimators that are advertised to be effective in a multiple signal context. Among these are the single and multiple signal F detectors of Shumway (1983), the estimator of Capon (1969) and the MUSIC estimator proposed by Schmidt (1979). We evaluate these alternative estimators here on a mixture of two signals constructed by adding the Small Fry nuclear explosion, contrived to arrive at 225 degrees to an interfering signal arriving at 135 degrees. Figure 1 shows the primary event in the left hand column and the interfering invent in the center, with the sum (SF+.5GS) shown in the last column. The event is assumed to be observed at the three component triangular array with approximately 1 km sides, as described by Shumway et al (1998).It can be seen that the mixture obscures the primary signal quite effectively.

RESEARCH ACCOMPLISHED

In order to investigate possible approaches to evaluating the effect of contaminating signals such as the one in Figure 1, we consider frequency domain version of single and multiple station signal detectors with velocity and azimuth parameterized by a probe vector at wavenumber $\boldsymbol{\theta} = (\theta_1, \theta_2)'$, i.e.,

$$\boldsymbol{x}(\boldsymbol{\theta}) = \left(e^{2\pi \boldsymbol{r}_1' \boldsymbol{\theta}}, \ldots, e^{2\pi \boldsymbol{r}_N' \boldsymbol{\theta}}\right)', \tag{1}$$

where the array coordinates in km, relative to a center sensor at the origin, are denoted by $\boldsymbol{r}_j = (rj1, rj2, j = 1, \ldots, N$; in Figure 1, there are $N = 3$ sensors.

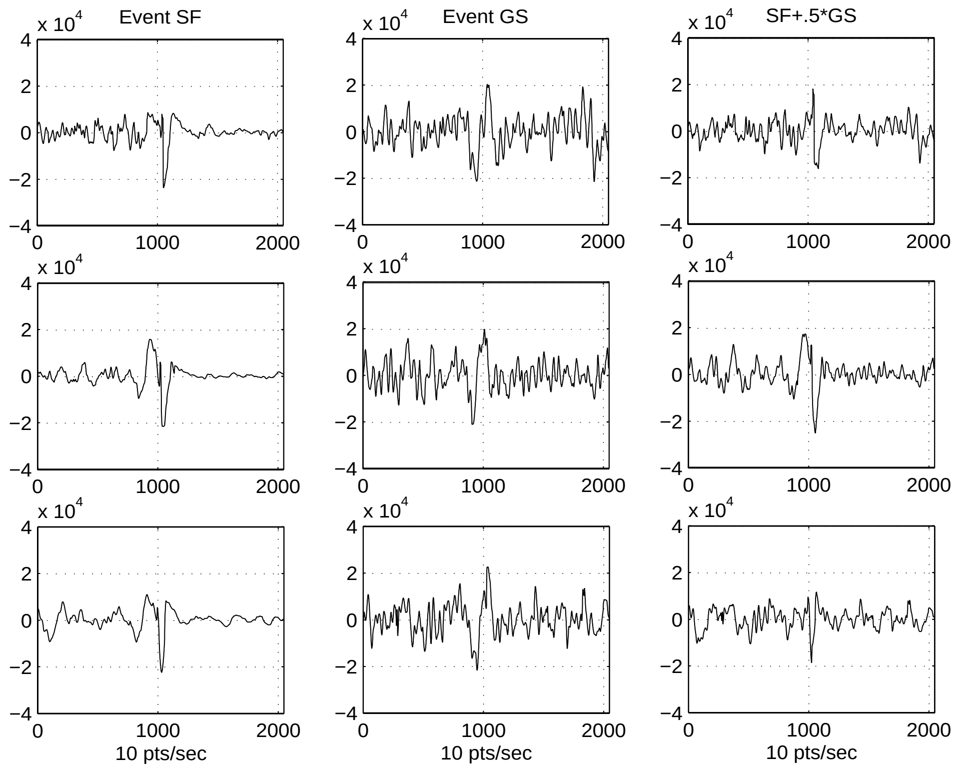

Figure 1. Infrasound explosive signals from 225° and 135° and a mixture sampled at 5 Hz at a triangular array with 1 km sides .

For the classical maximum likelihood solution in either the fixed or random signal case, the estimated wavenumber vector is the one maximizing the beam power, say

$$B(\boldsymbol{\theta}) = \boldsymbol{x}^*(\boldsymbol{\theta}) S\ \boldsymbol{x}(\boldsymbol{\theta}), \tag{2}$$

where

$$S = \sum_{k=1}^{K} \boldsymbol{Y}_k \boldsymbol{Y}_k^* \tag{3}$$

is the spectral matrix evaluated at K frequencies in the neighborhood of some assumed signal frequency f_0, and $\boldsymbol{Y}_k$ denotes the $N \times 1$ vector of discrete transforms of the observed waveform. It should be noted that (2) is realized simply online by filtering the signal in the neighborhood of the center frequency and delaying and summing at the velocity and azimuth matching the wavenumber vector. A power detector is then applied to the resulting beam to get an approximation to (2). All detectors here are expressed in terms of the spectral matrix S to provide a rough comparison to other detectors introduced below. Shumway et al (1999) have also derived the large-sample covariance matrix of the maximum likelihood estimator $\hat{\boldsymbol{\theta}}$, which can then be directly incorporated into the nonlinear least squares program for location (See Shumway, 2000).

The maximum likelihood detector, derived from a likelihood ratio test of no-signal in either the random or fixed signal case is the F-Statistic

$$F_1(\boldsymbol{\theta}) = \frac{\boldsymbol{x}^*(\boldsymbol{\theta}) S\ \boldsymbol{x}(\boldsymbol{\theta})/N}{\text{tr}(S) - \boldsymbol{x}^*(\boldsymbol{\theta}) S\ \boldsymbol{x}(\boldsymbol{\theta})/N}(N-1), \tag{4}$$

which has an F distribution with $2K$ and $2K(N-1)$ numerator and denominator degrees of freedom respectively when the wavenumber is correct (tr denotes trace). For online applications, one can filter the channels into the frequency band of interest and then use the ordinary time domain F-Statistic with $2BT$ and $2BT(N-1)$ degrees of freedom. The numerator is proportional to the output of a power detector and the denominator can be computed as the difference between the stacked power and the beam power in the numerator, when the data have been filtered into a frequency band of width B Hz and the sample length is T seconds. Among the advantages of this detector are (a) optimality under perfect signal correlation and spatially white noise,)(b) known statistical distribution that does not depend on signal and noise parameters and (c) existence of an easily applied online time version for monitoring applications. Potential disadvantages of the simple F detector are lack of robustness to signal correlation, investigated theoretically in Shumway et al (1999) and interfering signals. These latter two aspects are assessed here. We also study three alternative estimators that may have some potential for improved estimation in the mixed signal case.

The first is due to Capon (1969) and involves maximizing the quadratic form

$$C(\boldsymbol{\theta}) = \left[\boldsymbol{x}^*(\boldsymbol{\theta}) S^{-1} \boldsymbol{x}(\boldsymbol{\theta})\right]^{-1}. \tag{5}$$

over possible wavenumber vectors $\boldsymbol{\theta}$. Under the assumption that the theoretical spectral matrix of the vector $\boldsymbol{Y}_k$ is Σ, coupled with the Gaussian assumption, this statistic has a distribution proportional to chi-squared, where the proportionality constant is $\boldsymbol{x}^*(\boldsymbol{\theta})\Sigma^{-1}\boldsymbol{x}(\boldsymbol{\theta})$ (see Capon and Goodman, 1970, for the exact result). Hence, the distribution is known but depends on nuisance parameters in the spectral matrix Σ. Furthermore, asymptotic results for the estimator $\hat{\boldsymbol{\theta}}$ maximizing (5) are not available so there the inputs required for assessing global location uncertainty are not available. the statistic is sometimes referred to as a maximum likelihood estimator but the connection to maximum likelihood is tenuous at best. If S were the maximum likelihood estimator of the noise covariance matrix Σ (it is not) under the fixed signal model, $C(\boldsymbol{\theta})$ would be the maximum likelihood estimator of the variance of the best linear unbiased estimator.

An estimator based on the eigen vectors of S is the centerpiece of the Multiple Signal Characteristic (MUSIC) statistics suggested by Schmidt (1979). A good summary of the statistical properties of this estimator is Stoica and Nehorai (1989). If we let $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_N$ be the eigen vectors of S, then the MUSIC estimator is the value of $\boldsymbol{\theta}$ maximizing

$$M(\boldsymbol{\theta}) = \left[x^*(\boldsymbol{\theta})\left(\sum_{j=M+1}^{N} \boldsymbol{v}_j \boldsymbol{v}_j^*\right)\boldsymbol{x}(\boldsymbol{\theta})\right]^{-1}, \tag{6}$$

where $M < N$ is the assumed number of signals and $\boldsymbol{v}_{M+1}, \ldots, \boldsymbol{v}_N$ are the $N-M$ smallest eigen values. The estimator follows from the fact that for the spatially white noise model with correlated signals, the wavenumber vectors are orthogonal to the matrix containing the last $N-M$ eigen vectors as columns. Pre-multiplying by $\boldsymbol{x}^*(\boldsymbol{\theta})$ and taking the inverse leads to (6). Stoica and Nehorai (1989) have derived the large sample covariance matrix for a linear array.

A third possibility is a multiple fixed signal approach via likelihood ratio tests as proposed by Shumway (1983).This proceeds by minimizing the squared error

$$SSE2(\boldsymbol{\theta}) = \sum_{k=1}^{K} \|\boldsymbol{Y}_k - \boldsymbol{x}(\boldsymbol{\theta}_1)S_{1k} - \boldsymbol{x}(\boldsymbol{\theta}_2)S_{2k}\|^2$$

over $\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2')', S_{1k}, S2k, k = 1, \ldots, K$ corresponding to two signals at wave-numbers $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$. Denote the maximized value of the above by $SSE2(\hat{\boldsymbol{\theta}})$ and the maximized value with no second signal as $SSE1(\hat{\boldsymbol{\theta}}_1)$. Then, the multiple signal F-statistics for test the no-second-signal hypothesis is

$$F_2(\boldsymbol{\theta}) = \frac{N-2}{2}\frac{[SSE1(\hat{\boldsymbol{\theta}}_1) - SSE2(\hat{\boldsymbol{\theta}})]}{SSE2(\hat{\boldsymbol{\theta}}_2)}, \tag{7}$$

which converges to an F distribution with $2K$ and $2K(N-2)$ degrees of freedom.

We have tested the methods on the pure signal in the left-hand column of Figure 1 and on the mixture in the right-hand column. The result are shown in Figure 2, 3 and 4 and summarized in Table 1. All plots are constructed using a center frequency of .05 Hz, corresponding to the spectral peak of the observed data and $K = 13$ in the spectral matrix, which implies a bandwidth of $(13/1024)5 = .06$ Hz so that the frequency spans the interval $.03 - .09$ Hz.

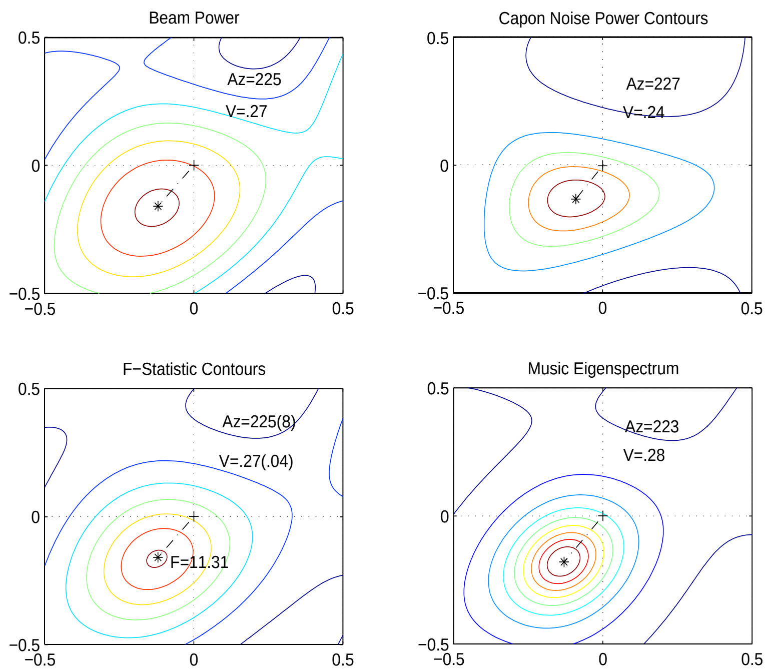

Figure 2. Detectors applied to pure infrasound explosive signal from 225°. Standard errors of the maximum likelihood estimators are in parentheses on the F-Statistic contour.

Figure 2 shows the wavenumber plots of the beam detector (2), F-Statistic (4), Capon detector (5) and the MUSIC detector (6) for the pure signal case. All statistics give azimuths within ±2° of the known azimuth. The standard deviation of the maximum likelihood azimuth estimator was 8°. F value of 11.31 exceeds the .001 significance point of the F-Statistic with $2(13) = 26$ and $2(13)(3-1) = 52$ degrees of freedom. The peaks in the beam power and Capon statistics are approximately the same width; the Capon contours are slightly distorted due to the non-diagonal spectral matrix S. The F-Statistic and MUSIC plots promise more resolution, due to narrower peaks.

We see in Figure 3 that narrow peaks do not necessarily translate into an improved ability to separate mixed signals. The detectors applied to the mixture still focus on the stronger signal. Estimators for the azimuth are biased, ranging from 215 to 217°, indicating that the contaminating signal has pulled these values toward its azimuth of 135°. Hence, there is no appreciable difference between the bias terms for the three methods. The standard deviations of the azimuth estimators are approximately doubled to 17° and the F value is reduced to 3.55, which still exceeds the .001 significance value. It is clear that the presence of the contaminating signal in this case increases the estimated variance, implying that this particular array will have less influence on the overall location.

wavenumber plots for the two multiple signal possibilities, MUSIC in (6) and the multiple signal F detector (7) are shown in Figure 4. The MUSIC estimator (6) in the bottom panel uses values of

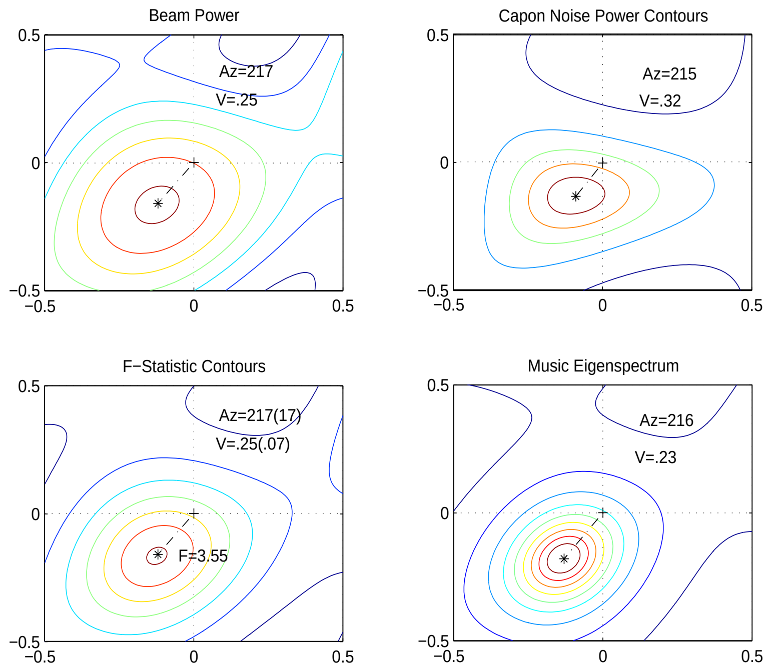

Figure 3. Single signal detectors applied to mixture of two pure infrasound explosive signals.

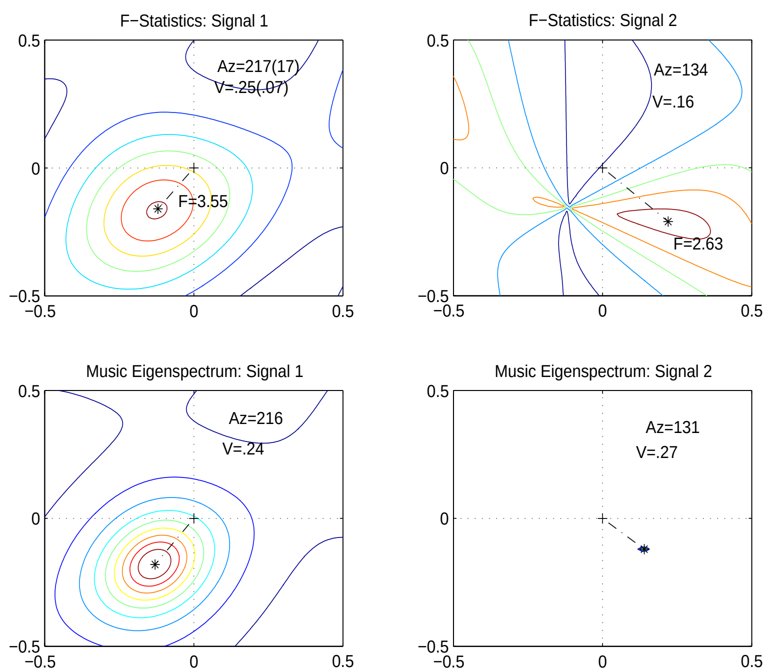

Figure 4. Multiple signal detectors applied to mixture of two infrasound explosive signals.

$M = 1$ for the single signal model and $M = 2$ for the two-signal version. Curiously, the two signal model focuses only on the imbedded signal velocity and azimuth, whereas the single signal model peaks for the stronger signal. The F-Statistics are taken as $F_1(\hat{\boldsymbol{\theta}}_1)$ in the left hand column and

$F_2(\hat{\theta}_1, \hat{\theta}_2)$ in the second column, where we assume the biased value for $\hat{\theta}_1$ in the computation of the statistic (7). The F-Statistic for the second signal, given the first signal at 217° is 2.63, which still exceeds the .01 false alarm level of 2.55 with $2(13) = 26$ and $2(13)(3-1) = 26$ degrees of freedom in the numerator and denominator.

A convenient summary of the results is given in Table 1 and we note the relative robustness of the single station F detector. Not also that despite a bias of about 8° in the maximum likelihood estimator, the increase in standard deviation still puts the estimated value within half a standard deviation of the ground truth azimuth of 225°.

Table1: Estimation by Different Methods for Single and Mixed Events Using Single and Multiple Signal Detectors

Event	SF225 Only	Mix	
Detectors	SF225	SF225	GS135
Single F	225(8)	217(17)	-
F-Values	11.31	3.55	-
Capon	227	215	-
Multiple F	225	217	134
F-Values	11.31	3.55	2.63
MUSIC	223	216	131

CONCLUSIONS AND RECOMMENDATIONS

We conclude that the performance of the likelihood based statistics under the assumptions of spatially white noise and possible signal decorrelation are relative robust to departures from these assumptions. We have also examined the Capon and MUSIC detectors as possible competitors to the F-Statistic and have note that they do not improve performance in the mixed signal case and do have convenient expressions for the variances of the estimated wavenumber parameters. We note that the likelihood based approaches produce large sample variances for the estimated wavenumber parameters (velocity and azimuth) that are needed for input into the nonlinear location simulations.

A reasonable number of calibration events are needed to give realistic inputs for the previously developed software that produce area uncertainty contours for the location of events detected by random configurations of detecting stations in the IMS network. for these inputs, we are currently downloading a number of events contained in the CMR R&D Test Bed Infrasound Waveform Library. Events available are a California earthquake, several gas pipe explosions, a Titan IV B launch and some bolides.

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