CONSTRUCTION OF 3-D EARTH MODELS FOR STATION SPECIFIC PATH CORRECTIONS BY DYNAMIC RAY TRACING

Vernon F. Cormier

University of Connecticut

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ABSTRACT

Three-dimensional models of the crust and mantle structure beneath Area 1 International Monitoring System stations in Eurasia are constructed for use with either asymptotic ray or numerical methods of waveform modeling. The models combine crustal and upper models of varying resolution specified on latitude and longitude grids having variable spacing. Model parameters are interpolated using Delaunay triangulation in 3-D (tetrahedra). Unless accurate narrow-angle crustal reflections and reverberations are required, ray bookkeeping is simplified by making all first-order crustal discontinuities narrow transition zones. Station-specific path corrections (SSPCÕs) for travel times are obtained in these models by dynamic ray tracing (DRT). DRT provides information on wavefront that can be used to accurately interpolate travel times by a paraxial approximation in the vicinity of end points of rays. By this method travel times are computed in 3-D models at dense grids specified around each IMS station. Each grid point will contain travel time and quantities needed to interpolate travel times spatially at finer intervals. These interpolating quantities can also be useful to event relocation. The grid of values is given as a binary, direct access file. The wavefront curvature information provided for each path includes information needed for path integrated attenuation (t*), geometric spreading, and ray-synthetic seismograms.

KEY WORDS: SSPCÕs, calibration, travel times, dynamic ray tracing, three-dimensional earth models

OBJECTIVE

Travel-time tables will be generated for paths of regional seismic phases within and to area 1 IMS stations in Eurasia using three-dimensional models of crustal structure. The task assigned to the University of Connecticut is to develop algorithms for constructing three-dimensional models of the crust and upper mantle and to compute travel times and ray theoretical waveforms for regional seismic waves propagating in these models. The results of this work will provide accurate regional phase travel times for improvements in the IDC's location estimates based on data from IMS stations in Eastern Asia. The incorporation of three-dimensional structure will correct for the effects of off-azimuth paths on travel times, which are pervasive throughout this region.

RESEARCH ACCOMPLISHED

Method of computing travel times and SSPC's

Under work completed by the co-Principal Investigator under an AFOSR contract, a three-dimensional dynamic ray tracing program was written to enable prediction of Lg from SmS ray paths in three-dimensionally varying crustal models. Rays were traced by integrating kinematic and dynamic ray tracing equations (Cerveny and Hron, 1980; Cerveny, 1985). The dynamic ray tracing system gives information needed to compute wavefront curvature, which can be used to compute geometric spreading and quantities needed for summation of Gaussian beams, a technique of seismogram synthesis closely related to the Maslov-WKBJ method. These techniques have been applied by the co-PI to a wide variety of applications requiring two-point ray tracing (e.g., Cormier and Beroza, 1987) and ray theoretical synthesis of seismograms in three-dimensionally varying media (e.g., Cormier and Anderson, 1999; Cormier, 1986).

Three-dimensional dynamic ray tracing

Station-specific path corrections (SSPC's) will be obtained from travel times computed by dynamic ray tracing (DRT). DRT provides information on wavefront that can be used to accurately interpolate travel times by a paraxial approximation in the vicinity of end pints of rays. By this method travel times will be computed in 3-D models at dense grids specified around each IMS station. Each grid point will contain a travel time and quantities needed to interpolate travel times spatially at finer intervals. These interpolating quantities can also be useful to event relocation. The grid of values will be given as a binary, direct access file, and will be the first deliverable product of the work. The wavefront curvature information provided for each path will also provide information needed for path integrated attenuation (t*), geometric spreading, and ray-synthetic seismograms.

Travel times for arbitrary 3-D paths

The second stage in this effort will consist of a simple code that retrieves the ray end-point quantities from that file and calculates the travel time of a specified phase from a source by a paraxial approximation. An important advantage of the paraxial approximation is the avoidance of expensive two-point ray tracing, which must find the exact ray-path connecting source and receiver to high accuracy. This will enable rapid retrieval of travel times for any arbitrary path in a fully three-dimensional structure, including the effect of three-dimensional path deviation from a great circle. In the theory of dynamic ray tracing, the paraxial approximation calculates the travel time from position x_o to position x by

$$T(x, x_o) = T(x) + \underline{p} \cdot \underline{\Delta x} + \underline{\underline{H}}\ \Delta x\ \underline{\underline{M}} \Delta x^t\, \underline{\underline{H}}^t ,$$

where p is a vector slowness at the ray end point, M is a 3 x 3 matrix of second derivatives of the travel time field in ray-centered coordinates at the ray end point, Δx is the Cartesian vector difference between the station location within a grid box and the ray end point in the grid box, and H is a 3 x 3 matrix whose columns consist of the vectors of the ray-centered coordinate basis. The algorithm of the proposed code simply evaluates the above equation using 25 quantities stored in grid boxes for each IMS station corresponding to paths from all other boxes not containing that station. It is simple to include among these quantities the path integrated attenuation (t*) for regional phases using measurements from models determined from the DSS data analyzed by the University of Wyoming group.

Ray paths need not precisely connect source and receiver. The paraxial equation above accurately corrects the travel time to the time that would be computed for the exact ray, provided that the source and receiver are not too distant from a frequency-dependent region of validity often termed the "paraxial vicinity." In tests of the algorithm for the models and ranges proposed here, use of the paraxial approximation at ray end points to extrapolate travel times within 25 km x 25 km boxes were found to produce travel times accurate to within 0.01 sec.

Previous work (Cormier and Anderson, 1999) modeled the Moho and the boundary between a sedimentary layer and hard-rock layer (basin topography) as surfaces interpolated by splines under tension. The Moho and basement surfaces were assumed to spatially continuous transition zones. With surfaces of discontinuities so modeled, all rays become either (1) turning rays and multiply surface reflected rays within the model or (2) rays that dive deeply, leaving the bottom of the model. This procedure eliminates the need for computing many reflection coefficients and complex descriptions of rays interacting with multiple first-order crustal discontinuities.

Starting three-dimensional models

Starting models for three-dimensional structure were assembled from published models for the crust and upper mantle of Eastern Asia (Fielding et al, 1992; Barazangi et al., 1996; Mooney et al, 1998; Ritzwoller and Levshin, 1998; Sambridge and Gudmundsson, 1998). Each of these models were constructed to satisfy different types of data and reported with differing degrees of resolution. Hence, there is no guarantee at the outset that a hybrid model incorporating features of all these models will be successful in reducing the travel time residuals from ground truth events calculated from laterally homogeneous reference models.

The Cornell model of Barazangi et al. (1998) reports the depth to the Moho and depth to a hard-rock basement in the crust at 0.1 x 0.1 deg resolution. Crust 5.1 by Mooney et al. (1998) incorporates many published local and regional refraction studies in a global 7 layered crust model reported at either a 1 x 1 deg resolution for crustal and basement thickness or a 5 x 5 deg resolution for the detailed 7 layered model. The University of Colorado Model (or CU model) of Ritzwoller and Levshin (1998) is obtained from surface wave group velocities and some Pn velocities. Except for the P velocity at the underside of the Moho constrained by Pn, the P/S velocity ratio is not well constrained. Although Sambridge and Gudmundsson's (1998) Regionalized Upper Mantle (RUM) model is based on a tectonic age regionalization (Jordan, 1981) at a spatially fine scale, it has been inverted from primarily teleseismic data. RUM, as it name implies, is a model of the upper mantle rather than of the crust.

Since previous work by Cormier and Anderson (1999) demonstrated the importance of fine scale moho (10 km and less) basement topography on the progagation of Pn and Lg, it was decided to incorporate into a hybrid model the high resolution model (0.1 deg x 0.1 deg) of basin thickness and Moho topography of the Cornell model. Since Crust 5.1 provides high detail in the vertical direction (up to 7 layers), it was used for structure above the Cornell high resolution Moho. (The high resolution basin structure of the Cornell model has yet to be included in test models.) The RUM model has been used for structure below the Cornell Moho.

Parameterization and interpolation of three-dimensional models

The varying types of data for crust and upper mantle structure, collected at widely different spatial scales, and the highly uneven distribution of ground truth events present a challenge to the parameterization a three-dimensional model for travel time computation. The chosen model parameterization should be flexible enough to be specified at high resolution where data is available and at lower resolution where it is not. Resolution should be high to describe features important to regional wave propagation, such as Moho and basin topography, but can be lower near interfaces having smaller velocity contrasts and lower with increasing depth in the mantle, where heterogeneity power decreases. To allow a smooth transition from the regional times and waveforms to teleseismic times and waveforms, the parameterization should incorporate the near sphericity of earth and its major structural discontinuities. The parameterization should also be economical in the specification of the number of knots.

The proposed algorithm of kinematic and dynamic ray tracing requires known first and second spatial derivatives of a 3-D model, which can be met by the use of spline interpolation using polynomials of order 3 or higher. Since the investigator had previously applied splines to interpolate model parameters in 3-D, tests were performed to determine whether this type of interpolation could also be applied to the 3-D structure beneath IMS stations. The routine previously used with dynamic ray tracing employed splines under tension at equally spaced knots in Cartesian co-ordinates (Cormier, 1986). The FITGRID[1] code previously employed under tension code was not easily adaptable to our IMS model. Its restriction to equally spaced knots requires cumbersome and constant resampling to incorporate both earth sphericity and spatially variable resolution. In future experiments, the software described by Wessel and Bercovici (1998), which may allow for irregularly spaced 3-D data, will be tested for application to dynamic ray tracing.

The most recent release of the NCAR graphics package[2] has a set of routines named CSAGRID[3] that can interpolate by splines on an irregularly spaced Cartesian grid. Unfortunately, tests showed that without an available tensioning factor in the current version of CSAGRID, the splines very poorly tracked realistic seismic velocity variations within layers. This deficiency could be remedied only be expensive densification of knot points, ultimately making its use as cumbersome as spline routines having regularly spaced grids.

The next and preferred parameterization and interpolation scheme tested is the one used and advocated by Sambridge et al. (1995) in the construction of the RUM model. This scheme parameterizes 3-D earth models by knots connected by tetrahedra. A linear gradient in velocity is assumed for the interpolated quantity within each tetrahedral element, making it possible to analytically integrate within each tetrahedron both the kinematic and dynamic (geometric spreading and wavefront curvature) ray tracing equations. A public-domain software package named qhull (Barber et al, 1996), maintained by the Computational Geometry Center at the University of Minnesota[4] , is available for performing the Delaunay tetrahedral tesselation of a model volume given a set of knots and model values. The file containing model values at knots may be given with knots occuring at any arbitrary order or spacing. Qhull returns a list of index numbers of the 4 knots describing each tetrahedron. Routines for navigating though the tetrahedra are available from Sambridge [5].

Figure 1 shows an example of a hybrid 3-D model constructed from qhull tetrahedrall tesselation using the Cornell Moho, the Crust 5.1 crust, and the RUM mantle. The example is for a 200 x 200 x 200 km block whose surface is centered at Nilore, Packistan. Note the thickening of the Moho to the north of Nilore, an effect of the crustal root that has grown from Indian-Eurasian plate collision. Some spatial variability is also seen in the low-velocity zone given by RUM. The sampling of the low velocities in the upper most crust 2.5-4km/sec needs to be densified to remove some artifacts associated with too broad tetrhedron facets at the surface of the earth.

Ray tracing

Although the tetrahedral tesselation of the model allows analytic integration of the ray tracing equations, intitial tests of travel times have used a Runge-Kutta numerical integration to shoot rays. Figure 2. Shows an example of ray end points shot at uniform increments vertical and azimuthal take-off angles from NIL. Note some effects of three-dimensional structure are evident in the scatter of rays at longer distances. A strong shadow zone is seen around 10-15 degrees. In this distance range, the first arrival is a Pn phase just grazing the underside of the Moho.

CONCLUSIONS AND RECOMMENDATIONS

Additional hybrid 3-D models will will be constructed for testing SSPC's for IMS using the a tetrahedral tesselation described in this report. A denser sampling will be incorporated in the upper 2-3 km of the crust to better describe the known three-dimensional variations in soft sediment cover. To speed-up forward modeling of travel times and dynamic ray tracing quantities, an analytic integration (Menke and West, personal communcations) for kinematic and, eventually dynamic ray tracing within tetrahedra, will be substituted for the numerical integration currently used. A test of the spline under tension parameterization of Wessel and Bercovici (1998) will also made to see if it offers any greater flexibility in model parameterization compared to the tetrahedral parameterization.

It will also be important to test the accuracy of travel time calculations, which may be affected by the choice of parameterization and the accuracy of paraxial approximations against other methods. A test will be performed with at least one model comparing the results of dynamic ray tracing and the results of the numerical eikonal solution method of Vidale (1988) being used by the MIT led consortium.

The model construction described in this report relies on hybrid models constructed from several global models of the crust and upper mantle. In work during the second year, University of Connecticut, in cooperation with the other members of our consortium, starting hybrid models will be refined to be consistent with ground truth travel times being assembled.

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Endnotes:

1. http://www.scd.ucar.edu/softlib/FITPACK.html
2. http://ngwww.ucar.edu/ng4.2/index.html
3. http://ngwww.ucar.edu/ngdoc/ng4.1/ngmath/csagrid/csahome.html
4. http://www.geom.umn.edu/locate/qhull
5. http://rses.anu.edu.au/seismology/projects/RUM/rum_download.html
6. http://coulomb.geol.uconn.edu/~cormier/pageophrays.pdf

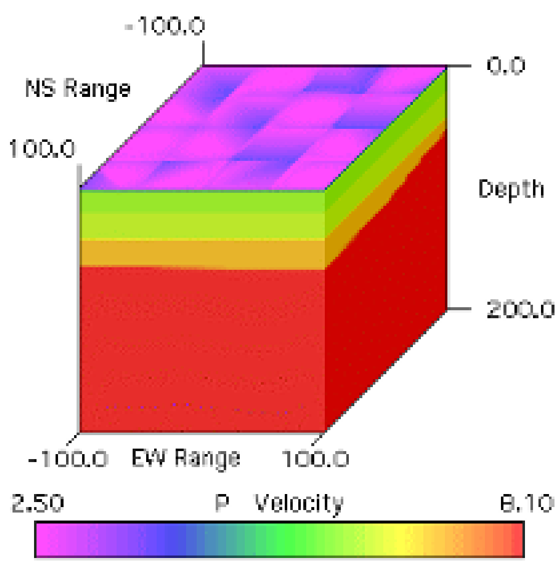

Figure 1. Three-dimensional crust and upper mantle structure beneath Nilore, Pakistan (IMS station NIL) assembled from the Cornell Eurasian Moho, Crust 5.1, and the RUM upper mantle. Note the tetrahedral parameterization in the uppermost 4 km needs to be densified to better describe shallow structure and eliminate artifacts.

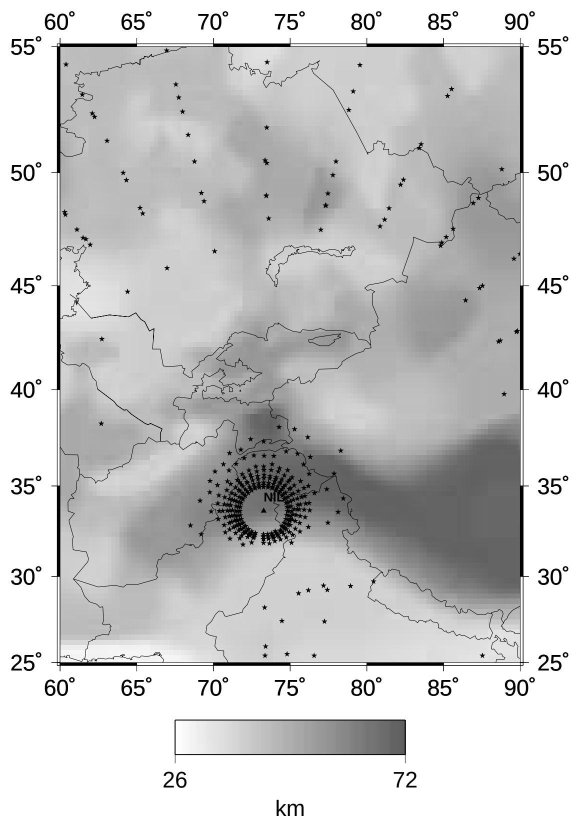

Figure 2. P Ray endpoints in the vicinity of NIL for rays shot at constant increments of vertical and azimuthal take-off angles. The Cornell Eurasian Moho topography is superposed. The hybrid 3-D model in which rays were shot was parameterized by tetrahedra of variable size and its size is 25° lat x 25° long x 700 km deep. Spacing of knots at tetrahedron vertices ranged from 10 to 50 km.