DEVELOPMENT AND INVESTIGATION OF TRANSPORTABLE REGIONAL DISCRIMINANTS

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ABSTRACT

In this project, we are investigating problems associated with the transportability of regional phase amplitude ratios, such as *Pn/Sn* or *Pn/Lg* ratios, for discrimination of explosions and earthquakes for nuclear-explosion and treaty monitoring in different tectonic regions. In one study, we have improved our algorithm, developed last year, for identifying and quantifying the causes of the variance of regional *P/S* ratios using analysis of variance. We have developed an expectation maximization (EM) algorithm for two-way or three-way analysis of variance (ANOVA2-ANOVA3) for partitioning the variance of *P/S* ratio measurements into source, site, and path effects. The EM algorithm can be applied to combinations of data sets where there may be missing data. The algorithm is being tested on Scandinavian array recordings of earthquakes, nuclear explosions and mine blasts. Also, a user interface has been developed in Matlab© for applying the EM ANOVA2-ANOVA3 algorithm to array data. We have initiated an investigation of a new regional magnitude scale, similar to the *mb(Lg)* magnitude, called *mb(Sn+Lg)* magnitude based on the sum total of the regional shear energy contained in the regional *Sn* and *Lg* phases. This study is motivated by previous observations of *Lg* blockage that might limit the use of *mb(Lg)* for discrimination. Because the *Sn* and *Lg* phases have sometimes been observed to trade off in amplitude in regions of blockage, the *mb(Sn+Lg)* scale may be more stable than a magnitude scale based on either phase individually. Initially, we have developed a magnitude scale based on measurements of the total energy in the *Sn* and *Lg* wavetrain, corrected for distance attenuation, and calibrated to teleseismic *mb*. We have tested the *mb(Sn+Lg)* scale with earthquake data recorded by the old ILPA array in northern Iran, Zagros events recorded at regional seismic stations in the Middle East, and earthquakes and explosions in China and Russia recorded at CDSN stations. Finally, we began statistical study of the transportability of *P/S* ratio discriminants using separability measures and optimum transformations in order to reduce dimensionality of multiple frequency *P/S* ratios. These transformations consist of calculating the intra-class and inter-class scatter matrices for *P/S* ratio discriminants and using the eigenvectors, corresponding to the largest eigenvalues, of the inter-class matrix to compute optimum transformation of discriminants that provide best separation. We are applying this analysis to distance-corrected discriminants in different regions (e.g., China, Eurasia, North America) in order to compare discriminant effectiveness for different regions and to evaluate the transportability of optimum discriminant decision surfaces.

OBJECTIVE

Introduction

We are investigating issues concerned with the transportability of regional discriminants developed in a region of a distinct tectonic type and applied to another region of different tectonic type. The best definition of a region must be based on homogeneous propagation characteristics for the seismic phases involved and not on political or geographical subdivision. This implies some commonality in the crustal and upper-mantle structures with regards to tectonic origin. It is rather pointless, for instance, to compare the performance of a *Pg/Lg* discriminant in Scandinavia and western North America. The '*Pg* phase' in these two regions is obviously not the same. In Scandinavia, the Pg is relatively small in amplitude compared to *Pn* and *Lg*, whereas in western North America, the *Pg* is the largest regional compressional phase. Thus, it is likely that the two do not represent the same 'phase' propagating in the similar waveguides.

A totally different issue is how to handle regions with notably anomalous propagation characteristics. It has been noted earlier (Baumgardt, 2001) that certain crustal discontinuities, such as sedimentary basins, are associated with blockages or high attenuation of various regional seismic phases, primarily *Lg*. Since such regions may be relatively small in dimensions, it may be difficult to predict their effect on the spectra and amplitudes of seismic phases with paths crossing them and correct for them. Nevertheless, many such regions have been identified. Examples of these are the southern end of the Caspian Sea, Tibet, some regions of Iran, and the Andes Mountain regions. Typically, these regions are often areas that have been subjected to recent tectonic deformation and occupy narrow bands. If appropriate corrections have not been made for such regions of blockage, it may be the best to avoid applying seismic phase measurements involving these in routine discrimination.

In this paper, we discuss research efforts conducted to investigate factors that affect discriminant transportability and to investigate possible seismic discrimination approaches that are more robust for transportability. As a follow-on to our previous study of single array variances (Baumgardt *et al*, 2001), we have continued to examine, by means of analysis of variance (ANOVA), the contribution of site effects on the variance of regional *P/S* amplitude. In this paper, we have applied our ANOVA approach to multiple arrays. The goal of this work is to quantify the residual variance of regional *P/S* amplitude ratios after correction for attenuation. In the second study, we have investigated the problem of dimensionality reduction for multi-band regional *P/S* ratios and the combination of features that produce the optimal discrimination. In the third study, we have investigated the utility of a new regional magnitude based on the sum total of the energy in the *Sn* and *Lg* wavetrain, which we call *mb(Sn+Lg)*, that may be more stable than *mb(Lg)* in regions of strong Lg blockage.

RESEARCH ACCOMPLISHED

Multi-Array Analysis of the Site Variance of Regional P/S Ratios

In our initial study (Baumgardt *et al*, 2001), we studied the variations in the Pn, Lg, and Pn/Lg amplitude ratios recorded at Scandinavian events from underwater explosions. A two-way analysis of variance method (ANOVA2) was applied to study the partitioning of the amplitude and ratio fluctuations between site and source effects. As a follow-up to this study, we have begun a study of multi-array ANOVA2. In ANOVA2, we fit the following model to the amplitudes and amplitude ratios:

$$y_{ijk} = m + a_{,j} + b_{i,} + e_{ijk}$$

where y_{ijk} are the array element amplitudes or amplitude ratios, m is the mean of all the data, $a_{,j}$ is the source term, $b_{i,}$ is the site term, and e_{ijk} is the error term. This model is fit to the logarithms of amplitudes and amplitude ratios where it is assumed that the amplitudes and ratios are log-normal and dependent on both the additive event and the site effects, both of which are specified to have a zero sum. The errors are assumed to be normally distributed. ANOVA2, in essence, tests the hypothesis that the data come from the same population with a common mean (Johnson and Wichern, 1988). In this study, we focus on the variations in the site terms and ignore for the time being the significance tests on the commonality of the mean.

As an example of this analysis for multiple arrays, we applied it to the Pn, Lg, and Pn/Lg amplitude ratios measured for three seismic arrays. The data for this study came from a set of underwater explosions in the Baltic that were recorded at three Scandinavian high-frequency seismic arrays, NORESS, FINES, and HFS, originally studied by Baumgardt and Der (1998). Our approach was to measure the amplitudes and their ratios, after correcting the amplitudes for distance. The results of the analysis in two frequency bands, 2-4 Hz and 4.5-9 Hz, are shown in Figure 1.

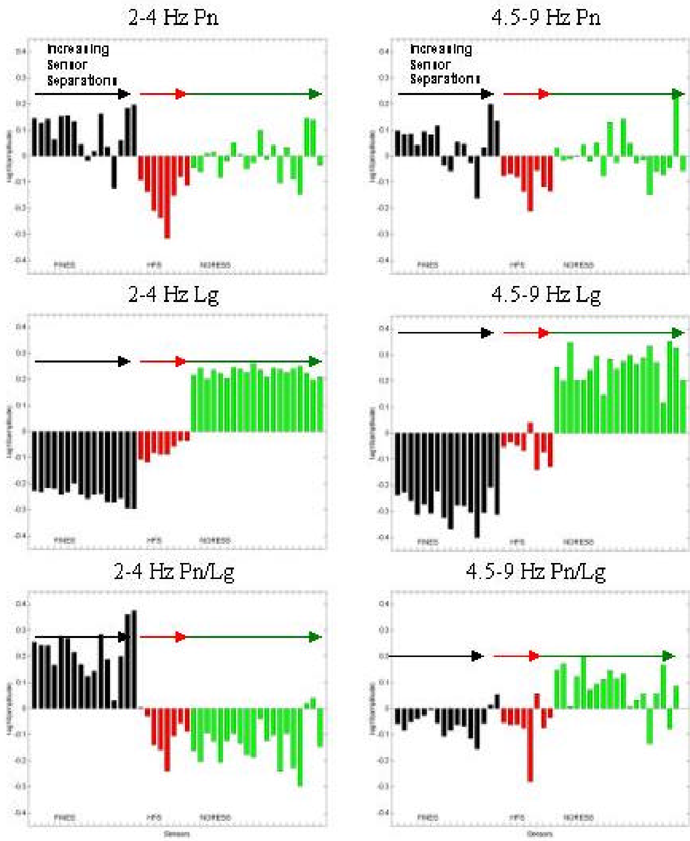


Figure 1. Examples of the effects of the arrays and sites in two frequency bands within arrays on the Pn/Lg ratio in two frequency bands obtained from ANOVA2. A generalized version of the EM algorithm was used to fill in a small number of missing observations. The site terms for each array are designated by different colors

Figure 1 shows that there are strong biases due to each array and site effects within each array. The ANOVA analysis of single arrays, described by Baumgardt *et al* (2001) gave Pn amplitude variations that controlled the variations in Pn/Lg site terms and the Lg site terms were more stable across the arrays. This is clearly evident in Figure 1 for the ANOVA applied to three arrays in combination, where the Pn site corrections have somewhat random variations of positive and negative values, with the exception of HFS in the 2- to 4-Hz band. The Lg variations, however, all have the same sign for each array. Thus, the random variations in the Pn site terms map into the Pn/Lg corrections.

The results of this study indicate that the principle site effect contribution to Pn/Lg variations is coming from the Pn site effects. However, when examining the variations between sites, the Lg variations are larger in magnitude than the Pn variations. Note that these amplitudes have been corrected for distance, so that these variations appear to be coming primarily from the site effects. Thus, when using the Pn/Lg ratio to identify seismic events, this variance needs to be factored into the estimates of the measurement precision for Pn/Lg ratios.

<u>Transportability Issues</u>

Assuming the existence of geophysically well-defined regions, the term *transportability* may mean the following different things:

1. Applicability of the same discriminant with statistics adapted to the specific region using available learning data sets and applying a consistent methodology to classify events (e.g. Fisk *et al*, 1996).
2. Applicability of a discriminant to various regions without any modifications, i.e. using statistics from other similar regions.
3. Application of the same discriminant with some adaptation of parameters but using assumed statistics for certain types of events (such as nuclear explosions) if learning sets for these are not available in the region.

In this paper, we prefer definition 1 while reconciling ourselves with the occasional necessity of applying definition 3. Option 2 is not likely to work because of the variability in the definition and physical nature of regional 'phases' from region to region. We shall also adapt the usual definitions of regional phases as they are commonly picked in the seismic bulletins from various regions, even though their excitation and propagation may be different from region to region. Given this regional variability, it seems certain the efficiency of discrimination will vary from region to region in a way that can be described in quantitative terms if sufficiently large learning data sets are available. In this paper we have not achieved this as yet, but we are attempting to set up a framework for worldwide transportability studies.

At regional distances, the most common discriminants used are distance-corrected spectral ratios of various wave groups. These constitute multi-dimensional discriminants because spectral ratio measurements are performed over numerous frequency bands and for multiple combinations of regional arrivals. Inspecting a typical plot of such data for a group of explosions and earthquakes, it is obvious that the distributions of the spectral ratios tend to be similar in various neighboring frequency bands, thus providing redundant information because of the strong correlation among the measurements in various bands and phase combinations. Typically, the separation between explosion and earthquake populations is poor at low frequencies (1-2 Hz) and improves with increasing frequency. Classification of a new event is commonly based on the positions of its many data points in such a graph relative to known earthquake and explosion data plotted in the same graph (e.g. Ryall *et al*, 1995).

Obviously, we do not need all the measurements in all spectral bands because some measurements may be more important than the others. Likewise, we may not need all the spectral ratios either. The question of how to combine such multiple measurements in the most effective way has not been studied extensively in regional seismology. Thus, the dimensionality of the data could be reduced considerably with appropriate data manipulation. Metrics of discrimination effectiveness are also needed to be able to compare various regions with regards to discrimination efficiency and transportability. What follows is a brief description of algorithms used in this study.

Handling of Missing Data-Generalized EM algorithm

In our data sets of spectral ratios, ratios in some bands may be missing. This may result from censoring of data because of low signal-to-noise ratios in either the numerator or denominator phases or other data problems. In order to utilize the available data fully, it is desirable to substitute estimated values into the slots of missing data and thus make the rest of the good data available to improve the correlation matrices used in discrimination.

We use the technique of expectation maximization (EM) to interpolate missing data using the available data values. We apply the following iterative procedure to compute the extrapolated data matrix:

a. Compute the first 'current' correlation coefficient matrix from the events that have complete sets of values in all frequency bands
b. Extrapolate the missing data points for all events that need them employing the 'current' correlation coefficient matrix using the formula below.
c. Compute the 'current' correlation coefficient matrix from the extrapolated data set; if the preset iteration number is reached go to d; otherwise, remove the extrapolated values and go to b.
d. Compute the correlation (not correlation coefficient) matrix for the reconstructed data set. These will be used in the separability analyses. Output the completed data matrix.

The extrapolated missing values a_i are computed as

$$a_i = \frac{\sum_k |C_{ik}|(a_k + m_i - m_k)}{\sum_k |C_{ik}|},$$

where the C_{ik} are the 'current' correlation coefficients, the summation over k occurs over the existing data values, and i is the index of the missing value. We are essentially correcting for the differences in means (m) between the existing and missing components and weight these according to the absolute values of the average correlations between them. As the iteration progresses, the values of the means, correlations, and extrapolated values will be optimized in some sense.

This completes the EM process. Typically a few iterations are quite sufficient. The approach described above is not the rigorous EM algorithm described in Duda *et al* (2001) but the one termed *generalized EM* algorithm by the same source, which includes a number of *ad hoc* methods for data extrapolation. The exact algorithm involves the iterative estimation of the population parameters from the existing data points without extrapolation of the missing values. It is cumbersome and slow computationally because numerous probabilities, usually assumed to be Gaussian, need to be computed. We feel that such complexity is not justified in this application. We go through this procedure in order to exploit the correlation information contained in the incomplete events and set up extrapolated data matrices that can be used in the handling of dimension reduction and separability issues.

Separability Measures and Optimum Transformations

Since spectral ratios of multiple regional phases in multiple frequency bands constitute discriminants with high dimensionality, dimensionality reduction is a desirable goal. Moreover, discrimination analysis will define the most efficient way to combine data and eliminate the measurements that contribute little to the solution.

Given the distributions of empirical data containing known event types, one can utilize several measures to examine how well a given category separates from others. The measures listed are based on the comparisons of the within-class scatter matrices $\boldsymbol{S}_W$ to the between-class scatter matrix $\boldsymbol{S}_B$. Different definitions of scatter matrices are given by Fukunaga (1990), Duda *et al* (2001), and Tou and Gonzalez (1974). The definition given to $\boldsymbol{S}_W$ by Duda *et al* (2001) for the two-class case is

$$\mathbf{S}_W = \mathbf{S}_1 + \mathbf{S}_2$$

and where

$$\mathbf{S}_i = \sum_{x \in D_i} (\mathbf{x} - \mathbf{m}_i)(\mathbf{x} - \mathbf{m}_i)^t,$$

where $\mathbf{m}_i$ is the mean vector of population i and the sum is over elements of class i . The between class scatter matrices $\boldsymbol{S}_B$ can be defined as

$$\mathbf{S}_B = (\mathbf{m}_1 - \mathbf{m}_2)(\mathbf{m}_1 - \mathbf{m}_2)^t$$

The solutions to the eigenvalue problem

$$\mathbf{S}_W^{-1}\ \mathbf{S}_B\ \mathbf{w} = l\ \mathbf{w}$$

from the equation above can be used to determine an optimum transformation for reducing the dimensionality of the problem (Duda *et al* 2001). The eigenvector associated with the largest eigenvalue, λ, gives an indication of the relative importance of the various orthogonal eigenvector components of the problem. The simplest transformation is the linear combination comprising the Fisher linear discriminant, expressed as,

$$\mathbf{w} = \mathbf{S}_W^{-1}(\mathbf{m}_1 - \mathbf{m}_2).$$

Thus, the optimum direction for best linear separation of the two populations is a projection of the vector connecting the means transformed by the matrix $\mathbf{S}_W^{-1}$. If this matrix is close to diagonal, the results should be similar to intuitive expectation based on the scatter diagrams. If not, the results may be hard to explain intuitively. Since we are striving for a simple optimum linear transformation, we shall use this transformation in our study. We have found that a simple linear discriminant derived from the transformation above works well for multi-band regional phase spectral ratios for both a single pair of phases or combinations of several of these. Fukunaga (1990) recommends that several eigenvectors associated with the largest m eigenvalues be used for discrimination in a lower dimensional subspace. He used scatter matrices different from those given above.

A number of separability measures are listed by Fukunaga (1990). A measure often used is the Bhattacharayya distance, expressed as,

$$B = 1/8(\mathbf{m}_1 - \mathbf{m}_2)^t \left[\frac{\mathbf{S}_1 + \mathbf{S}_2}{2}\right]^{-1}(\mathbf{m}_1 - \mathbf{m}_2) + \ln \frac{\left|\frac{\mathbf{S}_1 + \mathbf{S}_2}{2}\right|}{\sqrt{|\mathbf{S}_1||\mathbf{S}_2|}},$$

where S_1 and S_2 are the multivariate covariance matrices of the two populations and $\mathbf{m_1}$ and $\mathbf{m_2}$ are the means. Since the Bhattacharayya distance applies strictly to Gaussian populations, it may not be informative if the distributions are much different from the normal distributions (Duda *et al* 2001). The larger this measure, the better is the separability. Such measures are being used in this study to evaluate relative effectiveness of dicriminants in various regions.

Discrimination Results

We applied the above algorithms to two data sets. In both cases, the best known distance corrections were used (Jenkins *et al* 1998). The only exception is China where we have modified the Pg distance correction to correct for visible mismatches with our data.

China Data Set

The first data set, from China, contains both nuclear explosions and earthquakes. The data set consists of earthquakes around the WMQ region, Kazakh nuclear explosions, and one nuclear explosion at the Chinese test site, Lop Nor. The scatter plot of Pn/Lg ratios in four frequency bands is shown in Figure 2.

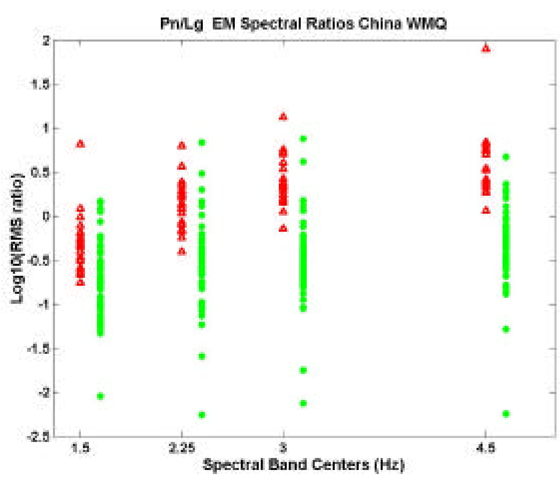


Figure 2. Scatter plot of the Pn/Lg spectral ratio data or four frequency bands from China. Red triangles denote explosions and green dots indicate earthquakes.

These data show poor separation generally, but the separation between explosion and earthquake populations improves with increasing filter band center frequency. After computing the various scatter matrices and subjecting the results to the eigenvalue-eigenvector analysis, the eigenvalues (Figure 3) show that we have at most two components, associated with the largest eigenvalues that can be used for effective discrimination.

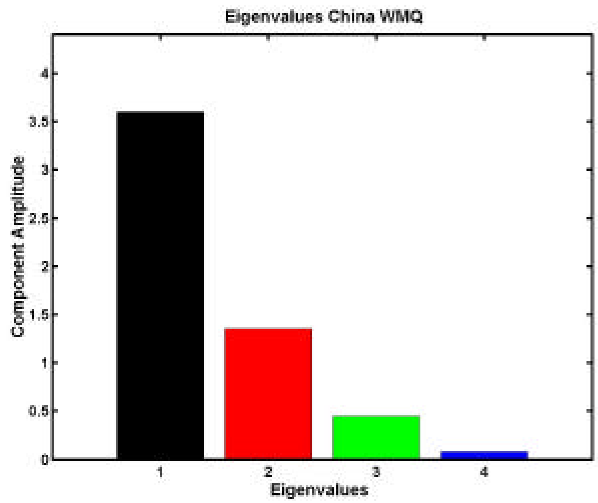


Figure 3. Eigenvalue spread derived from the analysis of Pn/Lg amplitude ratios for the Chinese station WMQ.

Plots of the eigenvectors in Figure 4 show how the Pn/Lg spectral ratio values in various bands can be combined to provide the discriminants in decreasing order of importance.

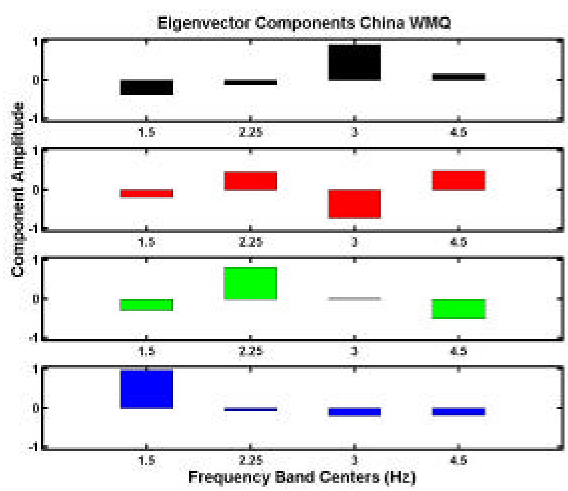


Figure 4. Plots of eigenvectors from the analysis of the China data. The first eigenvector has a dominant component for the 3-Hz band where the best separation exists. The last gives the largest weight to the first frequency band, the least effective discrimination.

Figure 5 shows plots of the ratios transformed with the eigenvectors shown in Figure 4. That is, linear combinations of original Pn/Lg ratios were computed using the weights shown in Figure 4, and these combinations are plotted versus eigenvector in Figure 5. The first eigenvector clearly produces the greatest separation. Explaining the details of the rest would be hard. Nevertheless, the first two combinations will give the best separation. There is a definite shift between the means of the projection on the second eigenvector that could also be used to complement the results from the first. The rest of the transformed ratios in Figure 5 clearly overlap completely.

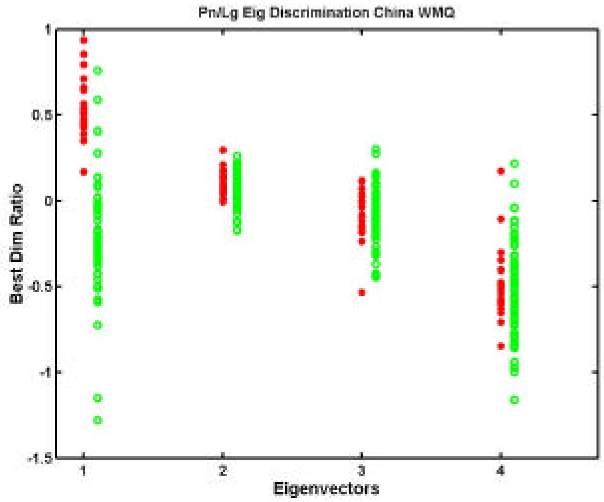


Figure 5. Discrimination plots for the China data set. Obviously, the best separation between the explosions (red dots) and the earthquakes (green dots) is achieved by the eigenvector #1 associated with the largest eigenvalue. The second eigenvector gives a small shift in means but large overlap. The remaining two overlap completely.

Nevada Test Site (NTS) Explosion vs. Skull Mountain Data Set

To compare this analysis with another region, we show the results of the analysis of Pn/Lg and Pg/Lg ratio data from the NTS and Skull Mountain earthquake data sets, originally studied by Walter *et al* (1995). These data sets consist of Skull Mountain earthquakes and NTS nuclear explosions as recorded at the Lawrence Livermore National Laboratory stations in Kanab, Utah, (KNB) and Mina Nevada (MNA). In this analysis, we combine Pn/Lg and Pg/Lg measurements into a 14-dimensional scheme.

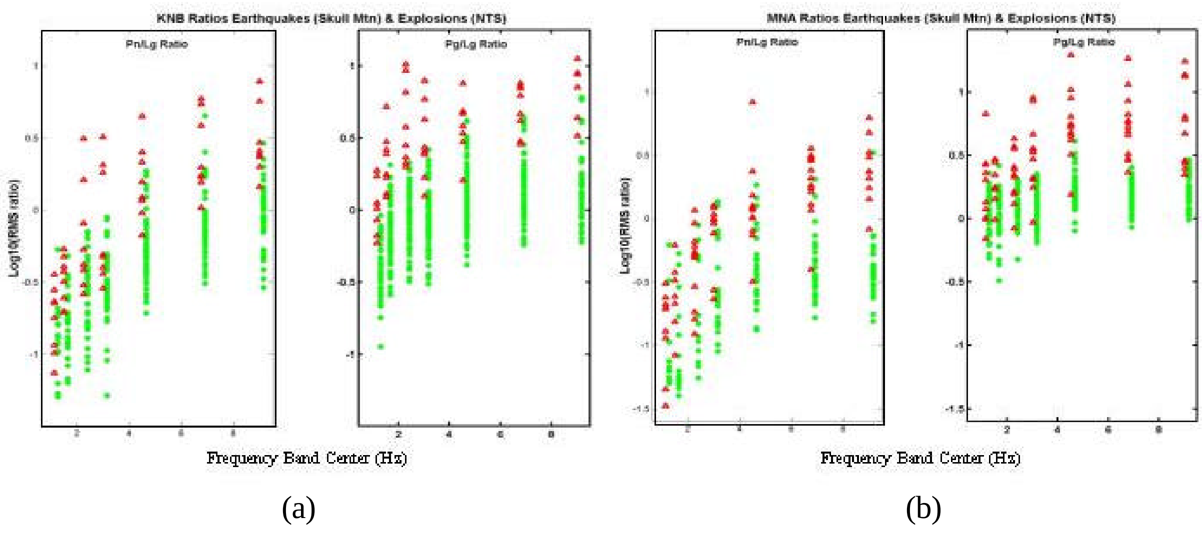
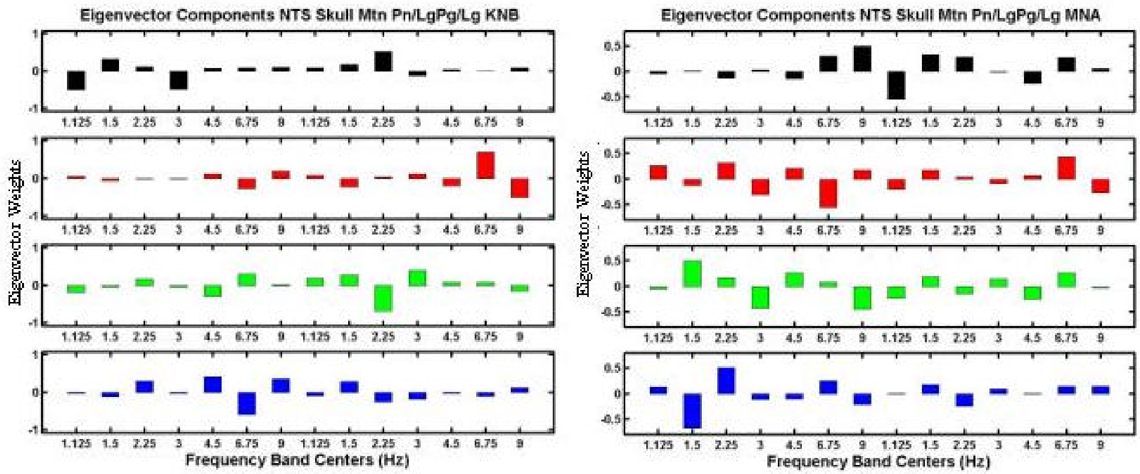


Figure 6. Scatter plots of Pn/Lg and Pg/Lg ratios for Skull Mountain earthquakes and NTS nuclear explosions at stations Kanab, Utah, (KNB) (a) and Mina, Nevada, (MNA) (b).

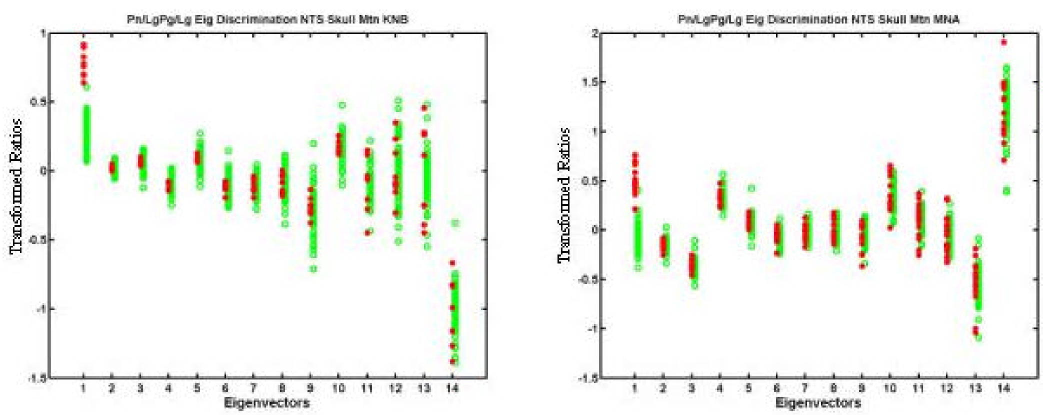
The evident improved scatter in both ratios at both stations with frequency has been pointed out by Walter *et al* (1995). It has been a generally accepted tenet of discrimination that the earthquake/explosion separation increases with frequency (e.g., Goldstein, 1995). The eigenanalysis was applied to the combination of Pn/Lg and Pg/Lg ratios, and the resultant eigenvector weights are shown in Figure 7.



(a) (b)

Figure 7. Eigenvectors for KNB (a) and MNA (b) computed from the amplitude ratios of earthquakes and explosions in Figure 6.

The weights for the principal eigenvalue, which are the first rows in Figure 7, to do not seem to be much larger for the high-frequency ratios. In fact, for KNB in Figure 7 (a), the weights seems strongest for lower frequency Pn/Lg amplitude ratios. Finally, Figure 9 shows the resultant transformed discriminants.



(a) (b)

Figure 8. Resultant transformed amplitude ratios after applying the weights in Figure 7 to the amplitude ratios in Figure 6.

The transformations clearly show the best separation on the first eigenvalue, although the separation is better for KNB than for MNA. Why the eigenanalysis seems to weight lower frequencies more than higher at KNB is not clear. Perhaps the actual means of the explosion and earthquake populations are separated more at low frequency than at high, even though there appears to be more overlap in the low-frequency bands.

Regional Magnitude Analysis

This study has been motivated by recent work of Patton (2001), who has argued that the regional mb magnitude scaled based on Lg, mb(Lg) originally developed by Nuttli (1973), is transportable between different tectonic regions. However, Baumgardt (2001) has shown many instances when Lg is blocked and thus mb(Lg) could not be used. Further, Baumgardt (2001) suggested that Lg and Sn amplitude blockages may trade off, that is, when Lg is blocked, Sn waves have stronger amplitudes. Discrimination studies in regions of Lg blockages, such as Novaya Zemlya, frequently use Sn in place of Lg in spectral amplitude ratios (e.g., Ryall *et al*, 1995). This suggest that perhaps Sn may be used in regions where Lg is blocked and visa versa.

In this study, we have investigated the relative stability of regional magnitudes based in Sn and Lg amplitudes and on the sum of the amplitudes. We have extended the method of Nuttli (1973) to develop mb(Sn) and mb (Sn+Lg) magnitudes. First, the Sn and Lg amplitudes are measured in various frequency bands. To correspond to standard practice for mb (Lg), the band from 0.5 to 1 Hz is measured but other higher frequencies have also been studied. We have applied this method on earthquakes in the Zagros Mountains of Western Iran recorded at surrounding regional stations that record both Sn and Lg. An example of this analysis is shown in Figure 9 for the station RAYN in Saudi Arabia. The plots show comparison of mb(Lg), mb(Sn), and mb(Sn+Lg) compared with the standard NEIS mb.

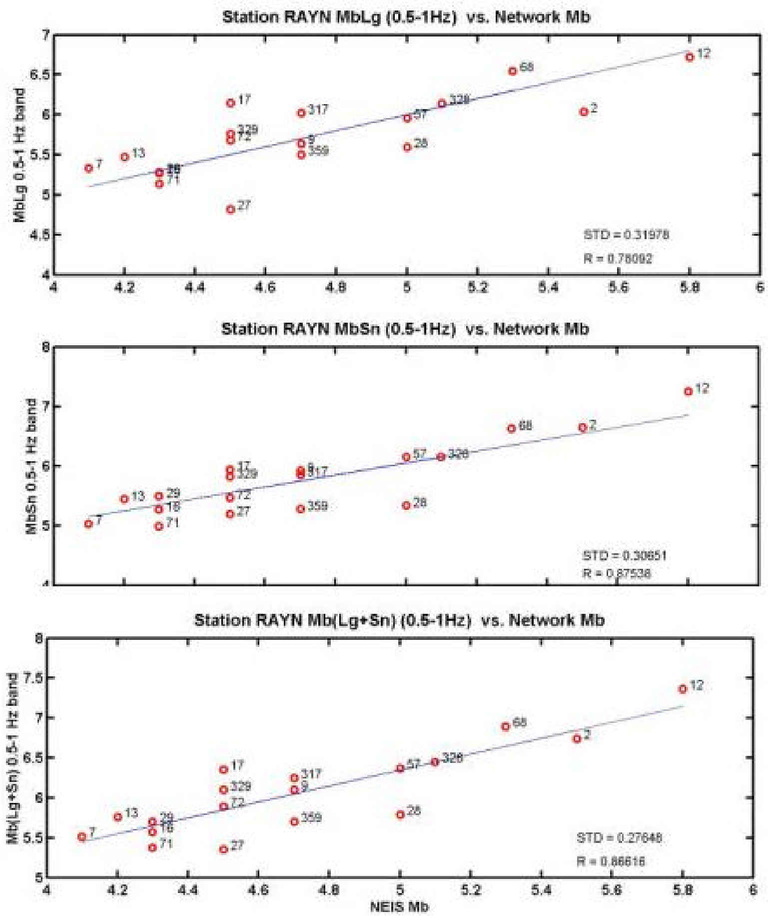


Figure 9: Plots of mb(Lg), mb(Sn), and mb(Lg+Sn) computed for earthquakes in the Zagros Mountains of western Iran recorded at the station RAYN against NEIS mb.

The line fitted to the data in Figure 9 has slope 1. These plots show that all three of these magnitudes scale well with NEIS mb. The scatter for the mb(Lg+Sn) was less than that of either mb(Lg) or mb(Sn). We are continuing to investigate these magnitudes in other regions where we have observed partial or complete Lg blockage.

CONCLUSIONS AND RECOMMENDATIONS

Multi-array analysis of site effects on regional phase spectral ratios for a close group of underwater explosions in the Baltic Sea show prominent biases apparently associated with each particular array. These biases are likely due to propagation and source radiation directionality effects. In addition, there are site-related variations within each array, which are likely to be associated with local geology. Such estimates of site effects may be used as corrections in discrimination work. We have investigated the problem of dimensionality reduction for multi-band spectral ratio data. We have found that a simple Fisher discriminant applied to multi-band regional spectral ratio data gives a good separation between earthquake and explosion populations comparable to or better than the best visible separation on scatter plots. We have initiated an investigation of a new regional magnitude scale, similar to the *mb(Lg)* magnitude, called *mb(Sn+Lg)* magnitude based on the sum total of the regional shear energy contained in the regional *Sn* and *Lg* phases. This study is motivated by previous observations of *Lg* blockage that might limit the use of *mb(Lg)* for discrimination. Because the *Sn* and *Lg* phases have sometimes been observed to trade off in amplitude in regions of blockage, the *mb(Sn+Lg)* scale may be more stable than a magnitude scale based on either phase individually.

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