ABSTRACT

When water in the cracks of low-porosity crystalline rock freezes, it affects the mechanical properties of the rock mass in two important ways. First, by immobilizing some cracks and bridging (and thus shortening) others, it reduces the average size and density of the initial cracks. O’Connell and Budiansky (1974) showed that a small reduction in this product of the average size of cracks times their volume density can produce a large increase in the elastic moduli of the rock. Second, ice in the cracks increases the fracture strength of the rock. Sammis and Biegel (2004) showed that this strengthening may be attributed to an increase in the effective coefficient of static friction for sliding on the cracks. By applying the micromechanical damage mechanics developed by Ashby and Sammis (1990), they were able to explain the strong temperature dependence of this strengthening in terms of the increase in the flow stress of the ice asperities with decreasing temperature below the freezing point. At the lowest temperatures, the strength of granite increased by a factor of 2 while the strength of limestone increased by a factor of 4. Since the Sammis and Biegel friction model is based on the flow stress of ice in the cracks, and this flow stress also depends on the strain-rate, these large increases in strength can be achieved at temperatures near the freezing point of water at high strain-rates in the explosion source. We used the “equivalent elastic medium model” for an explosive source developed by Johnson and Sammis (2001) to explore the effect of an increase in both elastic stiffness and compressive strength on the amplitude of far-field seismic radiation. An increase in the elastic moduli produces a decrease in the far field amplitudes. This is not surprising since it is well known that the apparent yield of an explosion decreases as the inverse of the shear wave velocity in the source rock. An increase in the coefficient of static friction, and consequent increase in compressive strength, also reduces the amplitude of the far-field seismic radiation. Our conclusion is that an explosion in frozen rock should have a smaller apparent yield than the same explosion in rock at temperatures above the freezing point. The effect should be larger in limestone than in granite.
OBJECTIVES

The objectives of this research program are to:

1. Build the micromechanical damage mechanics developed by Ashby and Sammis (1990) into source models for underground explosions.
2. Use this model to explore the influence of site effects such as rock type, ground water saturation, permafrost, and depth of burial on the seismic signature generated by the explosion.
3. Use this model to help interpret laboratory measurements and field experiments.
4. Explore the possibility that secondary radiation generated by the damage contributes to the regional seismic phases.

RESEARCH ACCOMPLISHED

Introduction

The Soviet test site at Novaya Zemlya at 73° North latitude lies well above the Arctic Circle. Rock at this site is probably below the freezing point of water to considerable depth. Permafrost thickness is greatest in non-glaciated polar regions like Siberia, where a record depth of 4900 feet to the permafrost base has been reported. Permafrost thickness in arctic Canada has been estimated to exceed 3000 feet and in arctic Alaska it may exceed 2000 feet. The question has therefore arisen as to how frozen water in cracks and pores might affect the seismic signature of an underground explosion.

Last year, Sammis and Biegel (2004) interpreted uniaxial compressive and tensile strength data on frozen rock from Mellor (1973) using the micromechanical damage model developed by Ashby and Sammis (1990). In this model, sliding on preexisting cracks in rock induces additional fracture damage and ultimate failure. The effect of ice in the damage model is to increase the effective coefficient of sliding friction on the preexisting cracks thus inhibiting the generation of new damage and strengthening the rock. In addition to strengthening, the damage model was also able to explain some of the subtler phenomenology in the frozen rock data such as the differences between porous and crystalline rock and the progressive strengthening observed to occur as the temperature was lowered from 0°C to −150°C.

We now use the “equivalent elastic medium source model” developed by Johnson and Sammis (2001) to investigate the effects of frozen water in the cracks of crystalline rock on the seismic radiation from an underground explosion.

The Mechanical Effects of Ice in the Cracks of Crystalline Rock

When water in the cracks of crystalline rock freezes, it affects the mechanical properties in two ways: 1) it increases the elastic moduli and 2) it increases the strength. Both effects can be understood in the context of the micromechanical damage mechanics developed by Ashby and Sammis (1990).

The Effect of Ice on the Elastic Properties

In the Ashby and Sammis (1990) damage mechanics, the size and density of fractures in crystalline rock are characterized by a single parameter called damage. The initial damage, before loading, is defined as

\[ D_0 = \frac{4}{\pi} (a \cos \chi)^3 N_f \]  

(1)

where \( a \) is the half-length of the cracks, \( N_f \) is the number of cracks per unit volume, and \( \chi \) is an angle describing their orientation (see Figure 1).
Figure 1. Crack geometry used in the Ashby and Sammis (1990) damage mechanics. Sliding on an inclined crack of length $2a$ nucleates tensile wing-cracks at its ends. Interaction between such cracks in an array of $Nv$ cracks per unit volume leads to failure and fragmentation.

The effect of ice in the inclined crack in Fig. 1 is to either totally immobilize it, thus reducing $Nv$ in eqn. (1), or, if saturation is not total, to form ice bridges thus reducing $a$ in eqn. (1). The net effect of both is to reduce the initial damage $Do$.

The elastic moduli of rock are extremely sensitive to $Do$. Figure 2, from O’Connell and Budiansky (1974) shows the effect of changing $Do$ on the P and S wave velocities in dry and water saturated rock. Note that the $x$-axis in Fig. 2 is $\varepsilon = N\left(\frac{a^3}{3}\right)$ where $N$ is the number of cracks per unit volume and $a$ is the half-length of the crack. Comparison with eqn. (1) shows that $\varepsilon$ can be written in terms of the initial damage as

$$\varepsilon = \frac{3}{4\pi} \left(\frac{1}{\cos \chi}\right)^3 D_o \approx 0.68D_o$$

The effect of freezing water in the cracks is to move to the left (toward lower damage) on the curves in Figure 2. This will produce an increase in elastic wave velocity. The effect is larger for S waves than for P waves in saturated rock.
Figure 2. The effect of fractures on the elastic wave velocities in wet and dry rock (from O’Connell and Budiansky, 1974). The crack density parameter $\varepsilon$ is closely related to the damage parameter in the Ashby and Sammis (1990) damage mechanics, differing only by a constant (see eqn. 2).

The Effect of Ice on the Strength

Sammis and Biegel (2004) used the Ashby and Sammis (1990) damage mechanics to explain measurements by Mellor (1973) of the uniaxial strength of saturated rock as a function of temperature from 20 to –197°C. They found that Mellor’s data for the compressive strength (replotted here as Fig. 3) could be fit by the Ashby-Sammis model if the coefficient of friction in the sliding cracks is temperature dependent. The required temperature dependence is shown in Figure 4. Sammis and Biegel (2004) then showed that a simple Bowden and Tabor (1950, 1964) asperity model consisting of a combination of rock and ice asperities could explain the temperature dependence of the coefficient of static friction in Figure 4 using the known temperature dependence of the creep strength of ice.
Figure 3. Strength of granite, limestone, and sandstone in uniaxial compression at low temperatures from Mellor (1973). Note that the strengths of saturated and air-dry granite samples are nearly the same at all temperatures indicating that the thin cracks in granite are saturated under air-dry conditions (From Sammis and Biegel, 2004).

Figure 4. Coefficient of friction on starter cracks required if the damage mechanics model is to explain the increase in compressive strength at low temperatures in Fig. 3 (From Sammis and Biegel, 2004).
In order to model the effect of ice on the damage generated by an explosion and on the seismic radiation, we require the principal stresses generated by the explosion as a function of distance and time. This is made difficult by the existence of the nonlinear processes between the cavity radius and the effective elastic radius, beyond which the assumptions of ordinary linear elasticity are valid. Sophisticated computer codes have been developed which include hydrodynamic effects, shock waves, and nonlinear equations of state (see, for example, Rodean, 1971; King et al., 1989; Glenn, 1993; Glenn and Goldstein, 1994, for discussion and further references). We use here an approximate method to calculate the stresses surrounding an explosion that is based on the equivalent elastic method developed by earthquake engineers to model the nonlinear behavior of soils that occurs during strong ground motion. The central idea is to make the material properties a function of the stress in the outward propagating pressure pulse and then to adjust these material properties in an iterative process until the appropriate values are present at all distances from the source. In effect, the nonlinear stress-strain behavior is approximated by a series of linear relationships that change with the level of stress. The present formulation, described by Johnson (1993), relates density and bulk elastic properties to the peak pressure and shear and anelastic properties to the maximum shear strain.

The details of this model are published in Johnson and Sammis (2001) and will not be repeated here. In that paper we modeled the 1 kt chemical explosion detonated in September 1993 as part of the Non-Proliferation Experiment (NPE) (see Denny, 1994). The results of this simulation and attendant damage calculations are summarized in the next section.

Simulation of the NPE Explosion

The first objective in modeling the 1993 NPE event was to check the ability of the equivalent elastic method to simulate the stress field around an explosion. As illustrated in Figure 5 the model gives reasonable values for the peak velocity as a function of distance out to about one kilometer beyond which the model predicts slightly higher velocities than those observed. The calculated waveforms were also comparable to those observed except for a bit of excessive reverberation.

Figure 5. Measured and calculated peak velocities for the NPE explosion. The open symbols are observations taken from Smith (1994) and Olsen and Peratt (1994), while the filled symbols were calculated with the equivalent elastic method (from Johnson and Sammis, 2001).
Having established that the equivalent elastic method gives a good approximation to the observed motions close to the NPE explosion, we now ask how these results would change if we increased the P and S wave velocities, the coefficient of friction, and the strength to simulate the mechanical properties if the cracks were filled with ice.

Figure 6 shows the effect of increasing the coefficient of friction on the radial distribution of fracture damage in the non-linear source region. For the higher coefficient of friction, damage is suppressed close in and enhanced further out. This is because the stresses do not fall off as fast with distance due to the smaller damage close in. Radial cracks are thus expected to extend out an additional 10 meters in the frozen rock (about an additional 20%).

![Figure 6. The effect on the radial distribution of damage when the coefficient of static friction is increased from $\mu = 0.6$ to $\mu = 1.0$ in the NPE simulation by Johnson and Sammis (2001).](image)

Figure 7 shows the effect of increasing the compressive strength on the radial distribution of damage. In the equivalent elastic source model the compressive strength is expressed as a reference strain defined as

$$\varepsilon_r = \frac{\tau_{\text{max}}}{\mu_o}$$  \hspace{1cm} (3)

In Figure 7, the reference strain is doubled (corresponding to ice in granite) and multiplied by 4 (corresponding to ice in limestone). The effect is very similar to that of increasing the coefficient of static friction – there is more damage and it extends an additional 10 meters.
Figure 7. The effect on the radial distribution of damage when the non-linear reference strain is increased by factors of 2 and 4 in the NPE simulation by Johnson and Sammis (2001). This is equivalent to increasing the compressive strength by a factor of 2 (simulating ice in granite) and 4 (simulating ice in limestone).

Figure 8 summarizes the effect of ice in the cracks on the amplitude of seismic radiation in the far-field. As argued above, ice is expected to increase the initial wave velocities and the strength of the source rock. These ice-induced changes all decrease both the amplitude and reduced velocity potential in the linear elastic regime.
CONCLUSIONS AND RECOMMENDATIONS

Inclusion of the mechanical effects of ice in the cracks of crystalline rock into the explosion source formulated by Johnson and Sammis (2001) has led to the following conclusions:

1. The increase in elastic wave velocities associated with ice in the cracks will decrease the seismic amplitudes in the far field resulting in an apparently smaller yield.

2. The increase in compressive strength caused by ice in the cracks will also decrease seismic amplitudes in the far field, also resulting in an apparently smaller yield.

3. At high loading rates, as in an underground explosion, the strength of granite is expected to increase by a factor of about 2, while that in limestone by a factor of about 4.
The following recommendations are based on the analysis in this paper and last year’s analysis of the frozen rock data (Sammis and Biegel, 2004).

1. The uniaxial data from Mellor (1973) should be supplemented with a full set of triaxial data in granite at low temperatures. Uniaxial data typically shows large experimental scatter, mostly because the strength is extremely sensitive to the initial flaw distribution in the absence of confining stress. A set of triaxial data would allow a more comprehensive assessment of the extent to which the damage model can represent the strength of rock at low temperatures.

2. The triaxial data set should be supplemented with measurements of the coefficient of friction as a function of temperature under saturated and air-dry conditions. These measurements can be made either on saw-cut samples as part of the triaxial set of experiments, or in Jim Dieterich’s double-shear apparatus at the United States Geological Survey laboratory in Menlo Park, California.

3. Both the triaxial measurements and friction measurements should be performed at different strain-rates to further test the hypothesis that the strengthening is associated with ice asperities.

4. Seismic velocities should be measured in frozen and thawed rock at the same field location as part of any pending field study of explosions in frozen rock.

5. The equivalent elastic source model of Johnson and Sammis (2001) should be improved to make the non-linearity depend explicitly on the damage. This non-linearity is now given by an analytic approximation that depends only on the peak stress (or, equivalently, on the reference strain).

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REFERENCES


