ABSTRACT

Recent applications of correlation methods to seismological problems illustrate the power of coherent signal processing applied to seismic waveforms. Examples of these applications include detection of low amplitude signals buried in ambient noise and cross-correlation of sets of waveforms to form event clusters and accurately measure delay times for event relocation and/or earth structure. These methods rely on the exploitation of the similarity of individual waveforms and have been successfully applied to large sets of empirical observations. However, in cases with little or no empirical event data, such as aseismic regions or exotic event types, correlation methods will likely fail due to the lack of previously observed similar waveforms.

This study seeks to use a suite of model-based signals computed for three-dimensional Earth models to form the basis for correlation detection using the subspace method. To demonstrate the method we modeled broadband regional seismograms for a moderate (M~5) earthquake near the China-North Korea border. Synthetic seismograms are computed with the Spectral Element Method for a suite of long-wavelength (2 degree) seismic velocity models inferred with the Markov Chain Monte-Carlo (MCMC) method. MCMC uses stochastic sampling to fit multiple data sets but unlike conventional inversions that estimate a single “optimal” model, MCMC results in a suite of models that sample the model space and incorporate uncertainty through variability of the models. The variability reflects our ignorance of Earth structure, due to limited resolution, data and model errors. The variability in 3D earth models produces variability in the seismic waveform response for the paths of interest. Model-based signals are combined using the subspace method where the set of synthetic signals are decomposed into an orthogonal basis by singular-value decomposition (SVD). The observed waveforms are represented with a linear combination of eigenvectors (signals) associated with the most significant eigenvalues of the SVD. We demonstrate the ability of model-based signals to represent intermediate period (down to 10 s) seismograms. Further work will require higher frequency synthetic seismograms and the inclusion of shorter wavelength velocity structure, whether inferred from various seismic data sets or generated by forward calculations through realizations of a stochastic model.
OBJECTIVES

Current methods for seismic monitoring of underground nuclear explosions are heavily dependent on the use of measured quantities from seismograms and deterministic models of Earth structure. For example, seismic events are typically located by minimizing the difference between the observed and predicted arrival times of major P- and S-wave phases from one-dimensional Earth models and source-specific station corrections. This procedure works well when the event is large enough to be observed at a number of stations with high signal-to-noise ratio and the travel time predictions are accurate. However, detection of signals above ambient noise can be difficult for small magnitude events, even at regional distance where there may be at most a few stations recording the event with adequate signal-to-noise (SNR) ratios. Furthermore, if the regional structure is poorly known, as frequently occurs, large bias may result in the location from errors in the travel time predictions. This study attempts to advance the basic methodology of seismic monitoring by using a suite of model-based signals and coherent signal-processing to detect signals in noisy data.

In the past large (> 10 kT) underground nuclear tests at known test sites were detected, located and identified using observations at teleseismic distances (> 2500 km) and the wealth of knowledge gained from previous tests, often from nearly the same location. As new nuclear states emerge it has become increasingly important to monitor broad observations at teleseismic distances (> 2500 km) and the wealth of knowledge gained from previous tests, often from nearly the same location. As new nuclear states emerge it has become increasingly important to monitor broad regions without previous explosion observations. For aseismic regions, monitoring is challenged by the lack of empirical (earthquake- and/or explosion-derived) constraints on travel times, surface wave dispersion and amplitudes for traditional location and identification processing. Significant progress has been made in the development of earth models to predict seismic observables, such as travel times, surface wave dispersion and full waveforms. Recent studies have demonstrated the efficacy of three-dimensional earth models to improve event location using travel times (e.g., Johnson and Vincent, 2002; Ritzwoller et al., 2003; Murphy et al., 2005; Morozov et al., 2005; Li et al., 2006; Flanagan et al., 2006). Similarly, tomographic models of surface group velocity dispersion improve surface wave magnitude estimates through the application of phased-matched filtering (Pasyanos et al., 1999; Stevens and Adams, 2000).

Another challenge impeding the lowering of detection thresholds to monitor small explosions is the difficulty detecting signals with low SNR. Conventional short-term/long-term average (STA/LTA) detectors require elevated signal power in a particular frequency band relative to the preceding noise and are inherently limited when signal power approaches the ambient background noise level. However, waveform correlation methods make use of more information, namely the repeated temporal structure of the waveform. The similarity is measured by the correlation function between a template waveform and a continuous data stream and a high value is obtained when the streaming waveform is the similar to the template (van Trees, 1968). This process, also called a matched filter, is an optimal detector in the presence of white Gaussian background noise and offers great sensitivity. However, it requires perfect knowledge of the signal. Formal statistical analysis provides a rigorous quantification of the probability of detection versus false alarm (i.e. the receiver-operator curve) for a given correlation value.

The application of matched filter detectors to seismology is challenged by the complexity introduced by the source (e.g., focal mechanism, depth, magnitude) and path effects on the resulting waveform(s). Geller and Mueller (1980) concluded that waveforms for adjacent events can be highly correlated when filtered in a sufficiently low-frequency band and the events have similar source parameters and are separated by no more than a quarter of the dominant wavelength. Others (Harris, 1991) argue that waveform correlation is significant out to event separations of one to two wavelengths, which is consistent with correlation lengths observed in the reciprocal problem of waveform coherence for a single event across an array aperture (Mykkeltveit et al., 1984). This phenomenon has been exploited to identify nearby mining explosions (e.g., Israelsson, 1990; Harris, 1991) and measure precise relative arrival times of adjacent nuclear explosions (e.g., Thurber et al., 2001; Waldhauser et al., 2004). Recently, correlation methods have been applied to large earthquake data sets covering active faults with dense local network data (Waldhauser and Ellsworth, 2000; Schaff et al., 2004) or the entire Chinese territory with regional network data (Schaff and Richards, 2004). Gibbons and Ringdal (2006) reported reduction in the detection thresholds for events in the European Arctic using single-channel and array-based matched filters and showed how array frequency-wavenumber analysis provides additional screening power.

Matched filters work well to detect repeating events that produce identical signals. However, seismic sources often generate related events with variations in source mechanism, time history and location that produce similar, but not identical waveforms. As a consequence, there is some uncertainty about the signals to be detected. If a collection of
template waveforms can be assembled that spans the range of signals likely to be produced by a particular source, a subspace detector (Harris, 1989, 1997; Harris et al., 2006) can be developed to detect signals from that source. Subspace detectors operate by correlating a linear combination of a set of orthonormal basis waveforms against an incoming data stream to detect signals that lie in the subspace spanned by the basis (hence the name “subspace detector”). The linear combination is chosen to maximize the correlation at each time step. The basis is developed from the singular value decomposition of a matrix constructed from the representative template waveforms. The basis consists of the left singular vectors corresponding to the most significant singular values in the decomposition.

The objective of this study is to test a new method to monitor events in aseismic regions with detectors employing model-based subspaces rather than empirical subspaces. Our goal is to detect events at regional distances at lower thresholds. Subspace detectors with empirically-derived basis functions have successfully detected clusters of mining explosions and small earthquakes from swarms (Harris et al., 2006) at lower monitoring thresholds than can be obtained with conventional power (STA/LTA) detectors. In such applications, path effects are calibrated with previously-observed waveforms and signals uncertainty is due principally to source effects, such as emplacement or near-source material properties, source-time function and/or focal mechanism. The present application focuses on uncertainty introduced by path effects, in situations where no previously-observed event waveforms are available. In this application, a basis is generated from a collection of representative template signals computed from a suite of fully three-dimensional (3D) earth models with the source parameters kept fixed. While model-based signals for 3D earth models are presently expensive to calculate even for intermediate frequency ranges, this project represents an attempt to advance monitoring science through the use of model-based signals for waveform matching. We expect the present study could be extended in the future to include higher-frequency synthetics and higher resolution earth models. Future efforts may also attempt to include source variability as well.

**RESEARCH ACCOMPLISHED**

The purpose of this project is to explore the feasibility of applying the subspace detection algorithm to model-based signals. The investigation focuses on regional seismograms from an earthquake in eastern Asia. Models are derived from the Markov chain Monte Carlo algorithm (Pasyanos et al., 2006). Signals are computed for 3D models using the spectral element method (Komatitsch and Vilotte, 1998; Komatitsch and Tromp, 1999). This effort was started in October 2004 as a Laboratory Directed Research and Development (LDRD) Project. The LDRD Program at LLNL supports internally competed projects for the development of new capabilities.

**Study Area**

For this proof of concept study we considered a moderate sized earthquake occurring near the China-North Korea border. The event occurred on January 11, 2002 and had a moment magnitude (Mw) of 4.89. Figure 1 (left) shows the study area, the event and station locations.

![Map of eastern Asia showing the earthquake (red circle, focal mechanism) we studied and regional distance stations (white triangles) that recorded the event.](image)

*Figure 1. (left) Map of eastern Asia showing the earthquake (red circle, focal mechanism) we studied and regional distance stations (white triangles) that recorded the event. (right) Vertical component waveforms (filtered 0.0125-0.1 Hz) from the event at the four regional stations. The station names and epicentral distances are indicated next to each waveform.*
Focal parameters were determined by William Walter (see Acknowledgements). Also shown in Figure 1 are the vertical component waveforms. Broadband waveforms were obtained from four regional stations operated by the Chinese Digital Seismic Network (CDSN, stations BJT, MDJ, SSE) and the Global Seismic Network (GSN, station INCN). These have good SNR for the surface waves at the frequencies of interest for this study. The instrument response was removed, the waveforms were integrated to displacement and the horizontal components were rotated to radial and transverse components.

**Stochastic Earth Models**

Models of the study region were derived from the Markov Chain Monte Carlo (MCMC) algorithm using surface wave group velocity dispersion, body-wave travel times, receiver functions and gravity (Pasyanos et al., 2006). The MCMC approach is probabilistic in nature, relying on a prior distribution of all model parameters. It computes the difference between all observations and model predictions and then accepts or rejects a model based on a likelihood measure. Codes to implement the MCMC algorithm were developed at LLNL. The approach is computationally intensive and the calculations were performed on the MCR cluster operated by Livermore Computing. The MCMC algorithm results in a set of posterior models that map the solution space. The approach has several advantages over conventional deterministic inversions: it allows for multiple data sets, it maps data and model uncertainties through the process and does not rely on normal statistics.

![Crustal thickness for four MCMC models of the Yellow Sea-Korean Peninsula region. The event (red circle), stations (green triangles) and paths (black lines) of our data set are also shown.](image)

We used a suite of MCMC models for the study region to compute the response of the event at the four regional stations (Figure 1). Models specify the seismic compressional and shear velocities ($v_p$ and $v_s$, respectively) and density ($\rho$) on a regular 2° by 2° grid in longitude and latitude. The models represent five crustal layers of variable thickness spanning the surface of the solid earth to the Moho and are underlain by a mantle half-space. We smoothed the models and embedded them in the CUB2.0 model (Shapiro and Ritzwoller, 2002) which is registered...
on the same grid. Figure 2 shows the crustal thickness estimates of four models. Note that the large-scale features of these models are similar, such as the ocean-continent differences. However, details of the model at any specific point or along any path reflect uncertainty in our estimates of the true structure. These 3D model differences will result in different template waveforms when the complete response is computed.

**Synthetic Seismograms**

We computed synthetic seismograms for this study using the spectral element method (SEM) code SPECFEM3D developed by Komatitsch and Tromp (1999) and Komatitsch et al. (2002). SPECFEM3D is a parallel code for computing the response of a fully 3D earth model to moment tensor loading. It is based on a finite element (FE) mesh of the spherical earth and uses high order polynomials along the edges of the elements sampled at unevenly spaced points. The points are chosen judiciously so that orthogonality relations result in simple and exact expressions for the motions at each time step, avoiding the inversion of large a linear system common to FE algorithms.

We modified the SPECFEM3D code to use the MCMC models embedded in the CUB2.0 model. Calculations were performed to compute the response at the four regional stations keeping the source parameters fixed. Each run was made on 144 CPU’s of the MCR cluster and ran for approximately nine hours to compute the response from 0-0.2 Hz. We have since made improvements to allow for higher frequency calculations with the same resources. Figure 3 shows the resulting model-based waveform templates compared to the observed data. Generally the model-based signals show very consistent body-waves, with only slight variations in the timing of arrivals. However, the surface waves, especially the later arriving short-period energy, displays differences likely related to dispersion and scattering. Note especially the data and synthetics for station BJT. This path crosses the sedimentary structure of the Bohai Basin and the data reveal a complex response.

![Figure 3. Three-component observed (black) and synthetic (colored) waveforms for the four regional stations. There are synthetic signals for nine different models. Z, R, T corresponds to vertical, radial and transverse components, respectively. Data and synthetics are filtered 0.01-0.1 Hz. Note that the time scale is different for station SSE.](image)

**Subspace Analysis**

The subspace analysis was performed for each station separately. Each three-component waveform (observed and synthetic) was multiplexed in channel sequential order, with \( M \) total points. The \( N \) model-based templates were treated as vectors and formed a matrix of length \( M \) and width \( N \). Following the subspace methodology the matrix of template waveforms was decomposed into its singular vectors and sorted by most-significant singular value. For this study we computed model-based signals for nine models (\( N=9 \)). The length of signal time windows for each station varied such that the entire surface wave and coda were captured (400-600 s, similar to Figure 3).
The dimension of the subspace is a significant design parameter subject to a tradeoff between the probability of detection and the probability of false alarm. The larger the number of significant singular vectors used to represent the observed waveform the better will be the fit to potential signals and the higher the probability of detection. However, using a larger subspace introduces a possibility for misleading high noise correlations and consequent false alarms. Determination of the subspace dimension is made, in part, by considering the “energy capture”. This is the fractional energy of each template waveform represented by the singular vector basis of dimension 1 to N. When all singular vectors are used, each of the N design templates will be perfectly represented. The energy capture is computed for each of the N original template waveforms and plotted as a function of the subspace dimension. Figure 4 shows the energy capture for the waveform templates computed for station BJT and using three different frequency bands.

Figure 4. The energy capture for templates computed for station BJT in three different frequency bands 0.0125-0.1 Hz (left), 0.0125-0.067 Hz (center) and 0.0125-0.05 Hz (right).

The energy capture shown in Figure 4 indicates that for the low frequency case a subspace dimension of only one or two is needed to represent 95% of the power in each original template waveform. However, the subspace dimension needed to represent the basis signals increases as the bandwidth increases. For the bands 0.0125-0.067 Hz and 0.0125-0.1 Hz we use subspace dimensions of 3 and 5, respectively. The subspace dimension must increase as additional complexity is added in the broader bandwidth waveforms. The key for the subspace methodology to work effectively is for the subspace dimension to increase slowly as the bandwidth increases.

Figure 5. (left) Linear correlation between the observed waveforms and the nine individual MCMC model-based signals, the tenth model is the subspace result, using a subspace dimension of three. The linear correlation is plotted for the three-component (black circles) and individual components (colored circles). (right) The resulting fit between the observed (blue) and subspace detector (red) waveforms.
The subspace representation is then applied to the observed waveforms. To evaluate the performance of the subspace detector we compute the linear correlation between the three-component synthetics for the nine individual MCMC models and for the subspace. Figure 5 shows the linear correlations for the waveforms observed at BJT using the frequency band 0.0125-0.067 Hz and a subspace dimension of 3. The linear correlations between the observed and individual model-based signals (model indices 1-9) vary between about 0.0 and 0.7. The subspace results in an improved waveform fit over any individual model. While values greater than about 0.5 indicate fairly good waveform similarity the subspace result (about 0.7) should be compared with the average correlation for the individual models (about 0.4) because there is no reason to choose any single model from the MCMC model set. Notice that the resulting waveform for the subspace has the proper surface wave dispersion. For these frequencies the individual model-based signals do not reproduce the late-arriving scattered surface wave energy that was not present in the basis waveforms (Figure 3).

The performance of the subspace representation, as measured by the increase in the linear correlation between the observed and the individual model-based and subspace signals, improves as the bandwidth is increased. Figure 6 shows the three-component linear correlations between the model-based and observed signals for three frequency bands. Increasing the bandwidth introduces additional complexity in the observed and model-based signals and the improvement in linear correlation for the subspace is most dramatic for the broadest band comparisons (80-10 seconds).

![Figure 6. Linear correlations between model-based signals (individual, 1-9, and subspace) and the observed three-component waveforms at three stations INCN (left), BJT (center) and MDJ (right). For each station the analysis was performed in three different frequency bands, indicated by the different colors: 0.0125-0.1, 0.0125-0.067 and 0.0125-0.05 Hz corresponding to period bands 80-10 (red), 80-15 (green) and 80-20 (blue) s.](image)

Figure 7. (left) Linear correlation between the observed and individual model-based and subspace signals for station BJT (80-15 s) with the addition of noise. The observed waveform has a SNR to greater than 10:1. The addition of noise degrades the correlations, however, the subspace still improves the representation of the observed signal over the individual model-base signals. (right) The observed (blue) waveforms with noise added to overwhelm the signal and the subspace signal (red) show that the correlation is still possible.

![Figure 7. Linear correlation between the observed and individual model-based and subspace signals for station BJT (80-15 s) with the addition of noise. The observed waveform has a SNR to greater than 10:1. The addition of noise degrades the correlations, however, the subspace still improves the representation of the observed signal over the individual model-base signals. (right) The observed (blue) waveforms with noise added to overwhelm the signal and the subspace signal (red) show that the correlation is still possible.](image)
The most dramatic increases in linear correlation between the observed and model-based signals is seen for stations INCN and MDJ. The broadband (80-10 s) comparisons are quite poor for the individual model-based signals, but these all increase dramatically when combined with the subspace methodology.

Finally, we tested the effect of adding noise to the observed signals in order to lower the SNR and found that we can still obtain improvements in the linear correlation with the subspace representation over the individual model-based signals (Figure 7).

CONCLUSIONS AND RECOMMENDATIONS

While limited in scope, this study demonstrates the feasibility of using model-based signals and the subspace representation of signals to improve monitoring of aseismic regions and lower detection thresholds. Correlation methods promise to lower detection thresholds below the 2:1 SNR limitation of conventional STA/LTA methods. The subspace method combines model-based signals to reproduce the temporal structure of observed waveforms as is required for coherent processing of incoming data streams. Although we have not yet tested this strategy on a continuous data stream we intend to use codes developed for empirical template waveforms to test signal detection capabilities (e.g. Harris et al., 2006).

The synthetic seismograms used in this study incorporated fully three-dimensional variability of earth structure, however the long-wavelength representations can be improved to be more realistic. We are currently completing a suite of higher resolution MCMC models of the Yellow Sea-Korean Peninsula region. These will be 1° by 1° rather than the 2° by 2° models we used above. The finer structure will result in more complex synthetic seismograms. Geophysical models will improve in their resolution and accuracy as more data become available and inference methods evolve. The MCMC algorithm is ideal for improving knowledge of earth structure because it can incorporate different data types and map uncertainties through to predictions of observables, such as complete waveforms. When resolution of earth structure is limited to a certain scale, stochastic (random) heterogeneity could be added to 3D earth models to mimic the effects of complex unresolved small-scale structure.

We are also attempting to increase the frequency content of the model-based signals. A new elastic finite difference code is being developed at LLNL (Nilsson et al., 2006) and we are working to use this code for future synthetic seismogram calculations. The code runs in parallel and scales effectively from 1 to 1024 CPU’s. While the computation of synthetics seismograms used in this study is expensive, we envision that these calculations will become more accessible in the future.

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